AST 308
Homework Set 6
Due Tuesday Nov 22

1. Use the virial theorem to estimate the mass of the galaxy cluster Abel 2136. It has a mean radial velocity V = 62,400 km s^{-1}, internal velocity dispersion σ_r = 700 km s^{-1}, and angular radius θ = 11 arcmin. Use the HST key project value for H_o. You might want to use the fact that 1 radian = 3438 arcmin.

2. Is the Coma Cluster likely to be virialized? To answer that, first answer: What is the crossing time for a galaxy in the Coma cluster, in units of t_o? Use the measured parameters given in [CO Example 27.3.1 on pg. 1066]. Assume that a typical galaxy moves at a constant speed equal to the cluster's radial velocity dispersion. Why is t_o the interesting timescale to compare to the crossing time?

Being "virialized" means that the galaxies and other mass in the cluster meet the requirements to be described by the virial theorem. These requirements are given in rather formal terms in [CO 2.4]. What does the crossing time have to do with deciding whether these requirements have been met? Is the cluster even gravitationally bound?

3. The 70 million K X-ray emitting gas that fills the 1.5 Mpc radius of the Virgo cluster has an X-ray luminosity $L_x = 1.5\times10^{36}$ W.

(a) Use [CO eq. 27.20] to find the electron number density and the mass of the gas. Note that some early printings of [CO] had errors in equations (27.18) and (27.19). The powers of 10 in the coefficients should be 10^{-52} and 10^{-40}, respectively.

(b) The combined visual-passband luminosity from all of the stars in all of the 2000 galaxies in the Virgo cluster is $L_V = 1.2\times10^{12}\ L_\odot$. Estimate the luminous mass, using M/L $=3M_\odot/L_\odot$ as the ratio for a stellar population. What is the ratio of the mass in stars to the mass in X-ray emitting gas?

(c) If the gas has no source of heating and is losing energy through thermal bremsstrahlung (free-free emission), how long will it take for the gas to radiate away all of its energy? Use [CO eq. 10.17] for the average kinetic energy per particle (protons and electrons), and assume that the X-ray luminosity remains constant throughout the cooling process. Use $t_{cool} = E / (dE/dt)$. How does this cooling time compare to t_0 (use the WMAP age of the universe)?

4. Derive [CO eqns. 28.21, 28.22 and 28.23], for gravitational lensing. Use the geometry shown in Fig. 28.35, and the deflection given in Eq. 28.20.

Hint 1. Start by showing that, for the small angles involved, $$\frac{\sin(\theta-\beta)}{d_S - d_L} = \frac{\sin\phi}{d_S}$$

Hint 2. Don't forget the quadratic formula, and also the law of sines: $$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

5. What is the mass of the lensing galaxy that produces the Einstein ring MG1654+1346? The ring is measured to have a diameter of 2.1 arcsec, the redshift of the background source is z = 1.74, and the redshift of the lensing galaxy is z = 0.25.