PHYSICS 215 - Thermodynamics and Modern Physics

Fall 2011

Homework Assignment

Set 10 (due Tuesday, November 8)

The Hydrogen Atom (Thornton and Rex, Chapter 7)

7.3, 7.10, 7.20, 7.23, 7.24, 7.29, 7.30
Solve the azimuthal equation

\[ \frac{d^2 g}{d\phi^2} = -k^2 g \]

and show that \( k = 0 \) or a \( +/- \) integer.

Try \( g = Ae^{-ik\phi} \)

\[ \frac{dg}{d\phi} = -ikAe^{-ik\phi} \]

\[ \frac{d^2 g}{d\phi^2} = -k^2 Ae^{-ik\phi} = -k^2 g \]

\[ \therefore \text{this is a solution.} \]

We also require that

\[ g(\phi) = g(\phi + 2\pi) \]

\[ Ae^{-ik\phi} = Ae^{-ik(\phi + 2\pi)} \]

\[ \therefore e^{-ik2\pi} = 1 \]

\[ \therefore k = 0 \text{ or a } +/\text{- integer.} \]
For a 3p state give the possible values of \( n, l, m_l, \ell, L_x, \) and \( L_y. \)

3p state means \( n = 3 \)

\[ l = 1 \]

Possible values of \( m_l = -1, 0, +1 \)

\[ L = \sqrt{l(l+1)} \hbar = \sqrt{2} \hbar \]

\[ L_z = m_l \hbar = -1\hbar, 0, +1\hbar \]

\[ L^2 = L_x^2 + L_y^2 + L_z^2 \]

\[ L_x^2 + L_y^2 = L^2 - L_z^2 \]

\[ = 2\hbar^2 - 1\hbar^2 = \hbar^2 \]

\[ \text{or} \quad = 2\hbar^2 - 0 = 2\hbar^2 \]

This is the only constraint of \( L_x, L_y. \)
7.20 For hydrogen atoms in a 4d state, what is the maximum difference in potential energy when in \( B = 2.5 \, T \)?

4d state means \( n = 4 \)

\[ l = 2 \]

\[ m_l = -2, -1, 0, +1, +2 \]

\[ V_B = m_l \mu_B B \]

\[ \therefore \Delta V_B = 4 \times 9.27 \times 10^{-24} \frac{J}{T} \times 2.5 \, T \]

\[ = 9.27 \times 10^{-23} \, J \]

\[ = 5.79 \times 10^{-4} \, eV \]
For a hydrogen atom in the 6f state, what is the minimum angle between the \( \vec{L} \) vector and the \( \hat{z} \) axis?

\[ 6f \implies n = 6, \quad l = 3 \]

\[ \vec{L} = \sqrt{l(l+1)} \hbar = \sqrt{3.4} \hbar = \sqrt{12} \hbar \]

Possible values of \( m_l \) are

\[
\begin{align*}
3\hbar & \\
2\hbar & \\
1\hbar & \\
0 & \\
-1\hbar & \\
-2\hbar & \\
-3\hbar &
\end{align*}
\]

The minimum angle \( \theta \) is given by

\[ \cos \theta = \frac{3\hbar}{\sqrt{12}\hbar} = 0.866 \]

\[ \implies \theta = 30^\circ \]
7.24 A level ($\lambda = 656.5$ nm) splits into 3 lines with $\Delta \lambda = 0.04$ nm between adjacent lines. What is $B$?

\[ E = \frac{hc}{\lambda} \]

\[ \therefore \Delta E = -\frac{hc}{\lambda^2} \Delta \lambda \]

\[ \Delta E = m_e \mu_3 B = \mu_0 B \]

\[ \therefore \mu_0 B = -\frac{hc}{\lambda^2} \Delta \lambda \]

\[ \therefore B = -\frac{hc}{\mu_0} \frac{(-\Delta \lambda)}{\lambda^2} \]

\[ = \frac{1242 \text{ eV nm}}{5.79 \times 10^{-5} \text{ eV}} \frac{0.04 \text{ nm}}{(656.5 \text{ nm})^2} \]

\[ \therefore B = 1.99 \text{ T} \]
Write all possible quantum numbers for the $5f$ state of hydrogen. What is the total degeneracy?

\[ n = 5 \quad l = 3 \]

\[
\begin{align*}
    m_l &= -3 & m_s &= -\frac{1}{2} & m_s &= +\frac{1}{2} & 2 \\
    m_l &= -2 & " & " & " & 2 \\
    m_l &= -1 & " & " & " & 2 \\
    m_l &= 0 & " & " & " & 2 \\
    m_l &= +1 & " & " & " & 2 \\
    m_l &= +2 & " & " & " & 2 \\
    m_l &= +3 & " & " & " & 2 \\
\end{align*}
\]

\[ 14 \]
Write all possible quantum numbers for the 6d state of hydrogen. What is the total degeneracy?

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<th>ml</th>
<th>m_l</th>
<th>m_s = -1/2</th>
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n = 6, l = 2

Total degeneracy = 10