PHYSICS 215 - Thermodynamics and Modern Physics
Fall 2011

Homework Assignment

Set 7 (due Tuesday, October 18)

Atomic Structure (Thornton and Rex, Chapter 4)
5, 8, 11, 22, 23, 24, 30
46 What is the ratio of scattering $>1^\circ$ compared with $>2^\circ$?

Prob. of scattering thru' $\theta$ or greater is: 

\[ f = \pi nt \left( \frac{Z_1 Z_2 e^2}{8 \pi \varepsilon_0 K} \right)^2 \cot \frac{2\theta}{2} \]

\[ \therefore \frac{f_1}{f_2} = \frac{\cot^2 \frac{1}{2}}{\cot^2 \frac{3}{2}} = 4 \]

What is the ratio of the number scatt. at $\theta = 1^\circ$ compared with $\theta = 2^\circ$?

\[ N(\theta) = N_i nt \left( \frac{e^2}{4 \pi \varepsilon_0} \right)^2 \frac{Z_1^2 Z_2^2}{r^2 k^2} \sin^4 \frac{\theta}{2} \]

\[ \Rightarrow \frac{N(\theta=1^\circ)}{N(\theta=2^\circ)} = \frac{\sin^4 \frac{2}{2}}{\sin^4 \frac{4}{2}} = \left( \frac{\sin 1}{\sin \frac{1}{2}} \right)^4 = 2^4 = 16 \]
What is the ratio of scattering off Aluminum (Z = 13) compared with Gold (Z = 79)?

\[ N(\theta) = \frac{N_i n t}{16} \left( \frac{e^2}{4 \pi \hbar c} \right)^2 \frac{Z_1^2 Z_2^2}{r^2 K^2 \sin^4 \frac{\theta}{2}} \]

so, with all else being equal,

\[ \frac{N(Al)}{N(Au)} = \frac{13^2}{79^2} = 0.027 \]
4.5 Calculate the impact parameter for 7.7 MeV α-ray scattering from gold at a) $\theta = 1^\circ$ b) $\theta = 90^\circ$

\[ b = \frac{Z_1 Z_2 e^2}{8\pi \varepsilon_0 K} \cot \frac{\theta}{2} \]

a) \[ b = \frac{8.99 \times 10^9 \times 2.79 \times (1.6 \times 10^{-19})^2}{2} \left( \frac{\cot \frac{1}{2}}{7.7 \times 10^6 \times (1.6 \times 10^{-19})} \right) \]

\[ = 1.48 \times 10^{-14} \cot \frac{1}{2} \]

\[ = 1.69 \times 10^{-12} \text{ m} \]

b) \[ b = 1.48 \times 10^{-14} \cot 45 \]

\[ = 1.48 \times 10^{-14} \text{ m} \]
What fraction of 5 MeV α-rays scatter through angles ≥ 6° from gold (Z = 79, ρ = 19.3 g/cm³), t = 10⁻⁸ m?

\[ n, \ # \ of \ nuclei \ per \ m^3, = N_A \cdot \frac{1}{19.3 \cdot 10^6} \]

\[ \text{Molar mass of gold} \]

\[ \frac{cm^3}{m^3} \]

\[ \therefore n = \frac{6.02 \times 10^{23} \cdot 19.3 \cdot 10^6}{197} \]

\[ \therefore n = 5.90 \times 10^{28} \ m^{-3} \]

\[ f, \ \text{prob. of scattering through } \Theta \ \text{or more} \]

\[ = \pi n t \left( \frac{Z_1 Z_2 e^2}{8 \pi \varepsilon_0 K} \right)^2 \cot^2 \frac{\Theta}{2} \]

\[ = \pi \cdot (5.90 \times 10^{28}) \cdot (10^{-8}) \times \]

\[ \left( \frac{8.99 \times 10^9}{2} \cdot \frac{2.79 \cdot (1.6 \times 10^{-19})^2}{(5 \times 10^6) \cdot (1.6 \times 10^{-19})} \right)^2 \cot^2 \frac{6}{2} \]

\[ = 3.48 \times 10^{-4} \]
4.11 What kinetic energy is needed for a projectile to just touch the surface of a nucleus?

\[ KE = K \]

\[ \text{Radius} + R \]

Initial \( PE = 0 \)

\[ PE = \frac{1}{4\pi\varepsilon_0} \frac{Z_1 Z_2 e^2}{r + R} \]

Conservation of energy \( \Rightarrow \)

\[ K = \frac{1}{4\pi\varepsilon_0} \frac{Z_1 Z_2 e^2}{r + R} \]

For alpha, Al \( \Rightarrow \)

\[ \frac{Z_1}{r} = \frac{2}{2.6 \text{ fm}} \quad \frac{Z_2}{R} = \frac{13}{3.6 \text{ fm}} \]

\[ K = \frac{8.99 \times 10^9}{(2.6 + 3.6) \times 10^{-15}} \cdot 2.13 \cdot (1.6 \times 10^{-19})^2 \]

\[ = 9.65 \times 10^{-13} \text{ J} = 6.03 \text{ MeV} \]
For α, Au
\[ Z_1 = 2 \quad r = 2.6 \text{ fm} \quad Z_2 = 79 \quad R = 7.0 \text{ fm} \]
\[ \Rightarrow K = 23.7 \text{ MeV} \]

For p, Al
\[ Z_1 = 1 \quad r = 1.3 \text{ fm} \quad Z_2 = 13 \quad R = 3.6 \text{ fm} \]
\[ \Rightarrow K = 3.82 \text{ MeV} \]

For p, Au
\[ Z_1 = 1 \quad r = 1.3 \text{ fm} \quad Z_2 = 79 \quad R = 7.0 \text{ fm} \]
\[ \Rightarrow K = 13.7 \text{ MeV} \]
What is $\beta$ for electrons in the first 3 orbits of a hydrogen atom?

I have shown that $v^2 = \frac{ke^2}{mr}$

and $r = n^2 a_0$ \[a_0 = 5.3 \times 10^{-11} \text{ m}\]

$\Rightarrow v^2 = \frac{ke^2}{n^2 ma_0} \Rightarrow \frac{v}{c} = \frac{e}{nc} \sqrt{\frac{k}{ma_0}}$

\[\therefore \beta = \frac{v}{c} = \frac{1}{n} \frac{1.6 \times 10^{-19}}{3 \times 10^8} \sqrt{8.99 \times \frac{9.1 \times 10^{-31} \times 5.3 \times 10^{-11}}{n}}\]

$= \frac{7.28 \times 10^{-3}}{n}$

\[\therefore \beta_1 = 7.28 \times 10^{-3}\]
\[\beta_2 = 3.64 \times 10^{-3}\]
\[\beta_3 = 2.43 \times 10^{-3}\]

$\sqrt{7.28 \times 10^{-3}} = \frac{1}{137}$, is a dimensionless number called the fine structure constant, which appears often in atomic physics.
A hydrogen atom absorbs a photon of $\lambda = 434$ nm. What were the initial and final states of the atom?

The absorbed photon has an energy of

$$E = \frac{1240}{434} = 2.85 \text{ eV}$$

so the atom goes to a higher energy level.

\[ n=2 \rightarrow n=5 \]

\[ n=3 \rightarrow n=4 \]

\[ n=4 \rightarrow n=5 \]
A hydrogen atom emits a photon of $\lambda = 95$ nm. What are the initial and final states?

$\lambda = 95$ nm $\iff E = \frac{1240}{95} = 13.05$ eV

Here the energy is lost so the atom goes to a lower state. Obviously the final state is $n = 1$ ($E = -13.6$ eV).

The initial state must be $13.05 - 13.6 = -0.547$ which is $n = -\frac{13.6}{25}$

so initial state is $n = 5$

$\Rightarrow n = 5 \rightarrow n = 1$
EM radiation of $\lambda = 100$ nm is incident on hydrogen at $n = 1$. What is the highest state that can be excited?

$\lambda = 100$ nm $\iff E = \frac{1240}{100} = 12.4$ eV

$\therefore$ Energy of highest state possible is $-13.6 + 12.4 = -1.2$ eV

This is higher than $n = 2$ ($-3.4$ eV) and $n = 3$ ($-1.51$ eV) but lower than $n = 4$ ($-0.85$ eV).

$\Rightarrow$ Highest possible state is $[n = 3]$