Nuclear Properties

Thornton and Rex, Ch. 12

Irene and Marie Curie
(3 Nobel Prizes between them!)
A pre-history

1896  Radioactivity discovered - Becquerel
  - $\alpha$ rays + (Helium)
  - $\beta$ rays - (electrons)
  - $\gamma$ rays 0 (EM waves)

1902  Transmutation observed
  - Rutherford and Soddy

1909  $\alpha$ rays are Helium nuclei
  - Rutherford and Royds

1912  Nucleus is shown to have very small radius (a few x $10^{-15}$ meters) at the center of the atom (a few x $10^{-11}$ meters).
  - Rutherford (and Geiger and Marsden)
Artificial Transmutation

1919 - Rutherford succeeded in producing the first nuclear reaction in the laboratory.

By colliding Nitrogen with rays, Rutherford succeeded in creating Hydrogen and Oxygen.
We can write this reaction as:

\[ ^{14}\text{N}_7 + 4^4\alpha_2 \rightarrow ^{1}p_1 + ^{17}\text{O}_8 \]

Nuclear Mass
Number \( A = Z + N \)

Nuclear charge
(atOMIC number \( Z \))

\[ ^{14}\text{N}_7 \]

\( A \approx \text{atomic weight (not an integer)} \)
we will discuss the integer \( N \) in a bit

Both integers, \( A \) and \( Z \), are “conserved”.

A shorthand for this reaction:

\[ ^{14}\text{N}_7 (\alpha, p) ^{17}\text{O}_8 \]
The Structure of the Nucleus

An early theory:

nucleus composed of protons and electrons:
Protons to get mass right plus enough electrons to get charge right.

Example: Beryllium has atomic weight 9 and atomic number 4.

- 9 protons and 5 “nuclear” electrons. (in addition to the 4 electrons orbiting around the nucleus).
Problems with this model:

- Uncertainty principle => nucleus would have too much energy
- Predicts incorrect spin for nuclei

Rutherford suggested a new elementary particle in the nucleus, with roughly the same mass as the proton, and zero charge.

I.e., the “neutron”
The Discovery of the Neutron

1930 - Bothe and Becker discover penetrating new type of radiation while bombarding Beryllium with $\alpha$ particles.

Curie and Joliot showed that when this new radiation struck a paraffin target (which contains Hydrogen nuclei) it knocked out high energy protons.
James Chadwick - Using measurements of energy and momentum, showed that radiation must be a new (uncharged) particle of about the same mass as a proton.

Chadwick named it the neutron (symbol n).

The reaction for creating it was

\[ ^9\text{Be}_4 + ^4\text{He} \rightarrow ^1\text{n}_0 + ^{12}\text{C}_6 \]

or

\[ ^9\text{Be}_4 (\text{He},\text{n})^ {12}\text{C}_6 \]

The generic name for a neutron or proton is a nucleon.
The Structure of the Nucleus

The modern picture of the nucleus: protons and neutrons, but no electrons.

The charge is determined by the number of protons (same as number of electrons). The mass is the sum of the masses of the protons and neutrons.

Example, Beryllium (atomic weight 9, atomic number 4)
- 4 protons and 5 neutrons in the nucleus, 4 electrons orbiting around it.
# Properties of Elementary Particles (circa 1932)

<table>
<thead>
<tr>
<th>particle</th>
<th>symbol</th>
<th>charge (C)</th>
<th>mass (kg)</th>
</tr>
</thead>
<tbody>
<tr>
<td>electron</td>
<td>$^0e_1$</td>
<td>$-1.6 \times 10^{-19}$</td>
<td>$9.1 \times 10^{-31}$</td>
</tr>
<tr>
<td>proton</td>
<td>$^1p_1$</td>
<td>$+1.6 \times 10^{-19}$</td>
<td>$1.7 \times 10^{-27}$</td>
</tr>
<tr>
<td>neutron</td>
<td>$^1n_0$</td>
<td>0</td>
<td>$1.7 \times 10^{-27}$</td>
</tr>
</tbody>
</table>

We write

\[
A = \text{atomic weight} = N_{\text{protons}} + N_{\text{neutrons}}
\]

\[
Z = \text{atomic number} = N_{\text{protons}} = N_{\text{electrons}}
\]

\[
N = \text{neutron number} = N_{\text{neutrons}} = A - Z
\]
Some examples:

<table>
<thead>
<tr>
<th></th>
<th>A</th>
<th>Z</th>
<th>N</th>
</tr>
</thead>
<tbody>
<tr>
<td>$^1$H$_1$</td>
<td>1</td>
<td>1</td>
<td>0</td>
</tr>
<tr>
<td>$^4$He$_2$</td>
<td>4</td>
<td>2</td>
<td>2</td>
</tr>
<tr>
<td>$^{12}$C$_6$</td>
<td>12</td>
<td>6</td>
<td>6</td>
</tr>
<tr>
<td>$^{56}$Fe$_{26}$</td>
<td>56</td>
<td>26</td>
<td>30</td>
</tr>
<tr>
<td>$^{238}$U$_{92}$</td>
<td>238</td>
<td>92</td>
<td>146</td>
</tr>
</tbody>
</table>
Isotopes

The “name” of the element (Hydrogen, Iron, Lead, etc.) is determined by the number of electrons = number of protons = \( Z \). It determines the chemical properties of the element.

Some elements exist in forms with different numbers of neutrons. These are called isotopes of the element.

Example: 3 isotopes of Hydrogen:

<table>
<thead>
<tr>
<th></th>
<th>A</th>
<th>Z</th>
<th>N</th>
</tr>
</thead>
<tbody>
<tr>
<td>Standard</td>
<td>(^1\text{H}_1)</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>Deuterium</td>
<td>(^2\text{H}_1)</td>
<td>2</td>
<td>1</td>
</tr>
<tr>
<td>Tritium</td>
<td>(^3\text{H}_1)</td>
<td>3</td>
<td>1</td>
</tr>
</tbody>
</table>
Stable and Unstable Isotopes

Not all isotopes are stable.

“Atomic Weight” averages over observable isotopes, so not an integer.
THE STABILITY PLOT

IN A COMMON WAY OF VIEWING NUCLEI, INVENTED BY EMILIO SEGRE, THE NUMBER OF NEUTRONS IS PLOTTED AS A FUNCTION OF THE NUMBER OF PROTONS.

It can be seen that the lighter nuclei contain almost equal numbers of protons and neutrons, while heavier nuclei show a distinct excess of neutrons.

This has been attributed to the fact that, with increasing number of protons, the Coulomb's force of repulsion requires more and more neutrons (attractive) to overcome it.
Sizes of Nuclei

Nuclei can usually be approximated by spheres of radius $R$, where

$$R = r_0 A^{1/3}$$

and

$$r_0 = 1.2 \times 10^{-15} \text{ m}$$

(Note: Volume $R^3 \text{ A}$)

The unit $10^{-15} \text{ m} = 1 \text{ femtometer} \ (\text{fm})$, often called 1 fermi.
Shapes of Nuclei

One can describe the shape of a nucleus by its “charge distribution”.

1950’s - measured by Robert Hofstadter at Stanford using 500 MeV electrons ($\lambda_{\text{de Broglie}} \sim 2.5$ fm).

The distribution can be parametrized by

$$\rho(r) = \frac{\rho_0}{1 + e^{(r-R)/a}}$$

$t \sim 2.2$, $a \sim 0.5$ fm for this nucleus
The Nuclear Force

The nuclear (or “strong”) force is what binds the protons and neutrons together into nuclei.

Properties:

• It is attractive.

• Within the nucleus, it is about 100x stronger than the Electromagnetic force (and about $10^{38}$x stronger than gravity).

• It is very short range. Outside of the nucleus, the EM force dominates, and the nucleus behaves just like a positive charge.
• It is charge-independent; that is, it acts the same on protons and neutrons.

• It is spin-dependent.
The nuclear force binds nucleons (protons and neutrons) together. Work must be done to separate them.

Conversely, energy is released when nucleons join together to form a stable nucleus.

The energy difference between a nucleus and its separate constituent nucleons is called the **Binding Energy**.

\[
\text{Binding Energy} = (Z m_p + N m_n - A M_Z) c^2
\]
Example: Helium

\[ M(^1\text{H}) = 1.007825 \text{ u} \quad (\ast = m_p + m_e) \]

\[ m_n = 1.008665 \text{ u} \]

\[ M(^4\text{He}_2) = 4.002603 \text{ u} \quad (\ast = m_\alpha + 2m_e) \]

Atomic Mass Unit: \( u = 931.5 \text{ MeV}/c^2 \)

B. E. = \( (2 \cdot M(\text{^1H}) + 2 \cdot m_n - M(^4\text{He}_2))c^2 \)

= 0.030377 \text{ u } c^2

= 28.30 \text{ MeV} \\

*These are actually the atomic masses of Hydrogen and Helium, which include the electron masses. Note that the electron masses cancel out of the formula.*
Radioactive Decay

In any nuclear reaction, the following quantities are conserved:

1. Nucleon Number, \( A \)
2. Charge
3. Energy
4. Momentum

\[ {}^{227}\text{Th}_{90} \rightarrow {}^{223}\text{Ra}_{88} + {}^4\alpha_2 \]

The masses of the two nuclei on the RHS add up to about \( 1.1 \times 10^{-29} \) kg less than the mass on the LHS. The difference is made up in Kinetic Energy (using \( E=mc^2 \)).

Most of the energy (about 6 MeV) is taken by the \( \alpha \) particle.
Decay

$$^{24}\text{Na}_{11} \rightarrow ^{24}\text{Mg}_{12} + ^0\text{e}^{-1}$$

The electron takes away 1 unit of electric charge, increasing $Z$ of the nucleus by 1. Atomic mass $A$ is unchanged.

(It’s as if a neutron changes into a proton and an electron.)

Positive Decay

The anti-particle to the electron (the positron, predicted by the Dirac Equation) can also take part in $\beta$ decay.

$$^{13}\text{N}_7 \rightarrow ^{13}\text{C}_6 + ^0\text{e}^{+1}$$

(It’s as if a proton changes into a neutron and a positron.)
For $\beta$ decay (positive or negative), one would expect the electron to come out with a single energy, just as for $\alpha$ decay.

However, experiment showed a continuous spectrum of energies:

In 1930 Wolfgang Pauli solved this by suggesting that the electron energy was shared with a new particle, the neutrino.
The neutrino is chargeless and (almost) massless.

1956 - neutrino finally detected.

Difficult to detect because it does not interact through EM or strong force, only through the weak force.

1998 - evidence of a nonzero mass confirmed.

Its mass is of order $10^{-3}$ eV/c$^2$ or less (compared to an electron mass of $m_e = 5.11 \times 10^{-5}$ eV/c$^2$).

We can now write the $\beta$ decays as:

\[ ^{24}\text{Na}_{11} \rightarrow ^{24}\text{Mg}_{12} + ^0\text{e}_{-1} + \bar{\nu} \]

\[ ^{13}\text{N}_{7} \rightarrow ^{13}\text{C}_{6} + ^0\text{e}_{+1} + \nu \]
Electron Capture

\[ _{26}^{55}\text{Fe}_{26} + ^0\text{e}_- \rightarrow _{25}^{55}\text{Mn}_{25} + \nu \]

For higher-Z nuclides, it is possible for electron capture to occur.

The effect is the same as for positive \[ \beta \] decay: a proton is converted to a neutron.

When electron capture occurs, the hole left by the captured inner electron will be filled by an outer electron which drops down, while emitting an X-ray of characteristic wavelength.
Decay

Just like an atom, a nucleus can also have excited states. Often, an α or β decay of a radioactive nucleus will leave the daughter nucleus in an excited state.

The excited nucleus will then decay to the ground state with the emission of a high energy photon (γ ray).

\[ ^{225}\text{Th}^*_90 \rightarrow ^{225}\text{Th}_{90} + \gamma \]

Obviously, this leaves A and Z unchanged.
A radioactive decay “chain” (Segre chart)

- Uranium
- Protactinium
- Thorium
- Thorium
- Actinium
- Francium
- Astatine
- Bismuth
- Lead
- Thallium
- Lead
- Astatine
- Polonium
- Radon
- Radium
- Polonium
- Bismuth
- Lead
- Neutron Number N
- Atomic Number Z
There are three radioactive series that eventually decay to lead isotopes.

<table>
<thead>
<tr>
<th>Mass Numbers</th>
<th>Series Name</th>
<th>Parent</th>
<th>$t_{1/2}$ (y)</th>
<th>End Product</th>
</tr>
</thead>
<tbody>
<tr>
<td>$4n$</td>
<td>Thorium</td>
<td>$^{232}_{90}\text{Th}$</td>
<td>$1.40 \times 10^{10}$</td>
<td>$^{208}_{82}\text{Pb}$</td>
</tr>
<tr>
<td>$4n + 1$</td>
<td>Neptunium</td>
<td>$^{237}_{93}\text{Np}$</td>
<td>$2.14 \times 10^{6}$</td>
<td>$^{209}_{83}\text{Bi}$</td>
</tr>
<tr>
<td>$4n + 2$</td>
<td>Uranium</td>
<td>$^{238}_{92}\text{U}$</td>
<td>$4.47 \times 10^{9}$</td>
<td>$^{206}_{82}\text{Pb}$</td>
</tr>
<tr>
<td>$4n + 3$</td>
<td>Actinium</td>
<td>$^{235}_{92}\text{U}$</td>
<td>$7.04 \times 10^{8}$</td>
<td>$^{207}_{82}\text{Pb}$</td>
</tr>
</tbody>
</table>
Decay times

Radioactive decay is a probabilistic event.

For large numbers of nuclei, the number that decay in a short time will be proportional to the total number $N$ and the time $\Delta t$:

$$ \Delta N = - \lambda N \Delta t $$

The solution to this differential equation is

$$ N(t) = N(0) e^{-\lambda t} $$
Activity

The Activity of a radioactive substance is defined as the number of decays per unit time:

Activity: \( R = - \frac{dN}{dt} = \lambda N(t) \)

The SI unit is the Becquerel:

\[ 1 \text{ Bq} = 1 \text{ decay/second} \]

An older, but still used, unit is the Curie:

\[ 1 \text{ Ci} = 3.7 \times 10^{10} \text{ decays/second} \]

1g of \(^{226}\text{Ra}\)

The activity falls also falls off exponentially:

\[ R(t) = R(0) e^{-\lambda t} \]

\( \lambda \) is the decay constant.
The half-life \( (t_{\frac{1}{2}}) \) is the time for the number of radioactive nuclei to drop by a factor of 2. It is easy to show

\[
t_{\frac{1}{2}} = \ln(2)/\lambda = 0.693/\lambda
\]

\[
N(t) = \left( \frac{1}{2} \right)^{t/t_{\frac{1}{2}}}
\]
Carbon Dating

Radioactive $^{14}C$ is continually produced in the atmosphere by bombardment of $^{14}N$ by neutrons produced by cosmic rays:

$$n + ^{14}N_7 \rightarrow ^{14}C_6 + p$$

There is a natural equilibrium ratio of $^{14}C$ to $^{12}C$ of $R_0 = 1.2 \times 10^{-12}$. This ratio occurs in the carbon in the CO$_2$ taken up by living organisms.

But when a living organism dies, the $^{14}C$ decays and the ratio of $^{14}C/^{12}C$ decreases, allowing us to date the time of death of the organism by

$$R(t) = R_0 e^{-\lambda t}$$
A radioactive sample decreases in activity by a factor of 5 in 1 hour. What is its half-life?

\[ R = R_0 e^{-\lambda t} \]

\[ \Rightarrow \frac{1}{5} = e^{-\lambda} \]

\[ \ln \frac{1}{5} = -\lambda \quad \Rightarrow \lambda = 1.609 \]

\[ T_{\frac{1}{2}} = \frac{0.693}{\lambda} = \frac{0.693}{1.609} = 0.431 \text{ hr} \]

OR

\[ \frac{R}{R_0} = \left( \frac{1}{2} \right)^n \quad \text{where} \quad n = \# \text{ of } \frac{1}{2}-\text{lives} \]

\[ \Rightarrow \frac{1}{5} = \left( \frac{1}{2} \right)^n \]

\[ \ln \frac{1}{5} = n \ln \frac{1}{2} \quad \Rightarrow n = 2.32 \]

So, if 1 hour = 2.32 \( \frac{1}{2} \)-lives

Then 1 \( \frac{1}{2} \)-life = \( \frac{1}{2.32} \) = 0.431 hr
Example 12.18

A bone is found to have a ratio of \( \frac{^{16}C}{^{13}C} = 1.10E-12 \). How old is it?

The \( \frac{1}{2} \)-life of \(^{14}C\) is 5730 years.

\[ \lambda = \frac{0.693}{5730} = 0.693 \times 10^{-3} = 1.21E-4 \]

\[ R = R_0 e^{-\lambda t} \]

\[ \frac{R}{R_0} = \frac{1.1E-12}{1.2E-12} = 0.917 = e^{-\lambda t} \]

\[ \ln 0.917 = -\lambda t \]

\[ t = \frac{-\ln 0.917}{\lambda} = \frac{-0.0870}{1.21E-4} \]

\[ = 719 \text{ years} \]
A TIME-DATING QUESTION (SEE EX 12.17)

\[ ^{238}U \rightarrow ^{206}Pb \]

Assume originally \( N_0 \) U and 0 Pb

After time \( t \)

\[ N_0 e^{-\lambda t} \rightarrow N_0 (1 - e^{-\lambda t}) \]

Ratio of \( \frac{Pb}{U} = \frac{N_0 (1 - e^{-\lambda t})}{N_0 e^{-\lambda t}} = e^{\lambda t} - 1 \]

If this ratio is measured, and if we know \( \lambda \), we can calculate \( t \).

\[
\lambda = 0.693 \quad \frac{1}{t_{\frac{1}{2}}} = \frac{0.693}{4.47 \times 10^9 \text{ years}} = 1.55 \times 10^{-10}
\]

12.47 If ratio is measured to be \( R = 0.76 \), what is \( t \)?

\[ e^{\lambda t} = 1 + R = 1.76 \]

\[ \therefore \lambda t = \ln(1.76) = 0.565 \]

\[ \therefore t = \frac{0.565}{1.55 \times 10^{-10}} = 3.65 \times 10^9 \text{ years} \]
ARTIFICIAL RADIOACTIVITY

This was discovered by Irene Joliot-Curie (the daughter of Mme. Marie Curie) and Frederic Joliot. They were awarded the Nobel Prize for Chemistry in 1935.

They bombarded many materials with energetic particles such as protons and α particles. These can force their way into a stable nucleus and create a new, unstable nucleus, which will then decay into another nucleus and other lighter particles. This is the phenomenon of artificial (or induced) radioactivity.

For example, Rutherford's experiment of 1919 was actually an example of artificial radioactivity.

The process really was:

\[ ^{14}_{7}N + ^{4}_{2}α \rightarrow ^{18}_{9}F \rightarrow ^{1}_{1}p + ^{17}_{8}O \]

The fluorine nucleus was an unstable isotope (stable fluorine is \(^{19}_{9}F\)) and it decays into oxygen and a proton. Rutherford did not have the instrumentation to observe the two-step process but, 15 years later, the Joliot-Curies did.
ACCELERATORS

Many other possibilities exist to initiate nuclear reactions (and so study nuclear forces). But they require colliding particles of sufficient energy to overcome the coulomb repulsion and so get close enough to the target nucleus.

This energy could be obtained with radioactive sources (which Rutherford, Chadwick and the Joliot-Curies had used) but they were often not intense enough to observe the infrequent reactions.

On the other hand, very intense sources of protons or alpha particles could be made available with discharge tubes. If some method could be used to increase the voltage of the discharge tube or somehow accelerate the particle to high energy with electric fields then nuclear reactions could be studied more easily.

A "race" was on to "split the atom" by artificial means.
ENTRANTS TO THIS RACE INCLUDED THE VAN DER GRAAFF GENERATOR, THE CYCLOTRON, AND EVEN NATURAL LIGHTNING.

THE WINNER WAS A COMPLICATED ARRANGEMENT OF TRANSFORMERS, DIODE RECTIFIERS, AND CAPACITORS ASSEMBLED AT THE CAVENDISH BY JOHN COCKCROFT AND ERNEST WALTON.

THE COCKCROFT-WALTON GENERATOR

A View of the Cockcroft-Walton Apparatus in late 1931
A Modern Cockcroft-Walton Machine (at Fermilab)
COCKCROFT AND WALTON WERE ABLE TO ACCELERATE PROTONS TO ENERGIES OF ABOUT 800 keV (800,000 VOLTS).

THEY ACCOMPLISHED THE TRANSMUTATION OF LITHIUM BY BOMBARDING IT WITH PROTONS OF THIS ENERGY.

THE REACTION WAS:-

\[
^7\text{Li}_3 + ^1\text{p}_1 \rightarrow ^4\text{He}_2 + ^4\text{He}_2
\]

AFTER THIS SUCCESS, ACCELERATORS WERE OFTEN CALLED ATOM SMASHERS.
A major breakthrough in particle accelerators was achieved by Ernest Lawrence at Berkeley in the early 1930s.

The basic idea is to position the particle in a magnetic field so that it moves in a circle, thus enabling the electric field (which causes the acceleration) to be used many times.
Behavior of a charged particle in a magnetic field

The radius of curvature is given by the Lorentz force supplying the centripetal force.

\[ r = \frac{mv}{qB} \]

If a particle performs a complete circular orbit inside a constant magnetic field, then the period of revolution of the particle is just the circumference of the circle divided by the speed.

\[ T = \frac{2\pi r}{v} = \frac{2\pi m}{qB} \quad \Rightarrow \quad f = \frac{1}{T} = \frac{qB}{2\pi m} \]

So the frequency (called the cyclotron frequency) is independent of the speed of the particle.
The D-shaped pieces (descriptively called “dees”) have alternating electric potentials applied to them such that a positively charged particle always sees a negatively charged dee ahead when it emerges from under the previous dee, which is now positively charged. The resulting electric field accelerates the particle. The radius of the trajectory is proportional to the momentum, so the accelerated particle spirals outward.
The cyclotron has been the workhorse of low energy nuclear physics to the present day. MSU’s cyclotron laboratory is a leader in nuclear research.

The facility consists of two “coupled” cyclotrons. The K500 accelerates ions up to 20 MeV/nucleon. The ions can then be transferred into the K1200 which can accelerate them up to 200 MeV/nucleon.

Typically then the beam will be made to hit a metal target, generating unstable isotopes for study.
NEUTRON ACTIVATION

There is a method of initiating nuclear reactions without the need for an accelerator. This is the neutron, which of course has the disadvantage that, being uncharged, it cannot be accelerated by electric fields (or bent by magnetic fields). On the other hand it has the great advantage that it can penetrate close to the nucleus without being repelled by the Coulomb force.

In the early 1930's the great Italian physicist Enrico Fermi (1901 - 1954) realized the importance of a careful study of the neutron bombardment of various nuclei. He bombarded a series of elements from hydrogen to uranium with neutrons and studied the radioactive transformations produced.

When he came to uranium Fermi was unable to identify the final products of the reactions. At about this time, he was forced to leave Italy because of the political situation under Mussolini. He and his family emigrated to the United States; accepting a position first at Columbia University and later moving to the University of Chicago.