PHYSICS 215 - Thermodynamics and Modern Physics

Practice Midterm Exam 2

1. [6 points] An interstellar spaceship travels from the earth to a distant star 12 light years away (as measured in the earth's frame). The trip takes 15 years as measured on the ship.

(a) What is the speed of the ship relative to the earth?

(b) When the ship arrives at its destination, it sends a radio signal back to the earth. How much time elapses on earth between the departure of the spaceship and the arrival of the signal?

2. [6 points] A beam of X rays has an energy of 10 keV.

(a) What is the wavelength of these X rays?

(b) The X rays are then Compton-scattered from electrons at rest. The scattered beam is observed at an angle of 90° relative to the incident beam. What is the wavelength of the scattered X rays?

(c) What is the energy of the scattered photons?

(d) What is the kinetic energy of the scattered electrons?

[Extra credit, 2 points] What is the angle of the scattered electrons?

3. [6 points] In an experiment done by scattering α particles on a thin gold foil, students find that 10,000 α particles are scattered at an angle of θ = 45°.

(a) How many α particles would they expect to see scattered at θ = 60°?

(b) If the gold foil (Z = 79) is replaced by an aluminum foil (Z = 13) with a similar number of scattering nuclei per unit area, how many α particles will now be scattered at 45°?

4. [6 points] An electron, initially at rest, is accelerated across a potential difference of 5 kV.

(a) What is its kinetic energy?

(b) What is its total energy??

(c) What is its momentum?

(d) What is its de Broglie wavelength?
Useful Constants:
Avogadro's number, \( N_A = 6.02 \text{E}23 \text{ mol}^{-1} \)
Speed of light, \( c = 3.00 \text{E}8 \text{ m/s} \)
Charge of an electron, \( -e = -1.6 \text{E} -19 \text{ C} \)
Coulomb's constant, \( k = 1/4 \pi \varepsilon_0 = 8.99 \text{E}9 \text{ Nm}^2/\text{C}^2 \)
Mass of the electron, \( m_e = 9.11 \text{E} -31 \text{ kg} = 511 \text{ keV/c}^2 \)
Mass of the proton, \( m_p = 1.67 \text{E} -27 \text{ kg} = 938 \text{ MeV/c}^2 \)
Planck's constant, \( h = 6.63 \text{E} -34 \text{ J.s} = 4.14 \text{E} -15 \text{ eV.s} \)
Planck's reduced constant, \( \hbar = h/2\pi = 1.05 \text{E} -34 \text{ J.s} = 6.58 \text{E} -16 \text{ eV.s} \)
Compton Wavelength of the electron, \( \lambda_c = h/m_e c = 2.426 \text{E}34 \text{ E} -12 \text{ m} \)

Useful Formulae:
Potential energy lost by a charge \( q \) in a potential difference of \( V \) is \( U = qV \)

Wave relation: \( v = v\lambda \) where \( v = \) velocity, \( \nu = \) frequency, \( \lambda = \) wavelength.

\[ \beta = v/c \quad \gamma = 1/\sqrt{1 - \beta^2} \]

Length Contraction: \( L' = L/\gamma \) \hspace{1cm} Time Dilation, \( T' = \gamma T \)

Addition of Velocities: \( v' = (v + u)/(1 + vu/c^2) \)

Relativistic Doppler Effect: \[ v' = \frac{\sqrt{1 - \beta}}{\sqrt{1 + \beta}} v \]

Momentum – Energy relations: \[ E^2 = p^2c^4 + m^2c^4 \]

\[ E = \gamma mc^2 \quad p = \gamma mv \quad K = E - mc^2 \]

Planck's Relation: \( E = h\nu \)

Einstein’s Photoelectric Law: \( h\nu = K + \phi \)

Compton Effect: \[ \Delta \lambda = \lambda' - \lambda = (1 - \cos\theta)h/m_e c \]

Bohr Quantization Relation: \( L = mv = n\hbar \)

Atomic Radii: \( r_n = n^2a_0/Z \)

where \( a_0 = 5.29 \text{E} -11 \text{ m} \)

Atomic Energies: \( E_n = -Z^2F_0/n^2 \)

where \( E_0 = 13.6 \text{ eV} \)

Impact parameter: \[ b = \frac{Z_1Z_2e^4}{8\pi\varepsilon_0 K} \cot(\theta/2) \]

\[ n = \rho N_A/A \]

Fraction of \( \alpha \)’s scattered through \( \theta \) or greater: \[ f = \pi b^2n t \]

Rutherford Scattering: \[ N(\theta) = \frac{N_0 n t e^4 Z_1^2 Z_2^2}{16(4\pi\varepsilon_0)^2 r^2 K^2 \sin^4(\theta/2)} \]
1. Let Earth measurements be unprimed
   spaceship measurements be primed

   \[ L = 12 \text{ ly.} \quad T' = 15 \text{ y} \]

   \[ T = \frac{L}{xT'} \quad \text{with} \quad x = \frac{1}{\sqrt{1 - \beta^2}} \]

   \[ \beta = \frac{L}{T} = \frac{L}{xT'} = \frac{12}{\frac{1}{\sqrt{1 - \beta^2}} \cdot 15} = \frac{12}{15} \cdot \sqrt{1 - \beta^2} \]

   \[ 15^2 \beta^2 = 12^2 (1 - \beta^2) \]

   \[ 15^2 \beta^2 + 12^2 \beta^2 = 12^2 \]

   \[ \beta^2 = \frac{12^2}{15^2 + 12^2} \quad \Rightarrow \quad \beta = \frac{12}{\sqrt{15^2 + 12^2}} = \frac{12}{19.2} = 0.625 \]

   \[ y = \frac{1}{\sqrt{1 - 0.625^2}} = 1.28 \]

   \[ \Rightarrow T \text{ (for outbound trip)} = 1.28 \times 15 = 19.2 \text{ y} \]

   \[ \Rightarrow \text{radio signal arrives after} \]

   \[ 19.2 + 12 = 31.2 \text{ y} \]
2. \( E_x = 10000 \text{ eV} \)

\[(a) \Rightarrow \lambda = \frac{hc}{E} = \frac{1240}{10000} = 0.1240 \text{ nm} \]

\[(b) \chi' = \lambda + (1 - \cos \theta) \frac{h}{mc} = 0.1240 + 2.4263 \times 10^{-3} = 0.1264 \text{ nm} \]

\[(c) \ E_x' = \frac{1240}{\chi'} = 9808 \text{ eV} \]

\[(d) \text{ CONSERVATION OF ENERGY } \Rightarrow \]

\[E_x = E_x' + KE_e \]

\[\Rightarrow KE_e = 10000 - 9808 = 191.6 \text{ eV} \]

EXTRA CREDIT:

\[E_e = 191.6 + 511000 = 511191.6 \text{ eV} \]

\[P_e c = \sqrt{E_e^2 - m_e c^4} = 13995 \text{ eV} \]

\[\text{CONSERVATION OF MOMENTUM } \Rightarrow \]

\[P_x' \sin 90^\circ = P_e \sin \phi \]

\[\Rightarrow \sin \phi = \frac{9808}{13995} \Rightarrow \phi = 44.5^\circ \]
3. **Using Rutherford's Formula** ⇒

(a) \( \frac{N'(\theta')}{N(\theta)} = \frac{\sin^4(\theta')}{{\sin^4}(\theta')} \)

⇒ \( N'(\theta') = 10000 \frac{\sin^4(45^\circ)}{\sin^4(60^\circ)} \)

⇒ \( N'(\theta' = 60^\circ) = 3431 \)

(b) \( \frac{N'(z'_2)}{N(z_2)} = \frac{z'_2^2}{z_2^2} = \frac{13^2}{79^2} \)

⇒ \( N'(z'_2 = 13) = 10000 \cdot \frac{13^2}{79^2} \)

= 271
4.  (a) $K = 5 \text{ keV}$

(b) $E = K + mc^2 = 5 + 571$ = 516 keV

(c) $pc = \sqrt{E^2 - m^2c^4} = 71.7 \text{ keV/c}$

(d) $\lambda = \frac{h}{P} = \frac{hc}{pc} = \frac{1240}{71.7E3}$

= 0.0173 nm

= 1.73E-11 m
Another Relativity Question

A spaceship traveling towards a distant star with a velocity of $v = 0.2\, c$ sends information back to Earth using yellow light of wavelength 580 nm. What is the apparent wavelength of the light when it is received on Earth?

\[ \lambda = \frac{\sqrt{1+\beta}}{\sqrt{1-\beta}} \lambda_0 \]

Note: the source is receding => longer wavelength

\[ \Rightarrow \lambda = \frac{\sqrt{1+0.2}}{\sqrt{1-0.2}} \lambda_0 = \sqrt{\frac{1.2}{0.8}} \cdot 580 \]

\[ \Rightarrow \lambda = 1.22 \cdot 580 = 710\, \text{nm} \]

What is the color of the observed light?

710 nm is red
Another Relativity Question

A spaceship is moving away from earth at a constant velocity of \(0.3c\) with respect to the earth. After some time, a shuttle is launched from earth in the direction of the spaceship. The shuttle has a velocity of \(0.8c\) with respect to the earth. What is the velocity of the shuttle as seen from the spaceship?

\[
u = \frac{u' - V}{1 - \frac{u'V}{c^2}} = \frac{0.8c - 0.3c}{1 - 0.8 \cdot 0.3}
\]

\[
\Rightarrow u = \frac{0.5c}{1 - 0.24} = \frac{0.5c}{0.76}
\]

\[
= 0.66c
\]
(b) Communications from the shuttle are on a frequency of 97.5 MHz. To what freq. does the receiver on earth have to be tuned?

\[ v = \frac{\sqrt{1 - \beta}}{\sqrt{1 + \beta}} \cdot v_0 \]

\[ \Rightarrow \quad v = \frac{\sqrt{1 - 0.8}}{\sqrt{1 + 0.8}} \cdot v_0 \]

\[ \Rightarrow \quad v = \sqrt{1 - \frac{0.2}{1.8}} \cdot 97.5 = 0.33 \cdot 97.5 \]

\[ \Rightarrow \quad v = 32.5 \text{ MHz} \]

(c) What frequency should the spaceship use?

\[ v = \frac{\sqrt{1 + \beta}}{\sqrt{1 - \beta}} \cdot v_0 \]

\[ \Rightarrow \quad v = \frac{\sqrt{1 + 0.66}}{\sqrt{1 - 0.66}} \cdot v_0 = 2.21 \cdot 97.5 \]

\[ = 215 \text{ MHz} \]
Rutherford Scattering

An α particle of kinetic energy = 9.1 MeV is incident on a silver foil (density $\rho = 8.6$ g/cc, $Z = 47$, $A = 108$ g/mole) of thickness $t = 10$ nm.

(a) How many silver nuclei are there per m$^3$?

$$n = N_A \cdot \frac{1}{A} \cdot \rho = 6.02E23 \cdot \frac{1}{108} \cdot 8E6$$

$$\Rightarrow n = 4.46E28 \text{ m}^{-3}$$

(b) If the impact parameter is zero, i.e. a “head on” collision, what is the angle of scatter?

$$b = \text{const} \cdot \cot \frac{\theta}{2} \Rightarrow b = 0 \Rightarrow \frac{\theta}{2} = 90^\circ$$

$$\Rightarrow \theta = 180^\circ$$

(c) In that case, what is the distance of closest approach?

Conservation of energy $\Rightarrow$

$$K + 0 = 0 + \frac{1}{4\pi\varepsilon_0} \frac{(Z_1e) \cdot (Z_2e)}{d}$$
(d) What fraction of the α particles are scattered through an angle of \( \theta = 90^\circ \) or greater?

\[ f = \frac{\pi n t \left( \frac{Z_1 Z_2 e^2}{8 \pi \varepsilon_0 K} \right)^2}{\cot^2 \frac{\Theta}{2}} \]

\[ \Rightarrow f = \frac{\pi \cdot 4.66E28 \cdot 10E-9 \cdot \left( \frac{8.99E9 \cdot 2 \cdot 47 \cdot (1.6E-19)^2}{2 \cdot 9.1E6 \cdot 1.6E-19} \right)^2 \cdot \cot^2 45}{\cot^2 45} \]

\[ \Rightarrow f = 8.08E-8 \]