PHY 251 Practical Lab #2
The Pendulum

Do not write on these pages, only on your answer sheet.

Objectives

- To investigate the functional dependence of the period (τ) of a pendulum on its length (L. The Greek letter tau (τ) is typically used to denote a time period or time interval
- use a pendulum to measure g, the acceleration due to gravity

Apparatus

Point masses and string, a digital timer, period gate, and meter stick will be used.

Theory

In the analysis of the motion of a pendulum we should realize that

1) The motion is part of a circle so angular acceleration (α) is a useful variable
2) The angular acceleration will not be a constant throughout the motion

Consider the pendulum shown in Figure 1. The weight at the end of the string is called the "bob" of the pendulum. The acceleration of the bob **tangent** to the arc "drawn" by the pendulum as it swings, a_t, is determined by $\mathbf{F_t}$, the force tangent to the arc. Since the tension in the string (T) always acts along the **radius**, it doesn't contribute to $\mathbf{F_t}$. Decomposing the gravitational force mg into components perpendicular and parallel to the string as shown in the diagram below, we find that

$$F_t = mg\sin\theta$$

Therefore the acceleration tangent to the circle is given by:

$$a_t = \frac{F_t}{m} = g\sin\theta$$

The angular acceleration α is then found by the relationship for circular motion

$$\alpha = -\frac{a_t}{r} = -\frac{g}{L}\sin\theta$$

Thus, as we have suggested, <u>the angular acceleration α is not a constant</u> but varies as the sine of the displacement angle of the pendulum.

For small angles (about $\theta < 0.5$ radian) angular accelerations can be shown (with a little calculus which we will skip) to lead to an oscillation of the angle θ by

$$\theta = \theta_0 \cos\frac{2\pi\ t}{\tau}$$

where θ_0 is the angle at time t = 0 (when we release the pendulum), and τ is the period of the motion. The period is the time it takes to complete one full cycle of the motion.

The period of a simple pendulum is given by:

$$\tau = 2\pi\sqrt{\frac{L}{g}} \qquad \text{or} \qquad \tau = \frac{2\pi}{\sqrt{g}}\sqrt{L}$$

This equation has the same form as the equation of a straight line y = mx + b, with an intercept of zero (i.e. b = 0). Notice in this equation, the period (τ) corresponds to y and $\sqrt{L}$ corresponds to x.

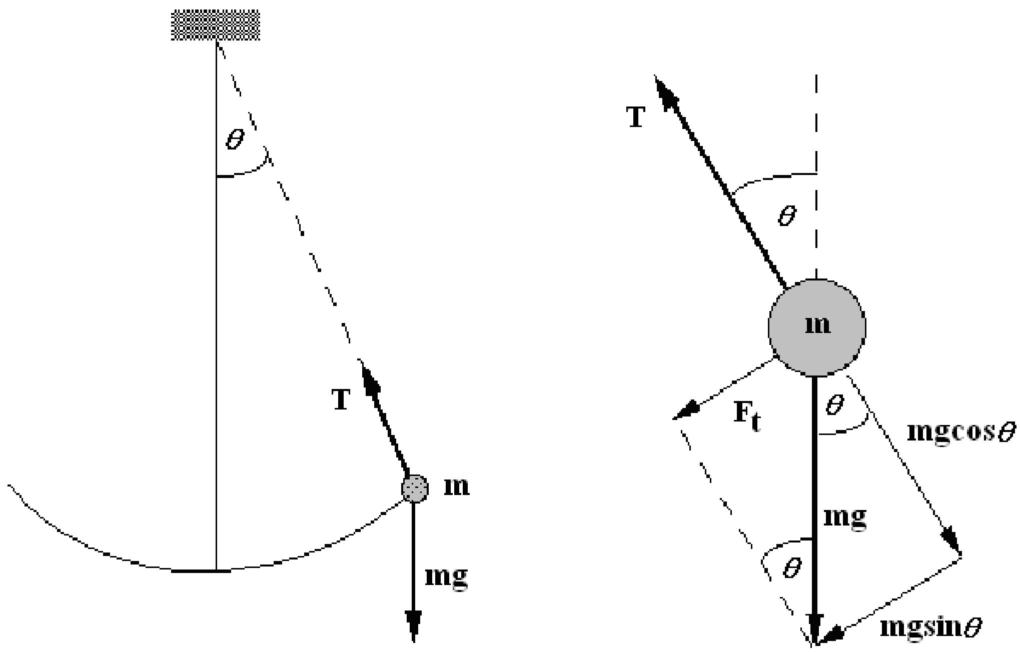

Figure 1

Procedure

The parameter of the system you will vary is L, the length of the pendulum from the support to the center of mass of the "point mass". The quantity you measure is the period τ.

You will use the PEND setting of the gate and measure the period several times for the lengths of the pendulum specified on your lab worksheet (you will get the worksheet when you do this practical lab). The PEND setting of the gate uses light and a photodetector in the following way: instead of timing until the next interruption of the light beam through the gate, it counts time until a *second* interruption of the light beam. That is because for the pendulum to complete a full oscillation, it must return to the same position, *and* be moving in the same direction. Half way through its oscillation, it will have swung back to the initial position but it will be moving in the opposite direction, so the PEND setting ignores this first interruption of the light beam, and waits until the second interruption, which will correspond to the full oscillation cycle. The time for the full oscillation is also called the period.

1. Adjust the length of the pendulum approximately to the first given length and assign a reasonable uncertainty to your measured value of this length.
2. With the timer in PEND mode, release the pendulum from a small starting angle (i.e. less than 30 degrees from the vertical) and measure the period of the pendulum. Enter this measured value in your spreadsheet.
3. Use Excel to calculate $\sqrt{L}$ and its uncertainty $\delta\left(\sqrt{L}\right) = \frac{\delta L}{2\sqrt{L}}$. The Excel formula for the square root is "=SQRT(CELL#)".
4. Repeat steps 1 through 3 for the other specified lengths.
5. Insert appropriate units for all quantities on the spreadsheet.
6. Transfer your data into Kaleidagraph and construct a graph of τ vs $\sqrt{L}$. Include horizontal error bars, the equation of the best fit line and the uncertainties in the slope and intercept of your best fit line.

Questions

1) What is the expected value of the intercept? Does the intercept of your graph agree with this expected value? Justify your response.
2) Use the slope of the graph of τ vs. square root of L to calculate g and its uncertainty. $\delta g = 2g\frac{\delta(slope)}{slope}$ Show your work:
3) Is your value of g consistent with 980 cm/sec^2?
4) If the mass at the end of the string is doubled, what will happen to the period of the pendulum? Explain your response.

<u>CHECKLIST</u>

1) the spreadsheet with your data and formula view of your spreadsheet
2) graph with best-fit line and equation of best-fit line and uncertainties
3) answers to the questions
4) other than specified in the questions, NO sample calculations are required

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measured or assigned	calculated

δL =		(insert units)

Trial	Length (L) (insert units)	sqrt(L) (units)	δsqrt(L) (units)	Period (units)
1				
2				
3				
4				
5				

USING UNCERTAINTIES TO COMPARE DATA AND EXPECTATIONS

One important question is whether your results agree with what is expected. Let's denote the result by r and the expected value by e. The ideal situation would be $r = e$ or $r - e = 0$. We often use Δ (pronounced "Delta") to denote the difference between two quantities:

$$\boxed{\Delta = r - e} \qquad \textbf{(1)}$$

The standard form for comparison is always *result - expected*, so that your difference Δ will be negative if your value is lower than expected, and positive if it is higher than expected.

This comparison must take into account the uncertainty in the observation, and perhaps, in the expected value as well. The data value is $r \pm \delta r$ and the expected value is $e \pm \delta e$. Using the addition/subtraction rule for uncertainties, the uncertainty in $\Delta = r - e$ is just

$$\boxed{\delta\Delta = \delta r + \delta e} \qquad (2)$$

Our comparison becomes, "is zero within the uncertainties of the difference Δ?" Which is the same thing as asking if

$$\boxed{|\Delta| \leq \delta\Delta} \qquad (3)$$

Equation (2) and (3) express in algebra the statement "*r* and *e* are compatible if their error bars touch or overlap." The combined length of the error bars is given by (2). $|\Delta|$ is magnitude of the separation of *r* and *e* . The error bars will overlap (or touch) if r and *e* are separated by less than (or equal to) the combined length of their error bars, which is what (3) says.