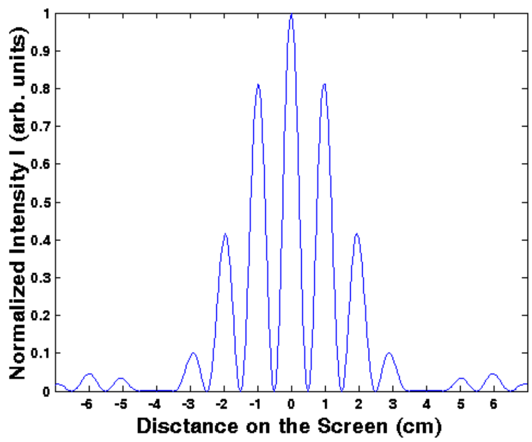
**PHY 431 Homework Set #9**

**Total 115%**

Due November 29 at the start of class

1) ***Fresnel/Fraunhofer Approximation (15%)***
Two long slits 0.10 mm wide, separated by 0.20 mm, in an opaque screen are illuminated by light with a wavelength of 500 nm. If the plane of observation is 2.5 m away, will the pattern correspond to Fraunhofer or Fresnel diffraction? How many Young's fringes will be seen within the central bright band?

2) ***The double slit (15%)***
A plane wave of light from a laser has a wavelength of 6000 A°. The light is incident on a double slit. After passing through the double slit the light falls on a screen 100 cm beyond the double slit. The intensity distribution of the interference pattern on the screen is shown. What is the width of each slit and what is the separation of the two slits?




3) ***Fraunhofer Diffraction (15%)***
   a. Consider the Fraunhofer diffraction pattern due to two unequal slits. Let *a* and *b* be the unequal slit widths and *c* the distance between their centers. Derive and expression for the intensity of the pattern for any diffraction angle $\theta$, assuming the arrangement to be illuminated by perpendicular light of wavelength $\lambda$.
   b. Use your formula from (A) to obtain expression for the pattern in the following special cases and make a sketch of those patterns:
      i. $a=b$,
      ii. $a=0$.

4) ***Spatial and Spectral Resolution (15%)***
Two stars have an angular separation of $1\times10^{-6}$ radian. They both emit light of wavelengths 5770 and 5790 A°.
   A. How large a diameter of the lens in the telescope is needed to separate the image of the two stars?
   B. How large a diffraction grating is needed to separate the two wavelengths present?

5) ***Gratings [15%]***
Green light of $\lambda$=530nm falls on a diffraction grating of length l=12 mm and period d=1.5 μm. Calculate the angular dimension of the primary maximum and the resolving power of the diffraction grating.

**6) Diffraction: Single Slit, Double Silts & Rectangular Apertures (40%)**

In a diffraction experiment, a collimated beam of laser from a green diode laser ($\lambda$=532nm) is to be used.

(A) The laser output beam is approximately circular with a diameter of 2 mm. You need a beam expander to produce a collimated beam of 10 mm for the diffraction experiments. You are given six lenses with the following focal lengths (negative lenses: f=-1 cm, -2 cm, and -5cm; positive lenses: f= +2cm, +10 cm, and +15 cm). **Choose** two lenses to set up such a beam expander (Galilean telescope type). **Specify** the distance between these two lenses and **Draw** a ray diagram to illustrate the beam expansion.

(B) In the 1st experiment, a hole of R=0.5 mm radius is used as an aperture.
**Explain** Fraunhofer and Fresnel diffraction regimes based on the Huygens-Fresnel principle. **Determine** whether Fresnel or Fraunhofer approximation applies when the screen-to-aperture distance z is (i) 10 cm (ii) 2 m.
[Hint: consider z vs. $\frac{\pi R^2}{\lambda}$ & $z^3$ vs. $\frac{\pi R^4}{4\lambda}$]

(C) In the 2nd experiment, the collimated beam falls normally on a slit 50 μm wide. A lens of 50 cm focal length placed just behind the slit focuses the diffraction light on a screen located at the focal distance.
**Calculate** the distance from the center of the diffraction pattern (central maximum) to the first minimum.

(D) In the 3rd experiment, a double slit is used as an aperture. It is found that the fourth secondary maximum is missing.
**Determine** the ratio of slit width ***a*** to slit separation ***b***. [consider Fraunhofer diffraction]

(E) A square aperture (width=a) is used for the final experiment. **Sketch** the diffraction pattern **and label the first zeros and maxima. Determine** the value of intensity ratio $I/I_0$ for the first diagonal maximum of the resultant Fraunhofer diffraction pattern. $I_0$ is the intensity at the central maximum. (The maxima occur at
$\frac{d}{d\alpha}\left(\frac{\sin\alpha}{\alpha}\right)=0,\quad i.e.\quad \alpha=\tan\alpha$. The first diagonal maxima occur at
$\alpha=\beta=1/2k\times a\times\sin(\theta)=\pm1.4303\pi$).