19-11. In a Snellen chart letters in the 20/20 row should subtend an angle of $\theta_l = 5' = 5/60° = 0.001454$ rad and details should subtend an angle $\theta_d = 1' = 0.0002909$ rad. Heights on the screen a distance of $D = 5$ ft from the eye are related to these angles by the relation $y = D\theta$. So

20/20 row: $y_l = D\theta_l = (60\,\text{in})(0.001454\,\text{rad}) = 0.0872\,\text{in}$, $y_d = y_l/5 = 0.0174\,\text{in}$.

20/15 row: $y_l = (15/20)y_{l,20/20} = 0.00654\,\text{in}$, $y_d = y_l/5 = 0.00131\,\text{in}$.

20/300 row: $y_l = (300/20)y_{l,20/20} = 1.308\,\text{in}$, $y_d = y_l/5 = 0.262\,\text{in}$.

20/100 row: $y_l = (100/20)y_{l,20/20} = 0.436\,\text{in}$, $y_d = y_l/5 = 0.0872\,\text{in}$.

20/60 row: $y_l = (60/20)y_{l,20/20} = 0.262\,\text{in}$, $y_d = y_l/5 = 0.0523\,\text{in}$.

19-12. Objects 25 cm from the eye should form images 125 cm from the eye. Since the lens is 25 cm from the eye this implies object and image distances of $s = 23.5\,\text{cm}$ and $s' = -123.5\,\text{cm}$.

(a) The needed power is then

$$P = \frac{1}{f} = \frac{1}{s} + \frac{1}{s'} = \frac{1}{0.235\,\text{m}} - \frac{1}{1.235\,\text{m}} = 3.45/\text{m} = 3.45\,\text{D}$$

(b) Assuming that the eye alone has a far point at infinity, the far point with the lenses is the object position at which the corrective lenses form an image at $-\infty$. The object distance s_{far} from the glasses is found as,

$$P = \frac{1}{f} = \frac{1}{s_{\text{far}}} + \frac{1}{-\infty} \Rightarrow s_{\text{far}} = f = 1/P = (1/3.45)\,\text{m} = 0.290\,\text{m} = 29.0\,\text{cm}$$

This corresponds to a distance of 30.5 cm from the eye. The range of clear vision is expanded if the correction is for a distance further than the normal near point.

19-13. (a) The lens should place the image of an object at infinity at the eye's far point. So.

$$P = \frac{1}{f} = \frac{1}{s} + \frac{1}{s'} = \frac{1}{\infty} + \frac{1}{s'} = \frac{1}{-0.5\,\text{m}} = -2\,\text{D}$$

(b) The new near point is the object position that places an image 15 cm from the eye:

$$P = -2/\text{m} = \frac{1}{s} + \frac{1}{s'} = \frac{1}{s} - \frac{1}{0.15\,\text{m}} \Rightarrow s = 0.214\,\text{m} = 21.4\,\text{cm}$$

(c) Adjusting the procedure for a lens 2 cm from the eye:

$$P = \frac{1}{f} = \frac{1}{s} + \frac{1}{s'} = \frac{1}{\infty} + \frac{1}{s'} = \frac{1}{-0.48\,\text{m}} = -2.083\,\text{D}$$
$$P = -2.083/\text{m} = \frac{1}{s} + \frac{1}{s'} = \frac{1}{s} - \frac{1}{0.13\,\text{m}} \Rightarrow s = 0.178\,\text{m} = 17.8\,\text{cm}$$

The needed power is -2.083 D and the near point is 17.8 cm from the glasses or 19.8 cm from the eye.

19-14. (a) -1.50 D to correct for myopia; -1.50 D to correct for astigmatism, cylinder axis horizontal

(b) -2.00 D to correct for simple myopia

(c) $+2.00$ D to correct for simple hyperopia

(d) $+2.00$ D to correct for hyperopia; -1.50 D for astigmatism, cylinder axis horizontal

19-15. The unaided far point is the image position for objects at infinity. The unaided near point is the image position for objects at the corrected near point.

(a) **Right eye far point:** $P==\frac{1}{f}-7\,\mathrm{D}=\frac{1}{s}+\frac{1}{s'}=\frac{1}{\infty}+\frac{1}{s'}\Rightarrow s'=f=\frac{1}{P}=\frac{1}{-7\,\mathrm{D}}=-\frac{1}{7}\,\mathrm{m}=-14.3\,\mathrm{cm}$

The unaided far point is 14.3 cm from the eye.

Right eye near point: $P=\frac{1}{f}=-7\,\mathrm{D}=\frac{1}{s}+\frac{1}{s'}=\frac{1}{0.15\,\mathrm{m}}+\frac{1}{s'}\Rightarrow s'=-0.0732\,\mathrm{m}=-7.32\,\mathrm{cm}$

The unaided near point is 7.32 cm from the eye.

Left eye far point: $P=\frac{1}{f}=-5\mathrm{D}=\frac{1}{s}+\frac{1}{s'}=\frac{1}{\infty}+\frac{1}{s'}\Rightarrow s'=f=\frac{1}{P}=\frac{1}{-5\,\mathrm{D}}=-\frac{1}{5}\,\mathrm{m}=-20\,\mathrm{cm}$

The unaided far point is 20 cm from the eye.

Left eye near point: $P=\frac{1}{f}=-5\,\mathrm{D}=\frac{1}{s}+\frac{1}{s'}=\frac{1}{0.15\,\mathrm{m}}+\frac{1}{s'}\Rightarrow s'=-0.0857\,\mathrm{m}=-8.57\,\mathrm{cm}$

The unaided near point is 8.57 cm from the eye.

(b) The corrected far point is the object position that places an image at the unaided far point. The corrected near point point is the object position that places an image at the unaided near point. With the wrong lens, then:

Right eye far point: $P=\frac{1}{f}=-5\,\mathrm{D}=\frac{1}{s}+\frac{1}{s'}=\frac{1}{s}+\frac{1}{-0.143\,\mathrm{m}}\Rightarrow s=0.502\,\mathrm{m}=50.2\,\mathrm{cm}$

The far point with the wrong lens is 50.2 cm.

Right eye near point: $P=\frac{1}{f}=-5\,\mathrm{D}=\frac{1}{s}+\frac{1}{s'}=\frac{1}{s}+\frac{1}{-0.0732\,\mathrm{m}}\Rightarrow s=0.115\,\mathrm{m}=11.5\,\mathrm{cm}$

The near point with the wrong lens is 11.5 cm.

Left eye far point: $P=\frac{1}{f}=-7\,\mathrm{D}=\frac{1}{s}+\frac{1}{s'}=\frac{1}{s}+\frac{1}{-0.20\,\mathrm{m}}\Rightarrow s=-0.5\,\mathrm{m}$

This answer is nonsense as the object position can not be negative for a single lens. This means that the eye must strain to see object at infinity. The relaxed eye will not see anything clearly. So the far point with the wrong lens is infinity but the eye must strain to see the far-away objects.

Left eye near point: $P=\frac{1}{f}=-7\,\mathrm{D}=\frac{1}{s}+\frac{1}{s'}=\frac{1}{s}+\frac{1}{-0.0857\,\mathrm{m}}\Rightarrow s=0.214\mathrm{m}=21.4\,\mathrm{cm}$

The near point with the wrong lens is 21.4 cm.

2D

(a) directly into the sun.

The sun subtends an angle of 0.5 degrees.

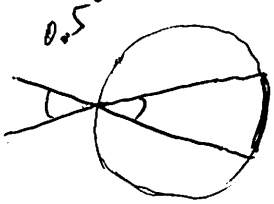
① The radius of the pupil is 1mm, so the area is

$$\pi \times (10^{-3})^2 \ m^2.$$

The intensity entering the eye is

$$P = 1\times10^3 \ W/m^2 \times \pi \times 10^{-6} \ m^2$$
$$\approx 3.1 \ mW$$

② The ~~diameter of~~ diameter of the image of the sun is about



$$22.5 \ mm \times \frac{0.5}{57.3} \ (\text{radians}) \approx 0.20 \ mm$$

(neglecting the refractive in the eye is ~ 1.34)

The area of the image is $\pi\times(0.1)^2 \ mm^2$

$$= \pi \times 10^{-8} \ m^2$$

The intensity at the eye retina is then about

$$I \approx \frac{3.1\times10^{-3} \ W}{\pi\times10^{-8} \ m^2} \sim 10^5 \ W/m^2$$
$$= 10 \ W/cm^2$$

(b) Into a 1mW He-Ne

The beam ~~spot~~ waist is about

$$W_0 \sim \frac{f_{eye}\times\lambda}{\pi \ W_{pupil}} \sim \frac{2.25 \ cm \times 0.632\times10^{-4} \ cm}{\pi \times 0.1 \ cm}$$
$$\sim 4.5\times10^{-4} \ cm$$

The average intensity at the retina is about

$$I_{ave} \sim \frac{P}{2\pi W_0^2} = \frac{10^{-3} \ W}{2\pi\times(4.5\times10^{-4} \ cm)^2} \approx 790 \ W/cm^2$$

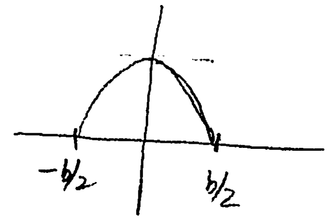
(c) Eye-damaging intensity is ~ 10 mW/cm². Both will damage your eye. But the LASER will cause MORE SERIOUS damage.

4. Optical Apodization

For far-field diffraction

$$U(x) \propto \int U(\xi)\, e^{-j2\pi f_x \xi}\, d\xi \qquad f_x = \frac{x}{\lambda z}$$

$$U(\xi) = \begin{cases} 0 & \xi < -b/2 \text{ or } \xi > b/2 \\ \cos\frac{\pi\xi}{b} & -b/2 < \xi < b/2 \end{cases}$$



Use $\boxed{\frac{e^{i\theta}+e^{-i\theta}}{2} = \cos\theta}$

$$U(x) \propto \int_{-b/2}^{b/2} \cos\frac{\pi\xi}{b}\, e^{-j2\pi f_x \xi}\, d\xi$$

$$= \int_{-b/2}^{b/2} \frac{e^{j\frac{\pi\xi}{b}} + e^{-j\frac{\pi\xi}{b}}}{2}\, e^{-j2\pi f_x \xi}\, d\xi$$

$$= \frac{1}{2}\int_{-b/2}^{b/2} \left(e^{j\pi(\frac{1}{b}-2f_x)\xi} + e^{-j\pi(\frac{1}{b}+2f_x)\xi} \right) d\xi$$

$$= \frac{1}{2} \left\{ \frac{e^{j\pi(\frac{1}{b}-2f_x)\xi}}{j\pi(\frac{1}{b}-2f_x)} \Bigg|_{-b/2}^{b/2} + \frac{e^{-j\pi(\frac{1}{b}+2f_x)\xi}}{-j\pi(\frac{1}{b}+2f_x)} \Bigg|_{-b/2}^{b/2} \right\}$$

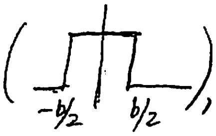
$$= \frac{1}{2} \left\{ \frac{e^{j\frac{\pi}{2}} \cdot e^{-j\pi f_x b} - e^{-j\frac{\pi}{2}} \cdot e^{j\pi f_x b}}{j\pi(\frac{1}{b}-2f_x)} + \frac{e^{-j\frac{\pi}{2}} e^{-j\pi f_x b} - e^{j\pi/2} \cdot e^{j\pi f_x b}}{-j\pi(\frac{1}{b}+2f_x)} \right\}$$

$$= \frac{1}{2} \left\{ \frac{\cancel{j} e^{-j\pi f_x b} - (-j) e^{j\pi f_x b}}{\cancel{j}\pi(\frac{1}{b}-2f_x)} + \frac{\cancel{-j} e^{-j\pi f_x b} \cancel{-j} e^{j\pi f_x b}}{\cancel{-j}\pi(\frac{1}{b}+2f_x)} \right\}$$

$$= - \frac{\cos(\pi f_x b)}{\pi(\frac{1}{b}-2f_x)} + \frac{\cos(\pi f_x b)}{\pi(\frac{1}{b}+2f_x)}$$

$$= \cos(\pi b f_x) \left[\frac{1}{\pi(\frac{1}{b}-2f_x)} + \frac{1}{\pi(\frac{1}{b}+2f_x)} \right]$$

zeros $\Rightarrow \pi b f_x = \frac{\pi b x_0}{\lambda z} = (n+\frac{1}{2})\pi$, $n \in 1, 2, 3, \cdots$

compare to simple $U(\xi) = \mathrm{rect}(\xi/b)$ (), $U(x) \propto \mathrm{sinc}\left(\frac{\pi b x}{\lambda z}\right)$

zeros $\Rightarrow \frac{\pi b x_0'}{\lambda z} = n\pi$, $n \in 1, 2, 3 \cdots$

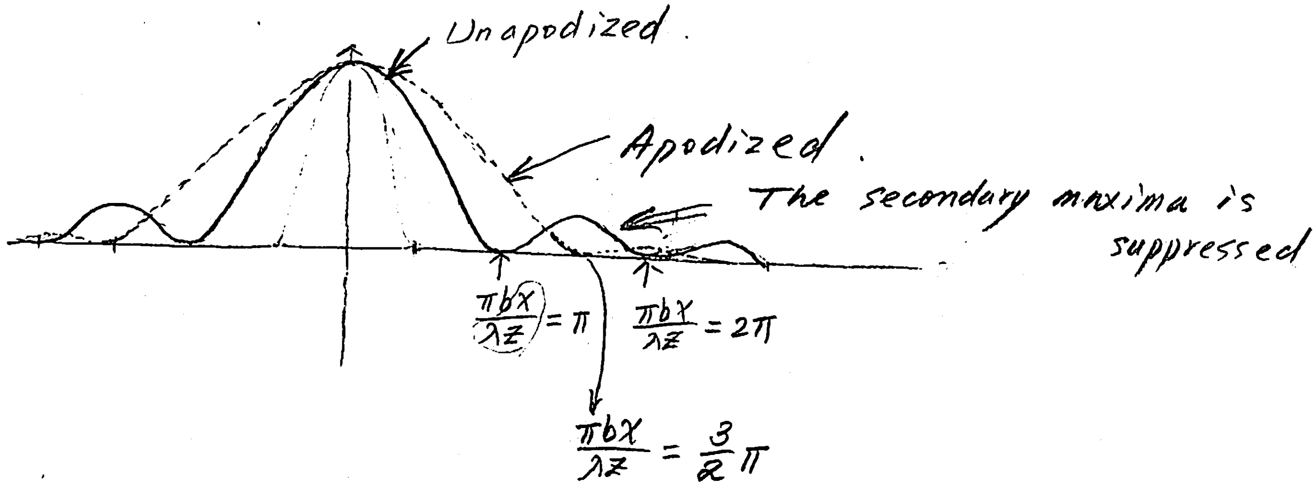
$\therefore$ The shift of zeros is $\left| \frac{\pi b x_0}{\lambda z} - \frac{\pi b x_0'}{\lambda z} \right| = \frac{1}{2}\pi$

Apodized

$$U(x) \propto \cos(\pi b f_x)\left[\frac{1}{\pi(\frac{1}{b}-2f_x)} + \frac{1}{\pi(\frac{1}{b}+2f_x)}\right]$$

Unapodized

$$U(x) \propto b\,\mathrm{sinc}(\pi b f_x) = b\left(\frac{\sin(\pi b f_x)}{\pi b f_x}\right)$$



"Apodization refers to the process of supressing the secondary maxima (side lobes) or feet of a diffraction pattern."

Compared to a regular slit, the cosine transmission indeed suppresses the secondary and higher order maxima (side lobes) though the central peak is broadened.

Gaussian Beam   Solutions of HW #10   C.W. Lai

A. $\theta_d \approx \frac{\lambda}{\pi W_0} = \frac{0.6328\,\mu m}{\pi \times 500\,\mu m} = 0.403 \times 10^{-3}\ \text{rad} = 0.403\ \text{mrad}$

B. $z_R = \frac{\pi W_0^2}{\lambda} = \frac{\pi \times (0.5mm)^2}{0.6328\,\mu m} = \frac{\pi\ (0.5mm)^2}{0.6328 \times 10^{-3}\,mm} = 1.241 \times 10^3\ mm$

$= 1.241$ meters

For $z = 50cm$

$W(lens) = W_0 \sqrt{1 + (z/z_R)^2} = 0.5(mm)\sqrt{1+\left(\frac{0.5}{1.241}\right)^2} \approx 0.539\ mm$

$R = z[1 + (z_R/z)^2] = 3.58016\ m = 358.016\ cm$

Note's convention

C. $\frac{1}{R'} = \frac{1}{f} + \frac{1}{R} = \frac{1}{10\,cm} + \frac{1}{-358.016\,cm}$

$R' = 10.287\ cm$

If you follow the convention on lecture notes, don't forget '-' sign for radius of curvature of the beam before the lens.

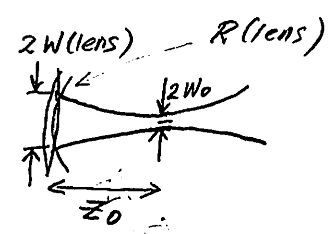
Another convention

$\frac{1}{R'} + \frac{1}{R} = \frac{1}{f} \qquad \frac{1}{R'} + \frac{1}{358.016} = \frac{1}{f}$ same results

(R is positive for object)

D. $W_0 \approx \frac{R_0 \lambda}{\pi W(lens)} = \frac{(10.287)[cm] \times 0.6328(\mu m)}{\pi \times 0.539\,mm} = 38.44\ \mu m$

$\begin{cases} R_0 = R' = 10.287\ cm \\ W(lens) = 0.539\ mm \end{cases}$



Derivation of $W_0 \approx \frac{R_0 \lambda}{\pi W(lens)}$

$\begin{cases} R(lens) = R(z_0) = z_0\left[1 + \left(\frac{z_R}{z_0}\right)^2\right] = z_0\left[1 + \left(\frac{\pi W_0^2}{\lambda z_0}\right)^2\right] \\ W(lens) = W(z_0) = W_0 \times \sqrt{1 + \left(\frac{z_0}{z_R}\right)^2} = W_0\sqrt{1 + \left(\frac{\lambda z_0}{\pi W_0^2}\right)^2} \end{cases}$

Because $R(lens)$ and $W(lens)$ are known, in principle, one can solve z_0 and W_0.

Use $\frac{z_R}{z_0} \ll 1$ will further simply the problem

$\begin{cases} R(z_0) \approx z_0 \\ W(z_0) \approx W_0 \times \frac{z_0}{z_R} \end{cases} \quad \therefore \quad \frac{R(z_0)}{W(z_0)} \approx \frac{z_R}{W_0} \approx \frac{\pi W_0}{\lambda} \Rightarrow W_0 \approx \frac{\lambda R_0}{\pi W(lens)}$

$$z_R = \frac{\pi W_0^2}{\lambda} = \frac{\pi \times (38.44\,\mu m)^2}{0.6328\,\mu m} = 7335.9\,\mu m \approx 7.336\,mm$$

E.

$$W(z=1m) = W_0 \times \sqrt{1+(z/z_R)^2}$$

$$\approx 38.44\,\mu m \times \sqrt{1+\left(\frac{10^3 mm}{7.336 mm}\right)^2}$$

$$\approx \boxed{5.24\ mm}$$

$$\theta_d \approx \frac{\lambda}{\pi W_0} = \frac{0.6328\,\mu m}{\pi \times 38.44\,\mu m} \approx 5.2\ mrad$$