Physics 431 Final Exam

THU, DECEMBER 15, 2011

3:00 – 5:00 P.M.

BPS 1308

- **Calculators, TWO letter-size sheets "handwritten notes" → OK**
- **Graded lab reports, Textbook, Handouts, Lecture Notes/Slides → OK**
- **HW solutions (allowed if written on TWO note sheets), laptops, smartphones, or similar electronic devices → NO**

The exam includes topics covered throughout the semester

The exam consists of problems totaling 250 pts.

Show all work on exam pages

Grades will be posted at BPS 4238 & Angel by 12 pm **Monday, December 19.**

Remember your "pass code" from the final exam if you intend to check your grades in my office.

Final exam problems are NOT limited to the following slides.
Check "Final Exam Topics". The exam is accumulative and covers all materials.
Review the example final exam and HW assignments.

Source: “Optical Physics”, Lipson^3

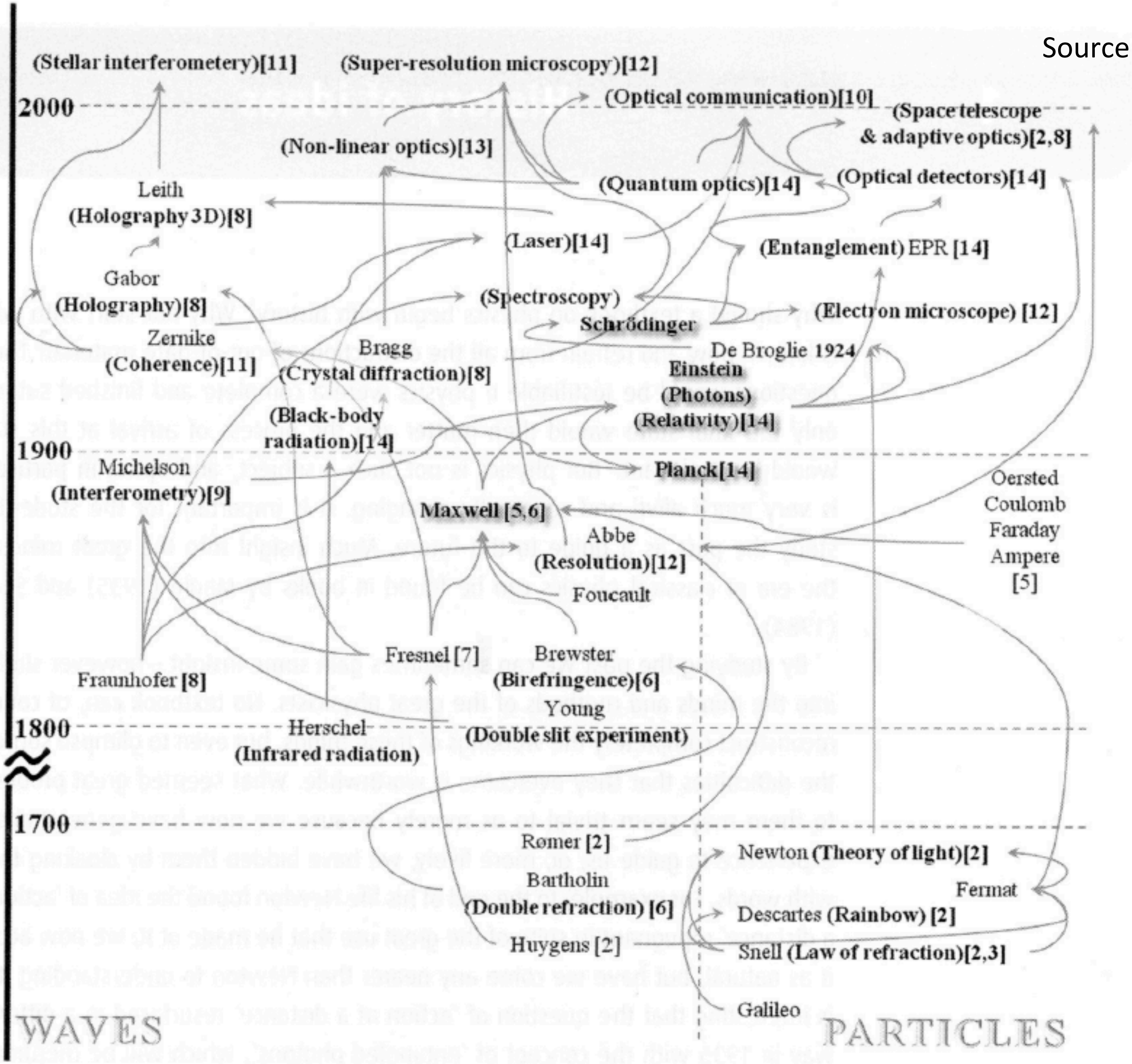


Figure 1.1 The development of optics, showing many of the interactions. Notice that there was little development in the eighteenth century, mainly because of Newton's erroneous idea of light particles. The numbers in square brackets indicate the chapters where the topics are discussed.

Index of refraction and 'speed' of light

The speed of light in vacuum is a physical constant.

$c = 299\ 792\ 458$ m/s (exact) ~ 3×10^8 m/s

In a medium, light *generally* propagates more slowly.

- in air: $v = c/1.0003$ $n_{air} = 1.00$
- in water: $v = c/1.33$ $n_{water} = 1.33$
- in glass: $v = c/1.52$ $n_{glass} = 1.52$

In general:

$v = c/n$ is the "***phase velocity***"

wavelength/n

frequency is the same (in linear optics)

n also depends on the wavelength → **dispersion**.

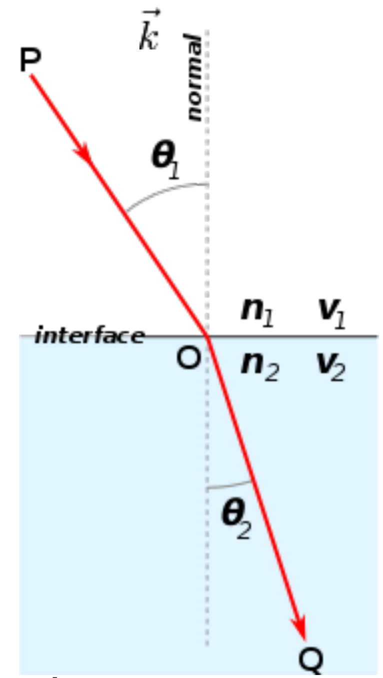
Snell's Law

$$n_1 \sin\theta_1 = n_2 \sin\theta_2 \ .$$

$$\frac{\sin\theta_1}{\sin\theta_2} = \frac{v_1}{v_2} = \frac{n_2}{n_1}$$

$$\frac{\sin\theta_1}{\sin\theta_2} = \frac{v_1}{v_2} = \frac{\lambda_1}{\lambda_2}$$

Frequency is the same.



1. Huygens' principle
2. Fermat's principle
3. Interference of all possible paths of light wave from source to observer — it results in destructive interference everywhere except extrema of phase (where interference is constructive)—which become actual paths.

4. Application of the general **boundary conditions of Maxwell equations** for electromagnetic radiation. ➔ amplitude of reflected and refractive waves [Chapter 23]

5. Conservation of momentum based on translation symmetry considerations

Derive Snell's Law by Translation Symmetry

A *homogeneous* surface can not change the transverse momentum.
The propagation vector is proportional to the photon's momentum.

$$E = pc$$

$$p = \hbar k$$

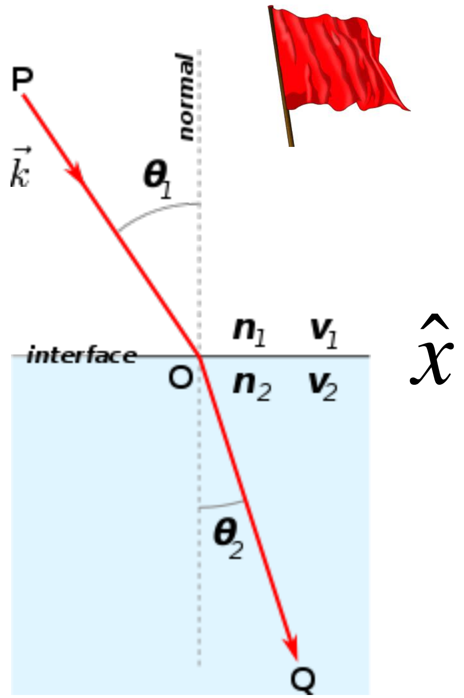
The transverse wave number must remain the same.

$$\vec{k}_1 \bullet \hat{x} = \vec{k}' \bullet \hat{x} = \vec{k}_2 \bullet \hat{x}$$

$$k_1 \sin\theta_1 = k' \sin\theta' = k_2 \sin\theta_2$$

$$k_1 = k' = \frac{2\pi}{\lambda_1} = \frac{2\pi}{\lambda_0 / n_1} = \frac{2\pi}{\lambda_0} n_1 = k_0 n_1$$

$$k_2 = \frac{2\pi}{\lambda_2} = \frac{2\pi}{\lambda_0} n_2$$

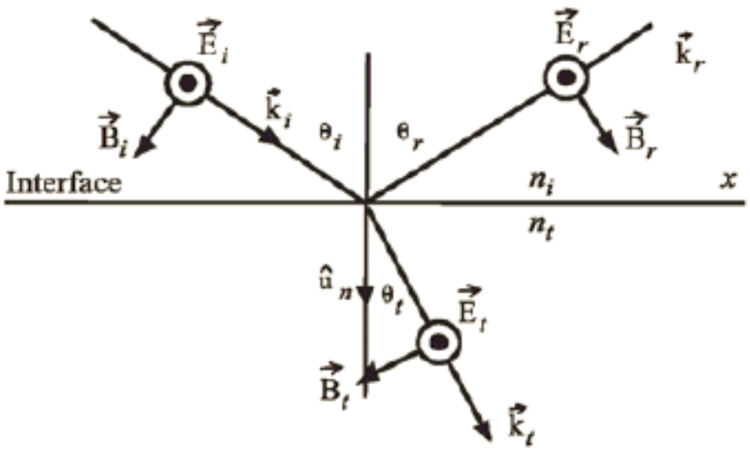


$n_1 k_0 \sin\theta_1 = n_2 k_0 \sin\theta_2$
$n_1 \sin\theta_1 = n_2 \sin\theta_2$

$$k_0 = \frac{2\pi}{\lambda_0} = \frac{\omega}{c}$$

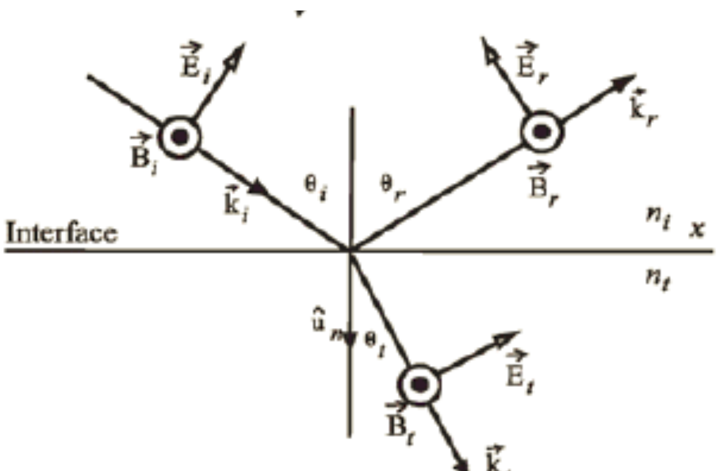
Reflection and Transmission (Fresnel’s equations)

Can be deduced from the application of boundary conditions of EM waves.



$$r_\perp = \left(\frac{E_{0r}}{E_{0i}}\right)_\perp = \frac{n_i \cos\theta_i - n_t \cos\theta_t}{n_i \cos\theta_i + n_t \cos\theta_t}$$

$$t_\perp = \left(\frac{E_{0t}}{E_{0i}}\right)_\perp = \frac{2n_i \cos\theta_i}{n_i \cos\theta_i + n_t \cos\theta_t}$$



$$r_\parallel = \left(\frac{E_{0r}}{E_{0i}}\right)_\parallel = \frac{n_t \cos\theta_i - n_i \cos\theta_t}{n_t \cos\theta_i + n_i \cos\theta_t}$$

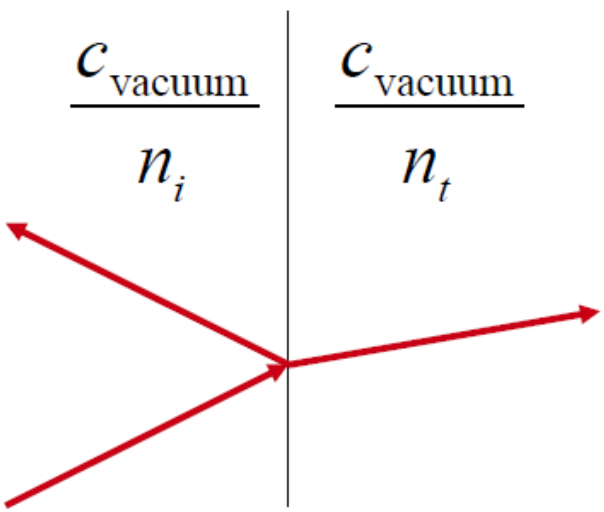
$$t_\parallel = \left(\frac{E_{0t}}{E_{0i}}\right)_\parallel = \frac{2n_i \cos\theta_t}{n_t \cos\theta_i + n_i \cos\theta_t}$$

Reflection and Transmission of Energy @ dielectric interfaces

Recall Poynting vector definition:

$$\|\mathbf{S}\| = c\,\varepsilon_0 \|\mathbf{E}\|^2$$

different on the two sides of the interface

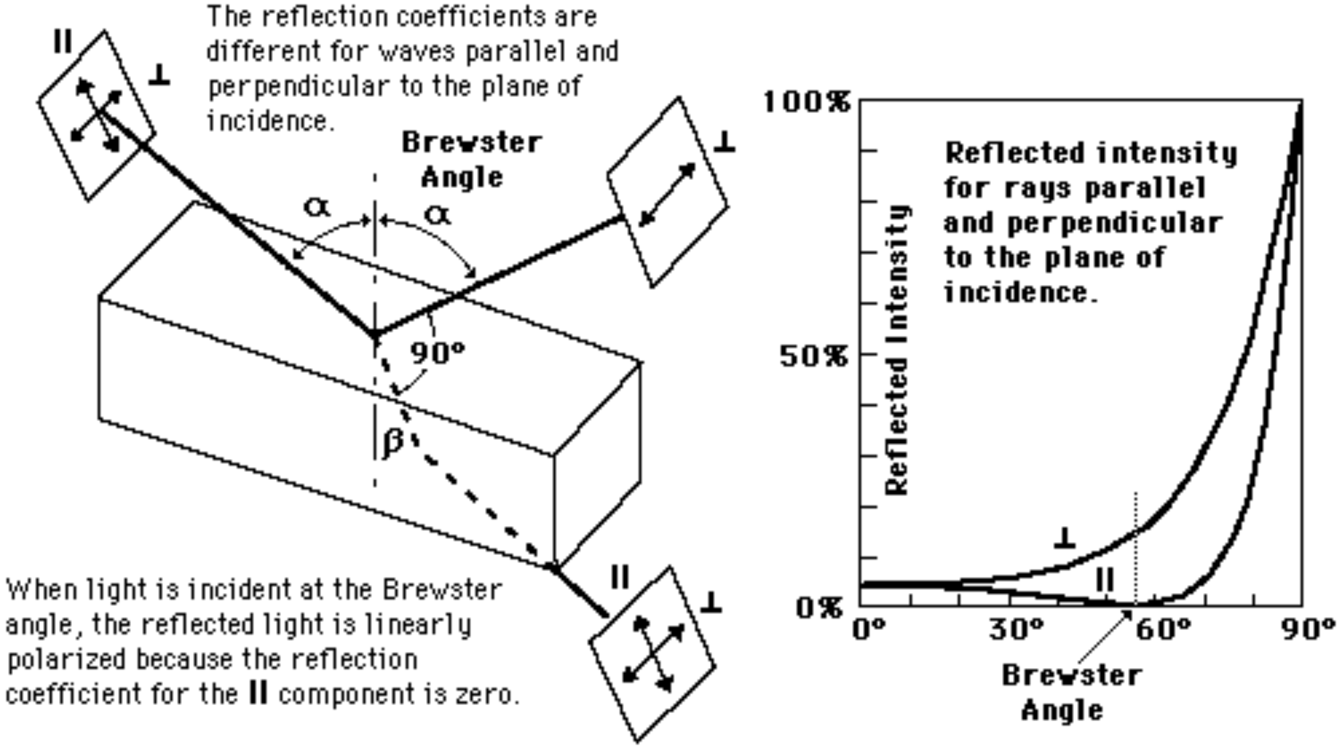


$$R = \left(\frac{E_{0r}}{E_{0i}}\right)^2 = r^2$$

$$T = \frac{n_t \cos\theta_t}{n_i \cos\theta_i}\left(\frac{E_{0t}}{E_{0i}}\right)^2 = \frac{n_t \cos\theta_t}{n_i \cos\theta_i} t^2$$

Polarization by Reflection

http://hyperphysics.phy-astr.gsu.edu/hbase/phyopt/polar.html



Polarization by scattering (Rayleigh scattering/Blue Sky)

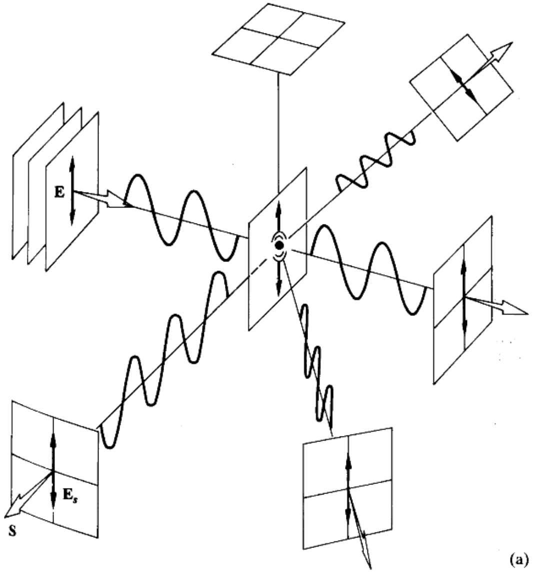


FIGURE 8.35a Scattering of polarized light by a molecule.

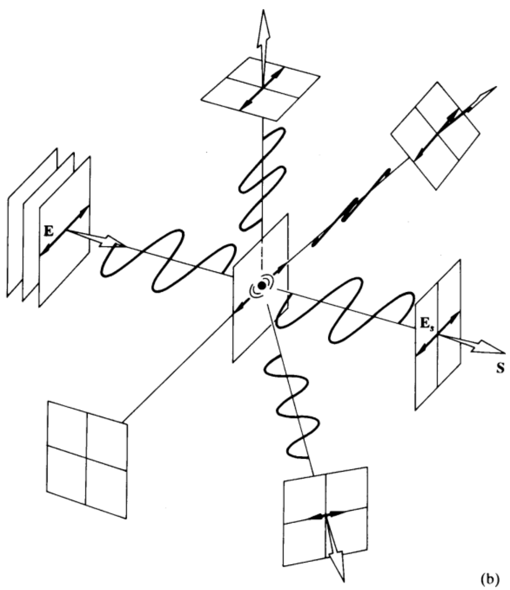


FIGURE 8.35b

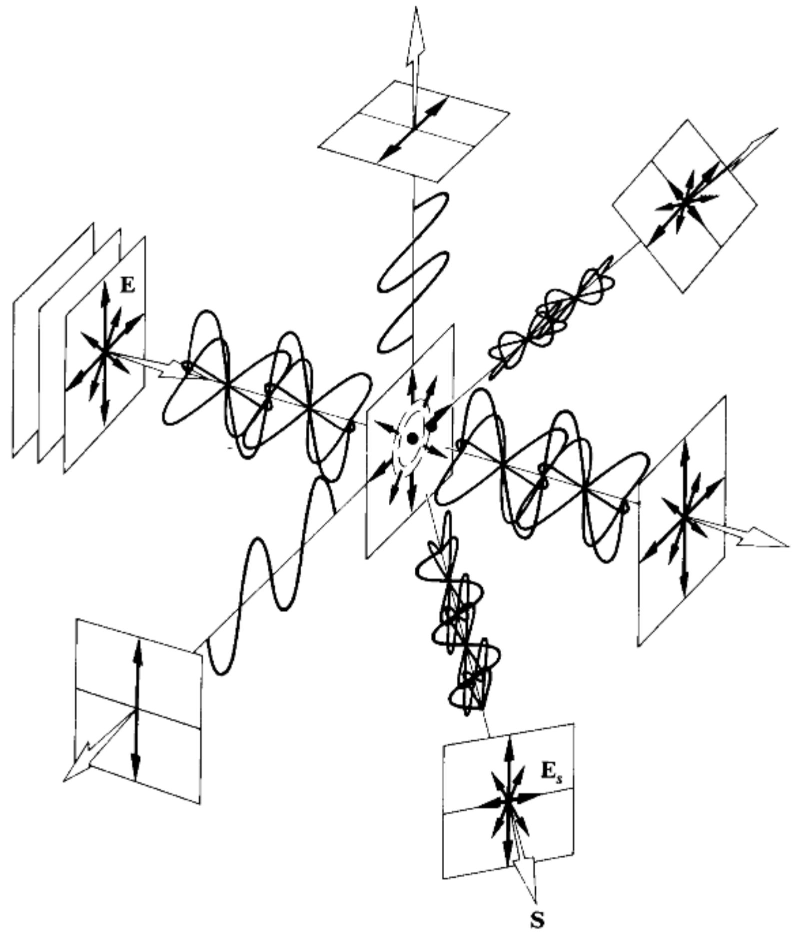


FIGURE 8.36 Scattering of unpolarized light by a molecule.

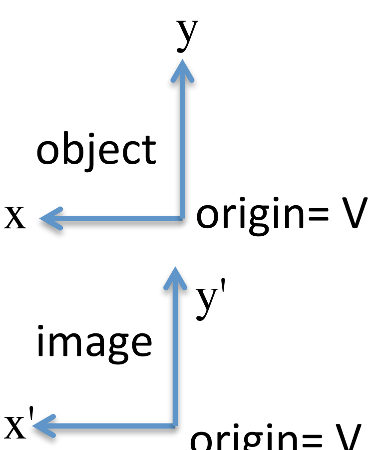
Ray Diagrams for Spherical Mirrors

$$\frac{1}{s}+\frac{1}{s'}=\frac{1}{f}, \quad f=-\frac{R}{2}$$

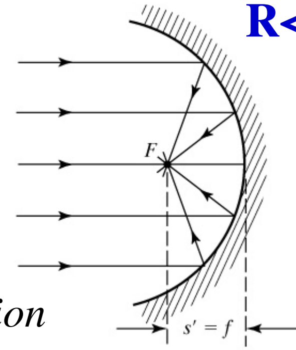
$$m=-\frac{s'}{s} \quad \textit{lateral magnification}$$

$$\frac{1}{s_o}+\frac{1}{s_i}=\frac{1}{f}, \quad f=-\frac{R}{2}$$

$$m=\frac{h_i}{h_o}=-\frac{s_i}{s_o}$$

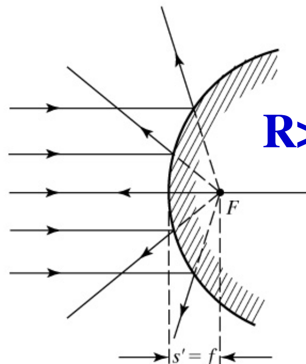


R<0

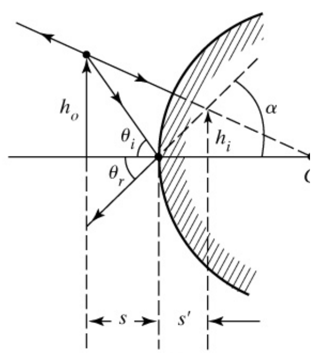


(a)

R>0

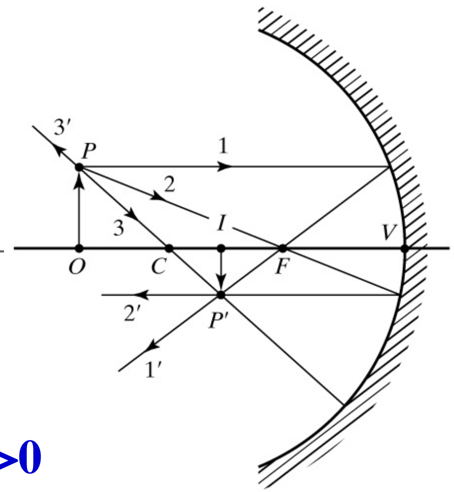


(b)



(c)

Ray Diagrams



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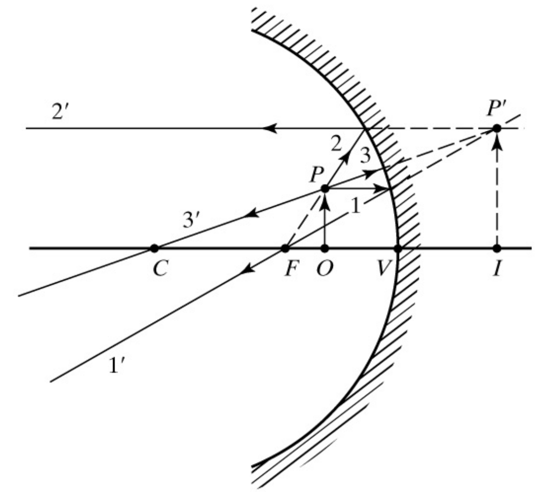

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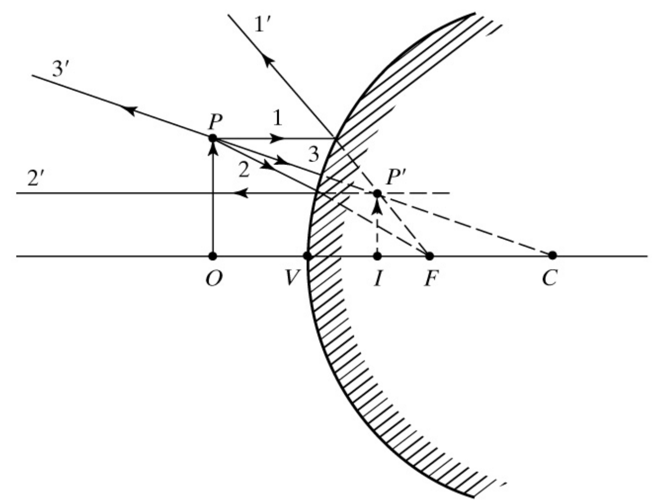
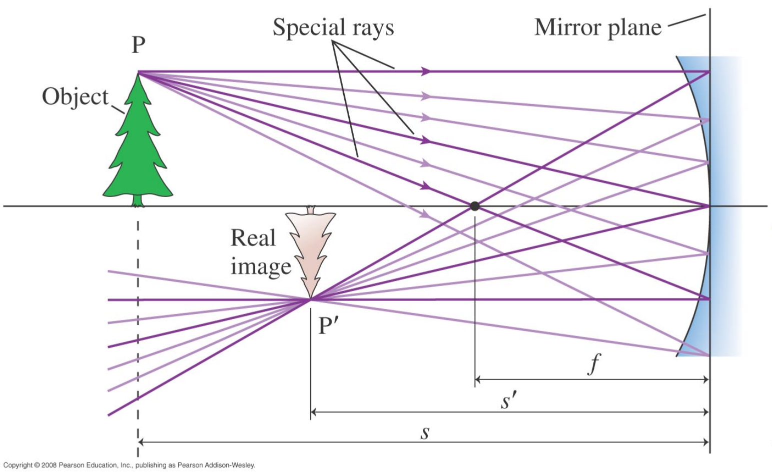
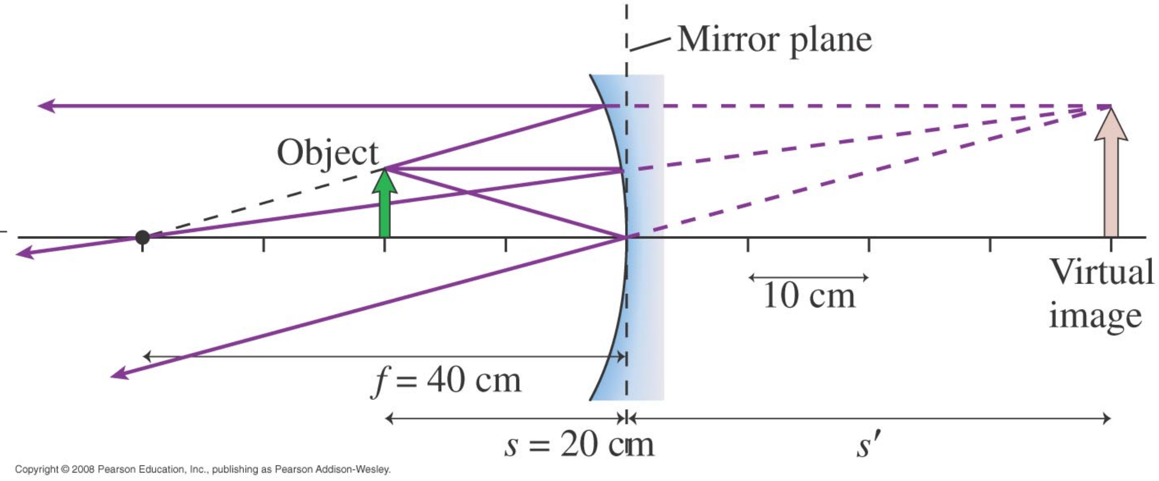
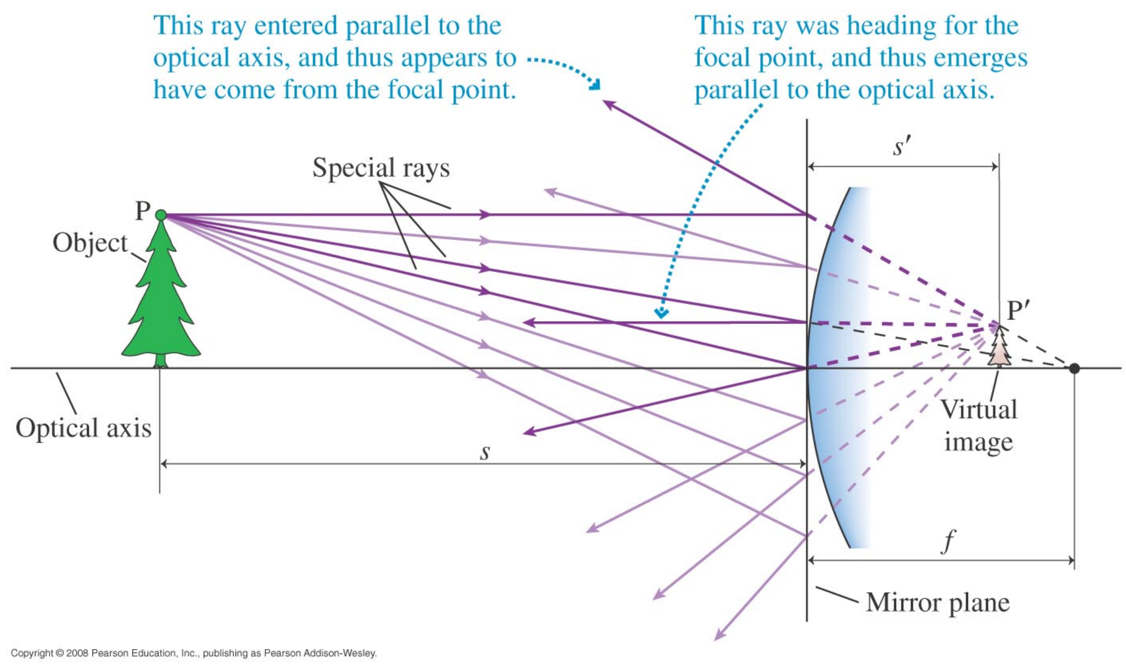


(c)

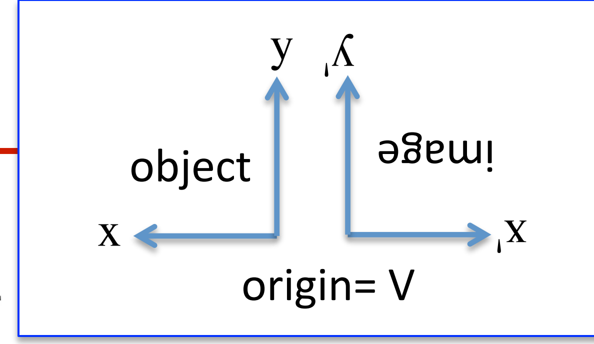
Imaging Formation by a Mirror

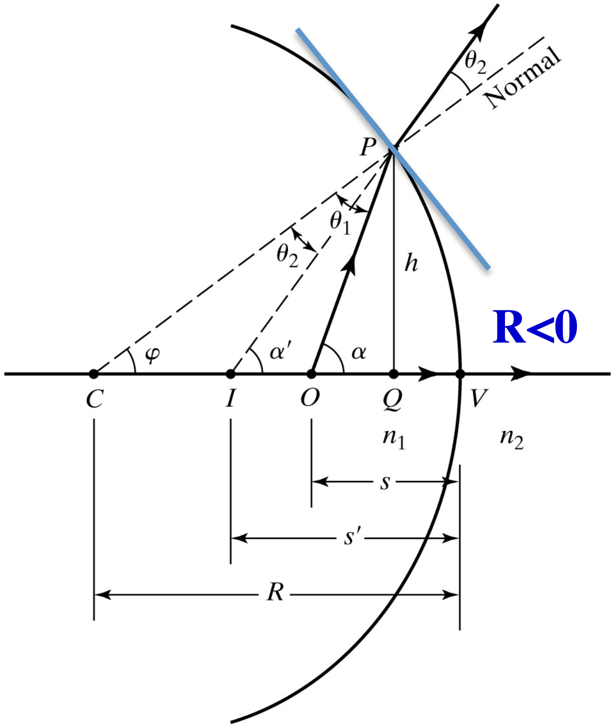


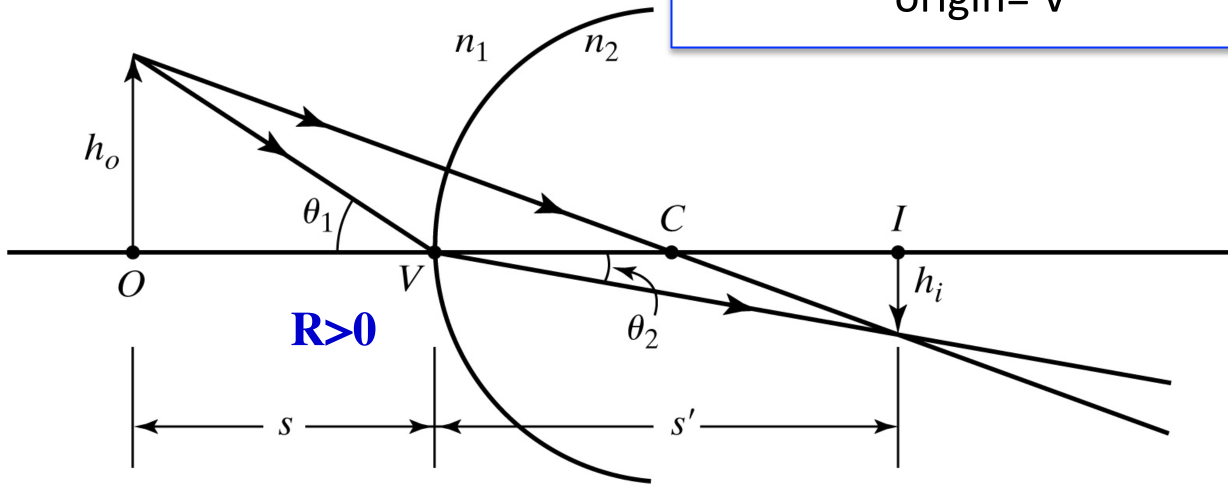




Refraction at a Spherical Surface







$$\frac{n_1}{s}+\frac{n_2}{s'}=\frac{n_2-n_1}{R}$$

$$m=\frac{h_i}{h_o}=-\frac{n_1 s'}{n_2 s}$$

When $R\to\infty$ (i.e. a plane surface)

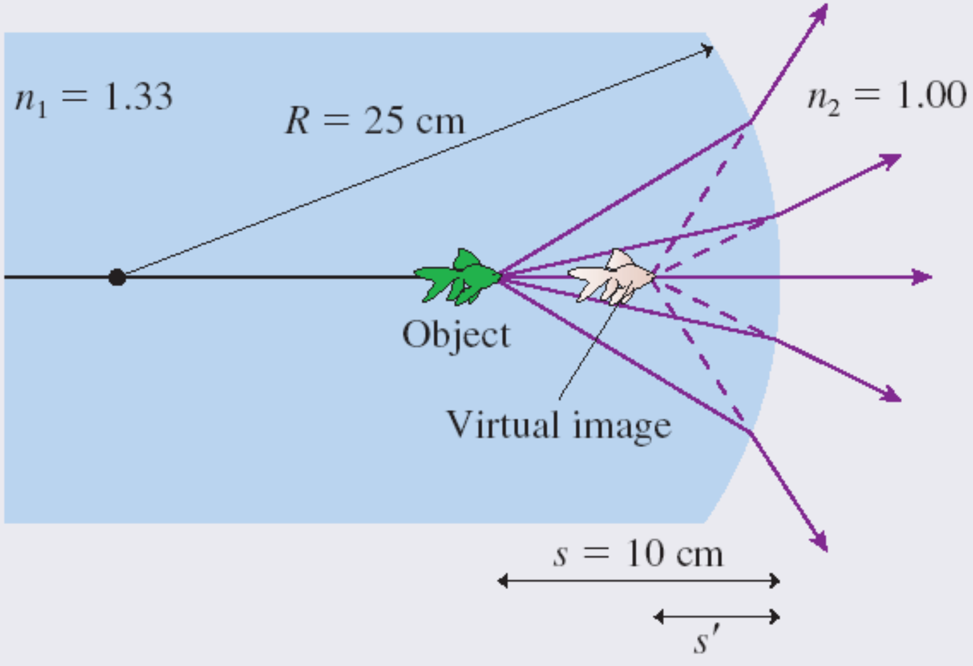
$$s'=-\left(\frac{n_2}{n_1}\right)s$$

$$m=+1$$

EXAMPLE A goldfish in a bowl

VISUALIZE **FIGURE 23.46** shows the rays refracting *away* from the normal as they move from the water into the air. We expect to find a virtual image at a distance less than 10 cm.

FIGURE 23.46 The curved surface of a fish bowl produces a virtual image of the fish.



EXAMPLE A goldfish in a bowl

SOLVE The object is in the water, so $n_1 = 1.33$ and $n_2 = 1.00$. The inner surface is concave (you can remember "concave" because it's like looking into a cave), so $R = -25$ cm. The object distance is $s = 10$ cm. Thus Equation 23.21 is

$$\frac{1.33}{10\ \text{cm}} + \frac{1.00}{s'} = \frac{1.00 - 1.33}{-25\ \text{cm}} = \frac{0.33}{25\ \text{cm}}$$

Solving for the image distance s' gives

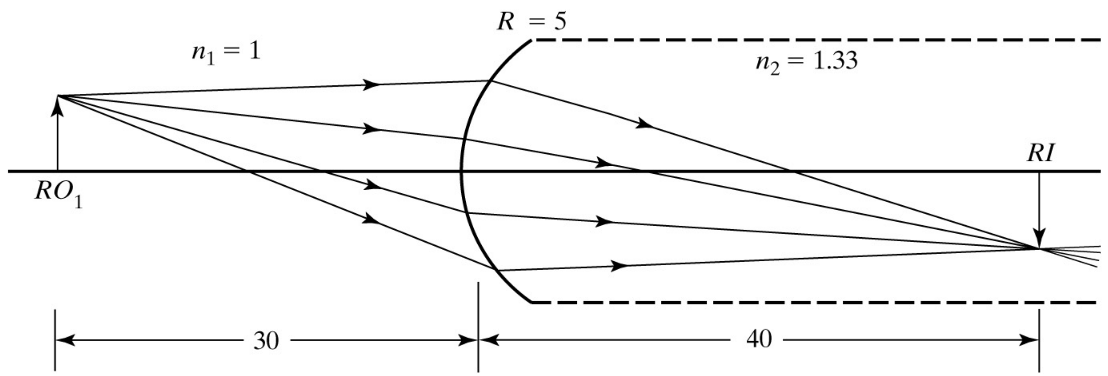
$$\frac{1.00}{s'} = \frac{0.33}{25\ \text{cm}} - \frac{1.33}{10\ \text{cm}} = -0.12\ \text{cm}^{-1}$$

$$s' = \frac{1.00}{-0.12\ \text{cm}^{-1}} = -8.3\ \text{cm}$$

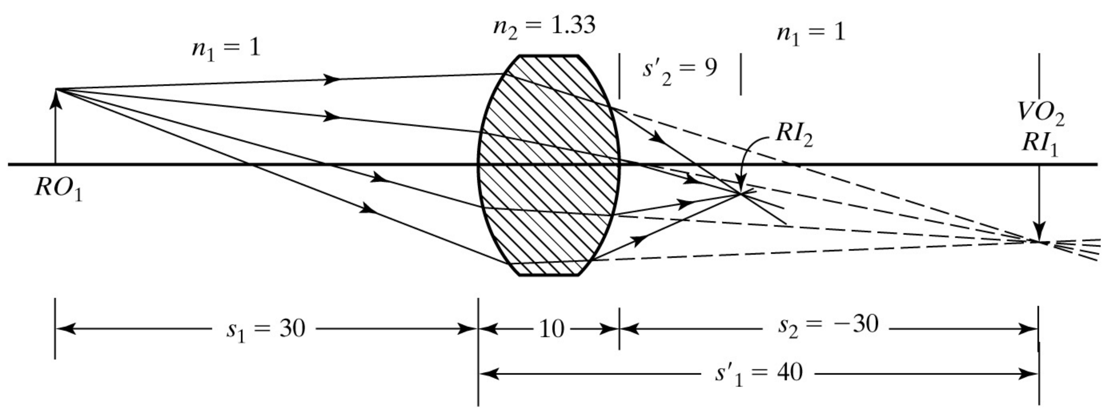
ASSESS The image is virtual, located to the left of the boundary. A person looking into the bowl will see a fish that appears to be 8.3 cm from the edge of the bowl.

Example of refraction by spherical surfaces

Explain the imaging formation by a cylinder filled with water.
Be specific (i.e. use 'real' values'), See Example 2-2 (page 34)

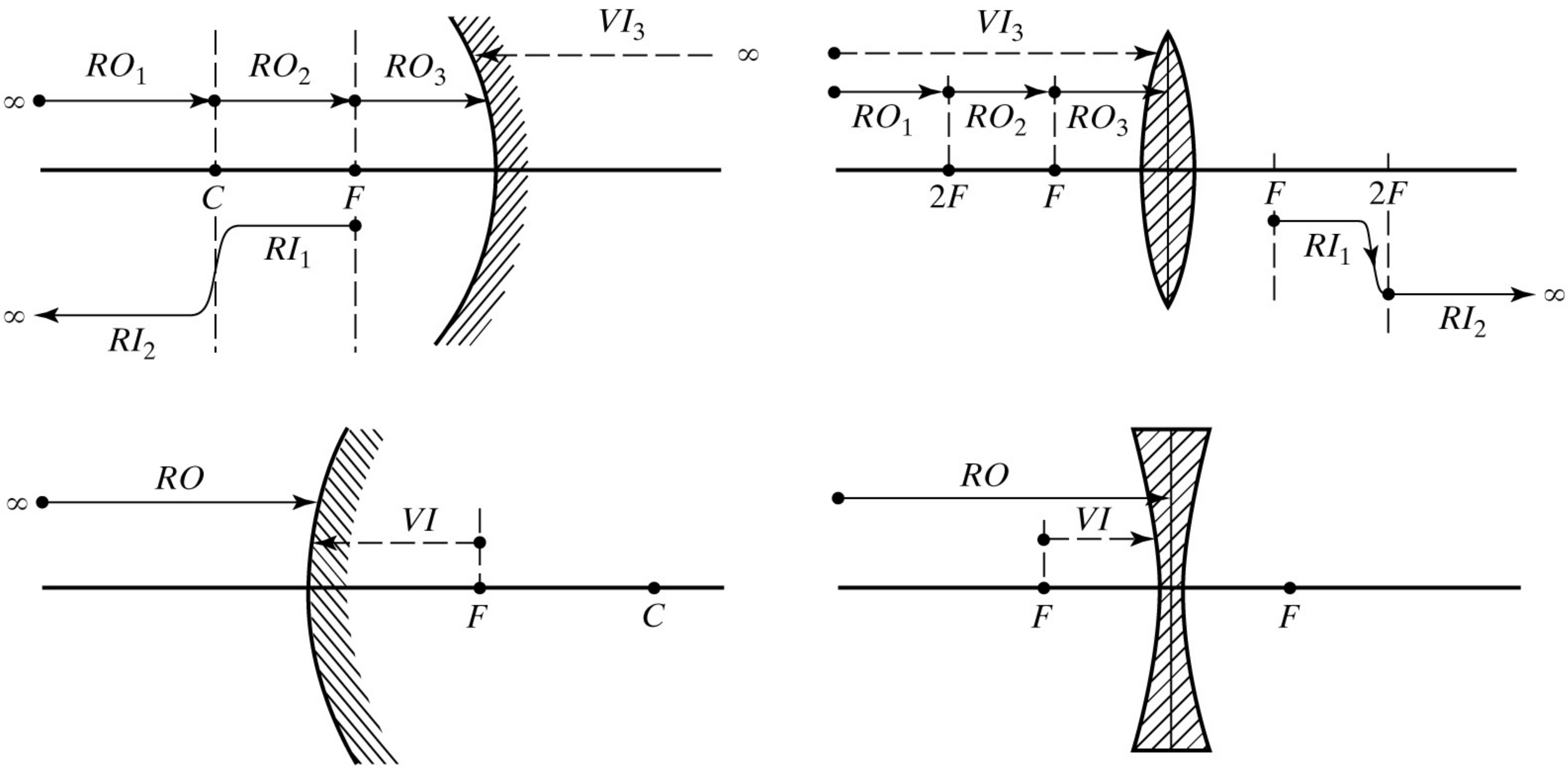


(a)

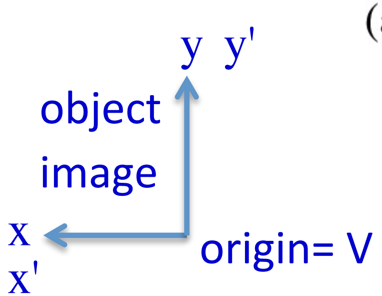


(b)

Summary of Image Formation: Spherical Mirrors & Thin Lenses



(a) (b)



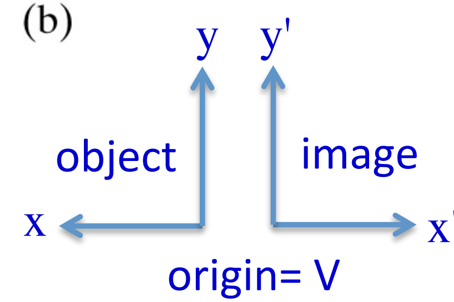
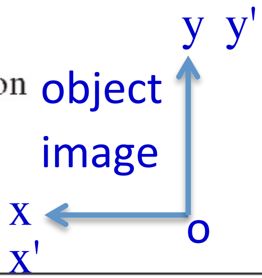
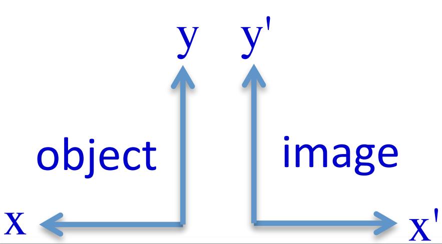
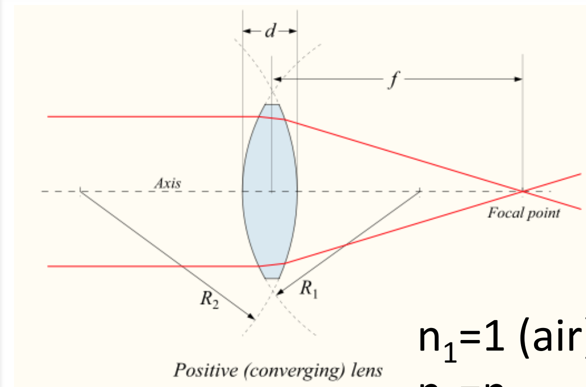


TABLE 2-1 SUMMARY OF GAUSSIAN MIRROR AND LENS FORMULAS

	Spherical surface	Plane surface
Reflection	$\frac{1}{s}+\frac{1}{s'}=\frac{1}{f},\ f=-\frac{R}{2}$	$s'=-s$
	$m=-\frac{s'}{s}$	$m=+1$
	Concave: $f>0,\ R<0$	
	Convex : $f<0,\ R>0$	
Refraction Single surface	$\frac{n_1}{s}+\frac{n_2}{s'}=\frac{n_2-n_1}{R}$	$s'=-\frac{n_2}{n_1}s$
	$m=-\frac{n_1 s'}{n_2 s}$	$m=+1$
	Concave: $R<0$	
	Convex : $R>0$	
Refraction Thin lens	$\frac{1}{s}+\frac{1}{s'}=\frac{1}{f}$	
	$\frac{1}{f}=\frac{n_2-n_1}{n_1}\left(\frac{1}{R_1}-\frac{1}{R_2}\right)$	the lensmaker's formula
	$m=-\frac{s'}{s}$	
	Concave: $f<0$	
	Convex : $f>0$	







n_1=1 (air)

n_2=n

The lensmaker's formula

$$\frac{1}{f}=(n-1)\left[\frac{1}{R_1}-\frac{1}{R_2}+\frac{(n-1)d}{nR_1R_2}\right]$$

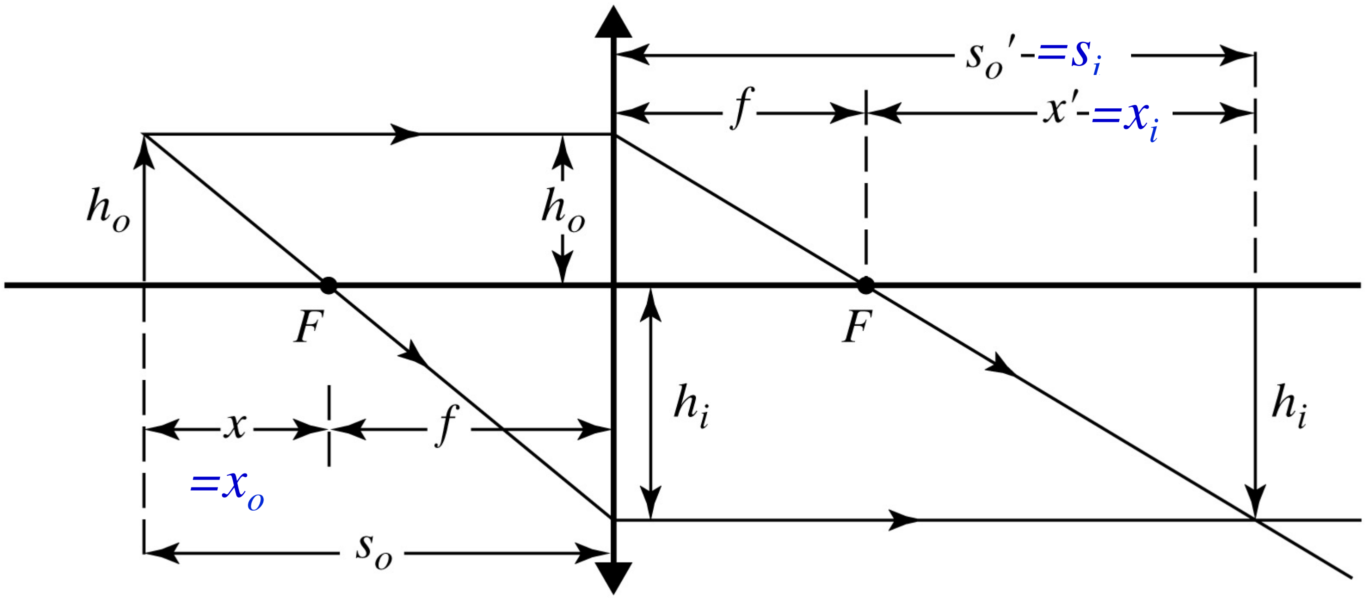
A "Thin" lens➔ d is negligible

$$\frac{1}{f}\approx(n-1)\left[\frac{1}{R_1}-\frac{1}{R_2}\right].$$

The refractive power of a lens of focal length f

$$D\,[\text{diopters}]=\frac{1}{f\,[\text{in meters}]}$$

Newtonian Equation for the Thin Lens



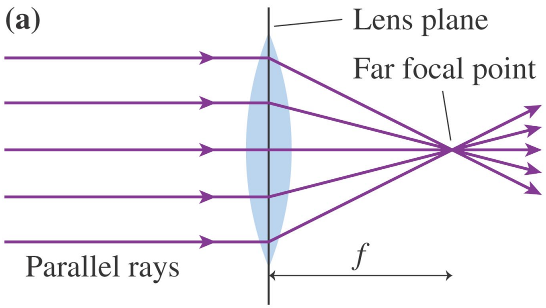
$$\frac{1}{f} = \frac{1}{s_0} + \frac{1}{s_i}$$

$$x_0 x_i = f^2$$

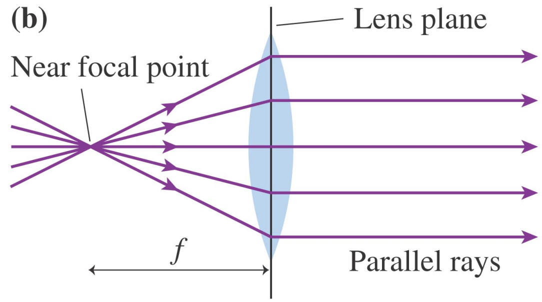
$$M_L \equiv \frac{y_i}{y_0} = -\frac{s_i}{s_o}$$ *lateral magnification*

$$M_T \equiv \frac{dx_i}{dx_0} = -\frac{f^2}{x_0^2}$$ *transverse magnification*

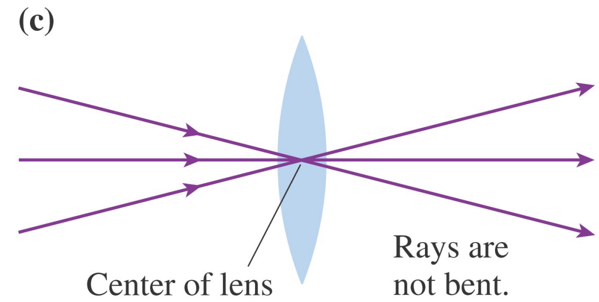
Major Rays



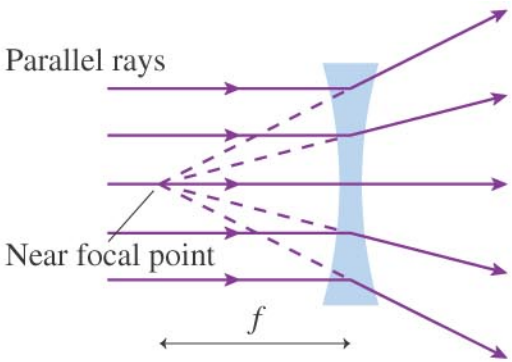
Any ray initially parallel to the optical axis will refract through the focal point on the far side of the lens.



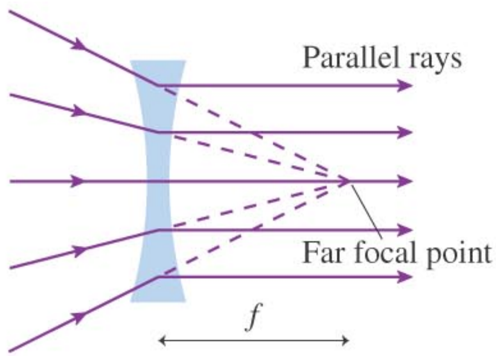
Any ray passing through the near focal point emerges from the lens parallel to the optical axis.



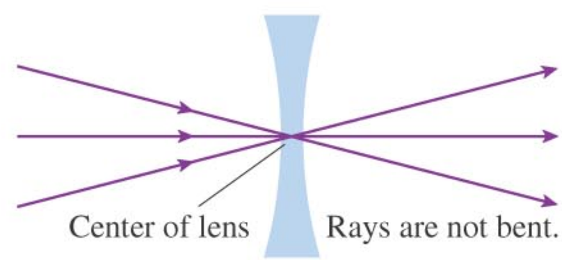
Any ray directed at the center of the lens passes through in a straight line.



Any ray initially parallel to the optical axis diverges along a line through the near focal point.

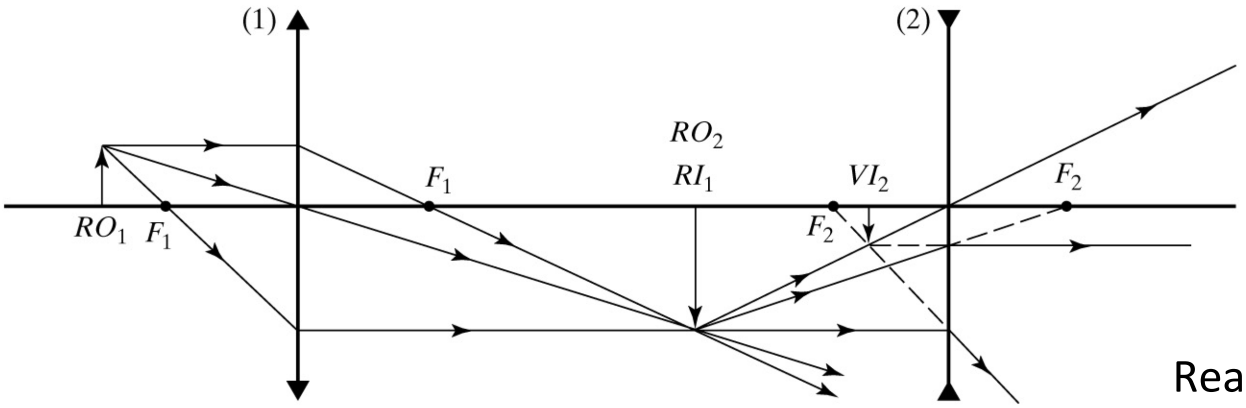


Any ray directed along a line toward the far focal point emerges from the lens parallel to the optical axis.

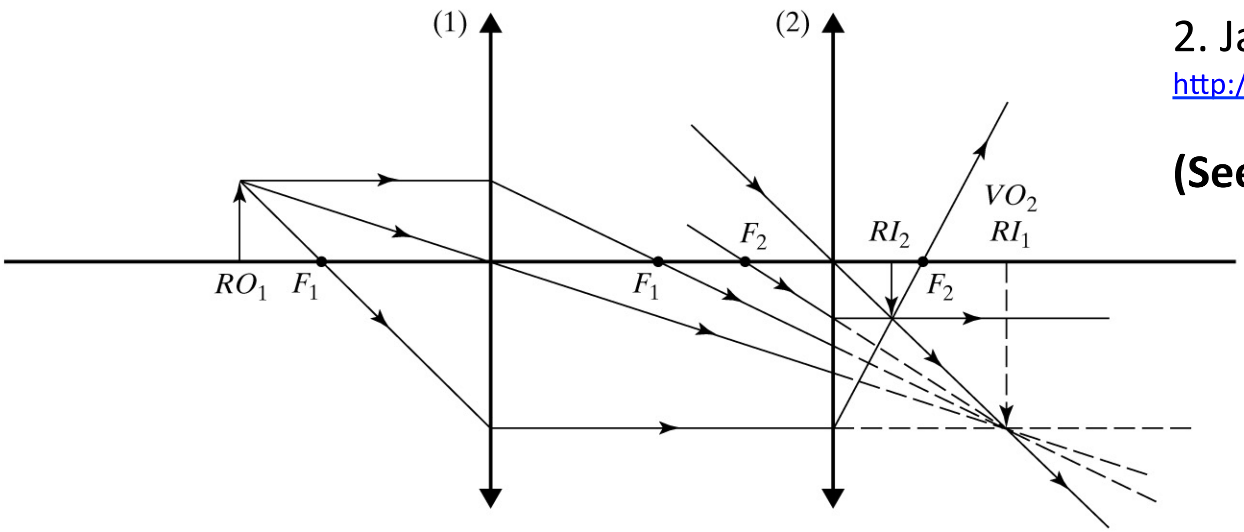


Any ray directed at the center of the lens passes through in a straight line.

Thin Lens Combination ➔ Sequential Imaging



(a)



(b)

Reading Assignments

1. Example 2-3 (page 38)

2. Java Applets
http://silver.neep.wisc.edu/~shock/tools/ray.html

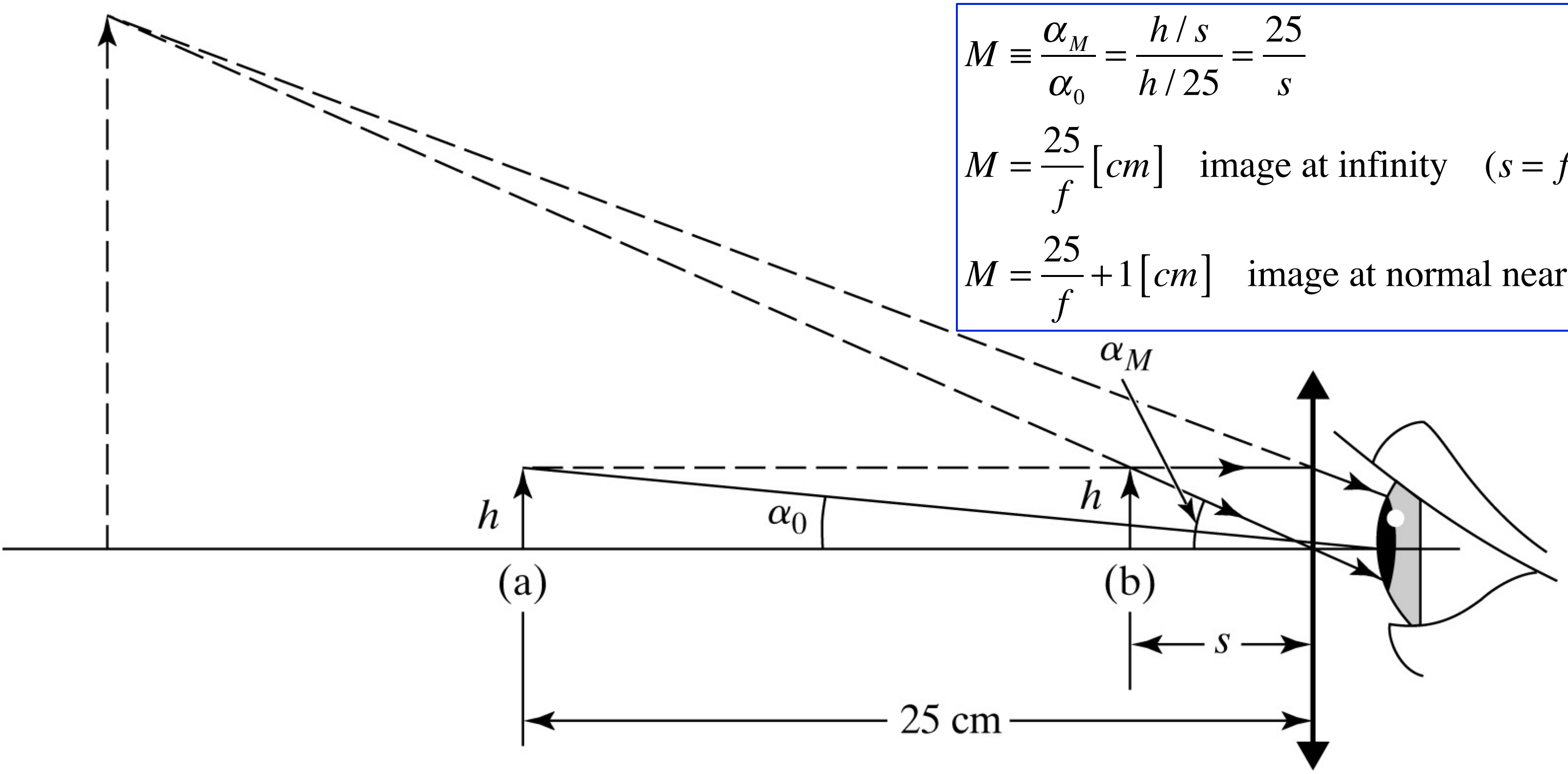
(See also Lab #1 Appendix)

Simple Magnifiers

$$M \equiv \frac{\alpha_M}{\alpha_0} = \frac{h/s}{h/25} = \frac{25}{s}$$

$$M = \frac{25}{f}[cm] \quad \text{image at infinity} \quad (s = f)$$

$$M = \frac{25}{f} + 1[cm] \quad \text{image at normal near point } (s' = -25cm)$$



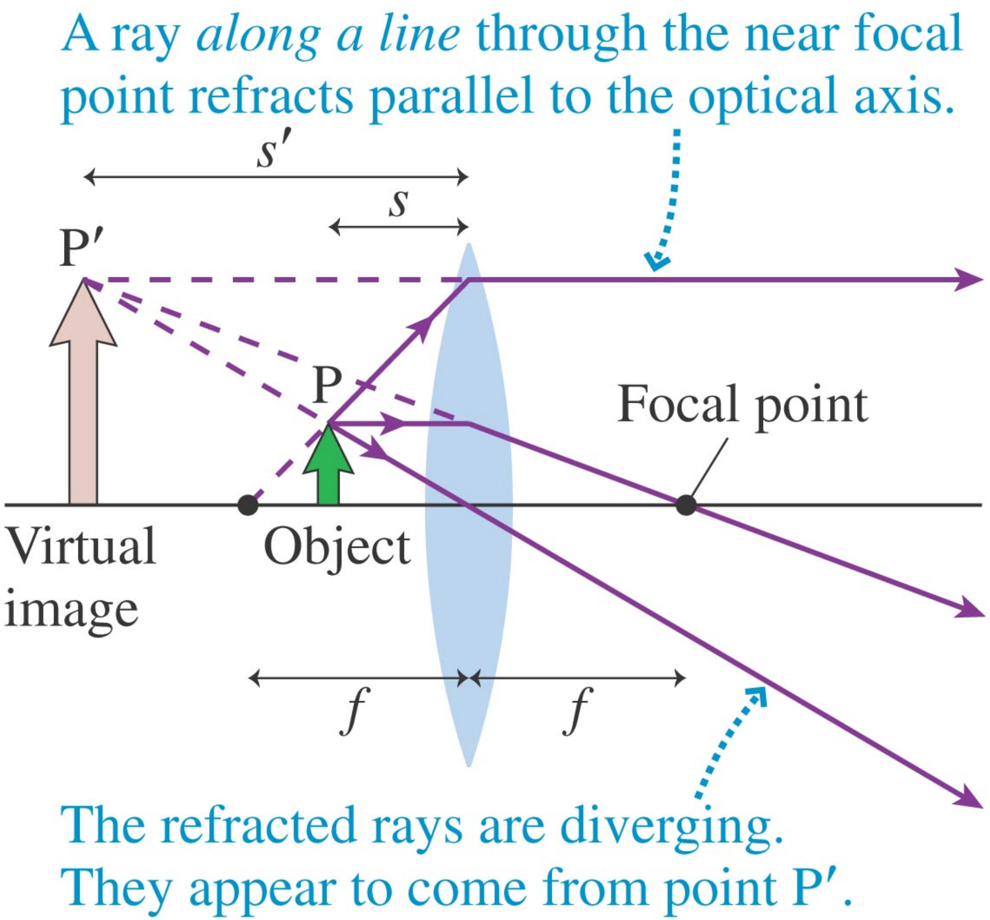
Magnifiers

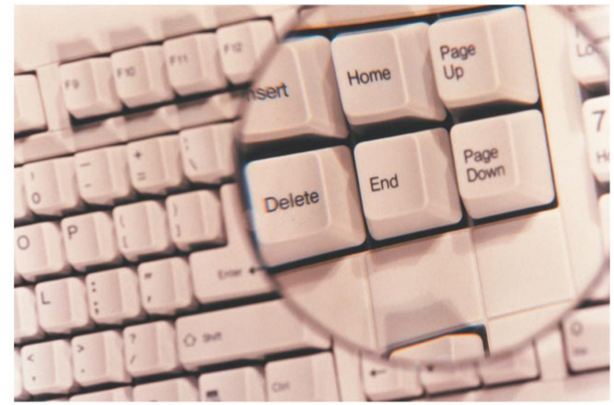
For relaxed-eye viewing, the angular magnification is

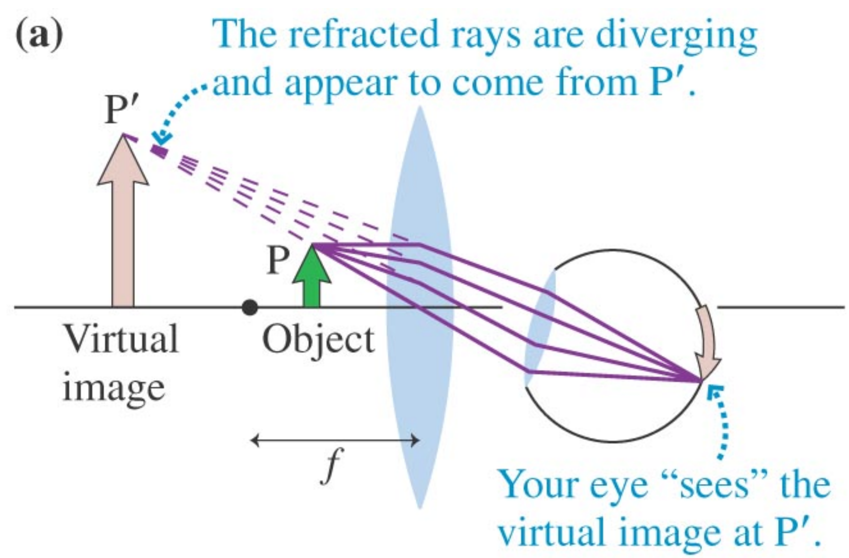
$$M = \frac{25 \text{ cm}}{f}$$

In microscopes and telescopes, the eyepiece acts as a magnifier to view the image of the objective.

Magnifying Glass/Virtual Image



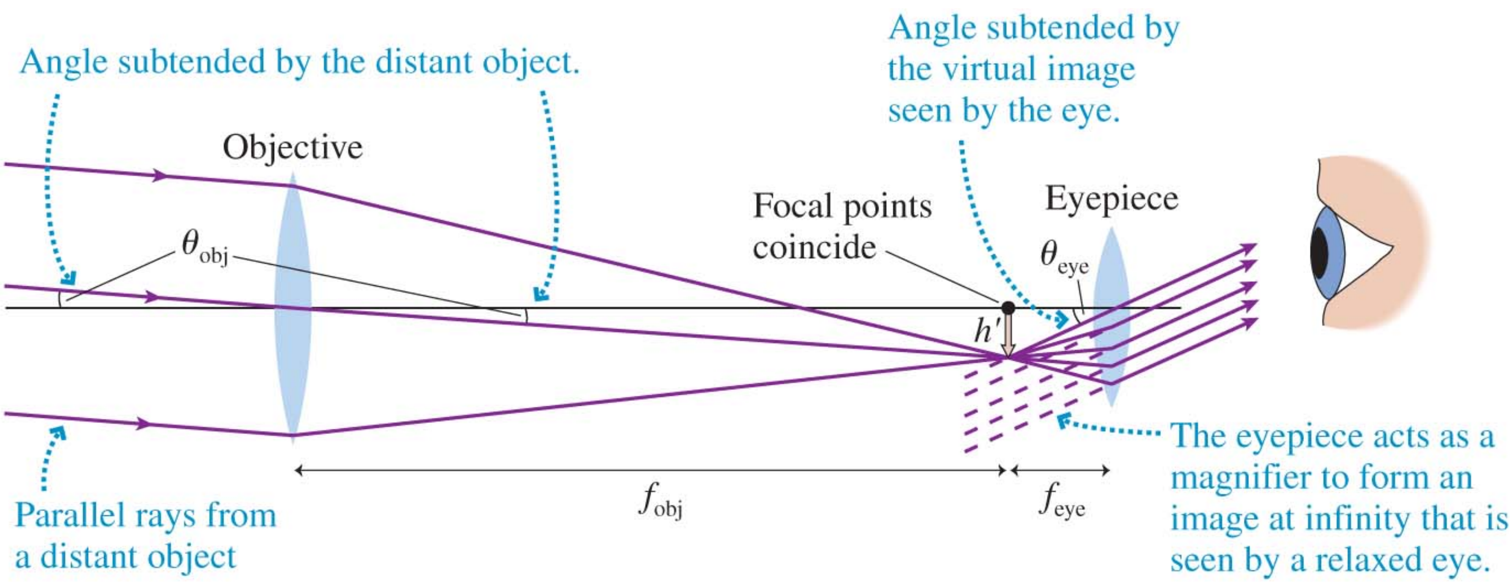




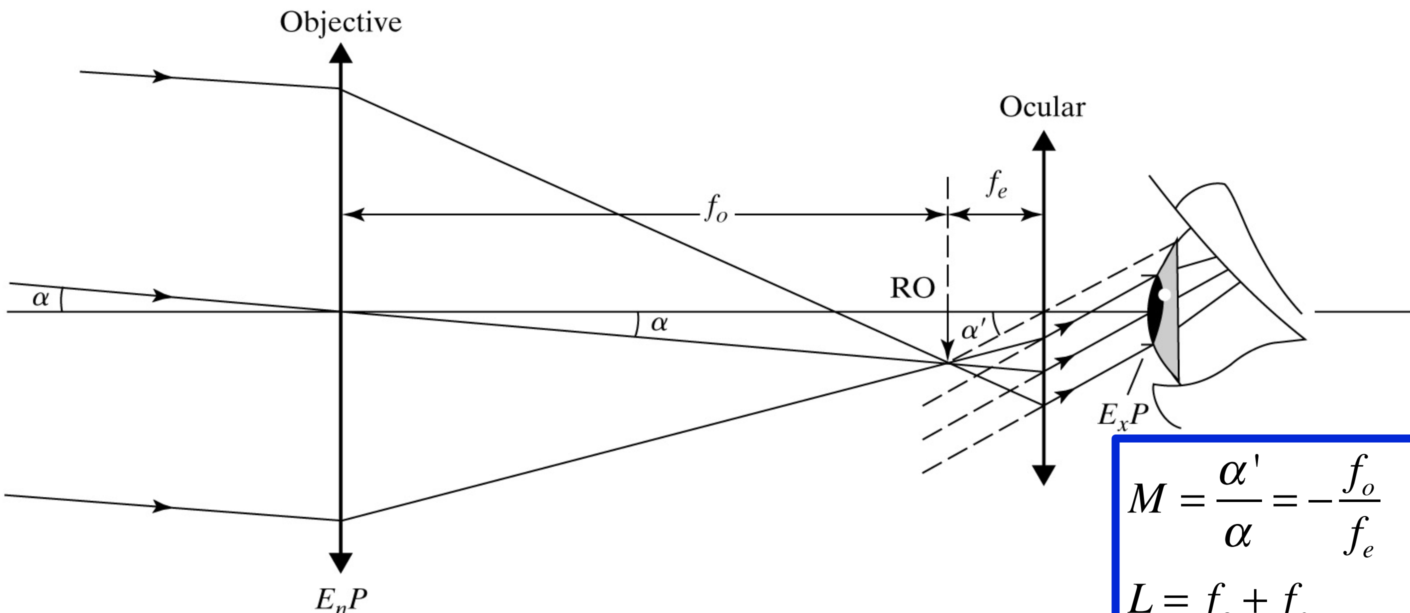
The Telescope

- A simple telescope contains a large-diameter objective lens which collects parallel rays from a distant object and forms a real, inverted image at distance $s' = f_{\text{obj}}$.
- The focal length of a telescope objective is very nearly the length of the telescope tube.
- The eyepiece functions as a simple magnifier.
- The viewer observes an inverted image.
- The angular magnification of a telescope is

$$M = \frac{\theta_{\text{eye}}}{\theta_{\text{obj}}} = -\frac{f_{\text{obj}}}{f_{\text{eye}}}$$



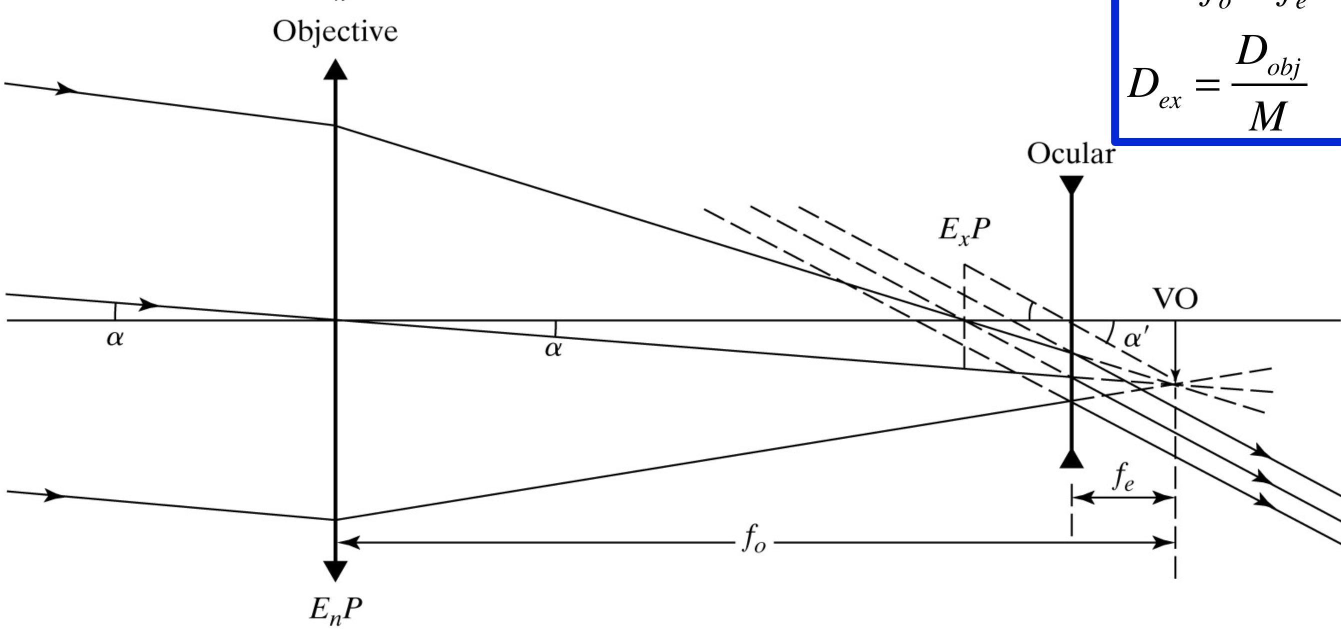
Telescope



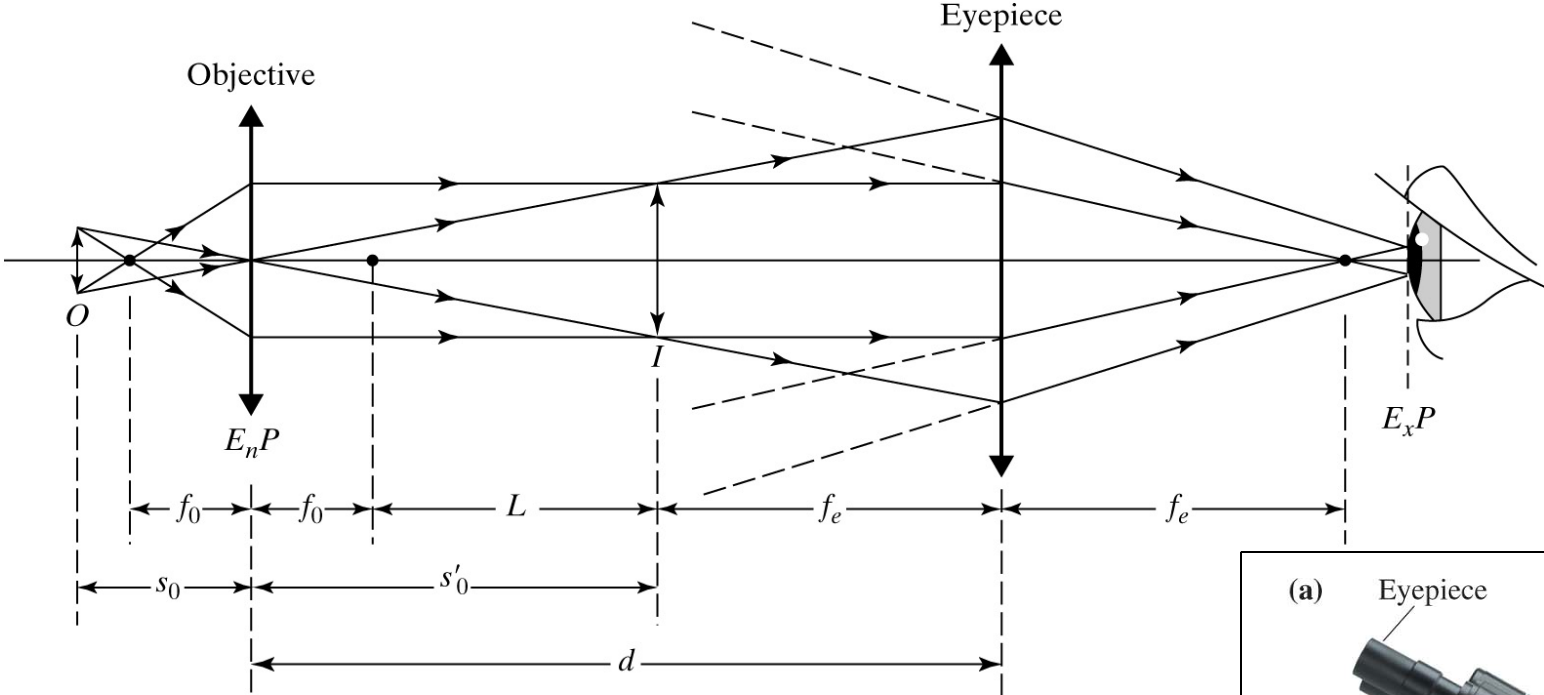
$$M = \frac{\alpha'}{\alpha} = -\frac{f_o}{f_e}$$

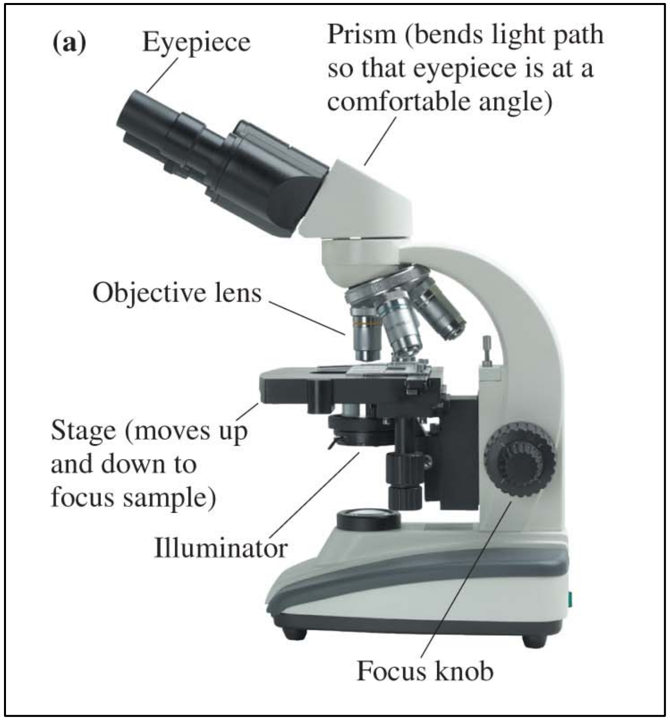
$$L = f_o + f_e$$

$$D_{ex} = \frac{D_{obj}}{M}$$ the diameter of the exit pupil



The Microscope





$$m_{\text{obj}} = -\frac{s'}{s} \approx -\frac{L}{f_{\text{obj}}}$$

The Microscope

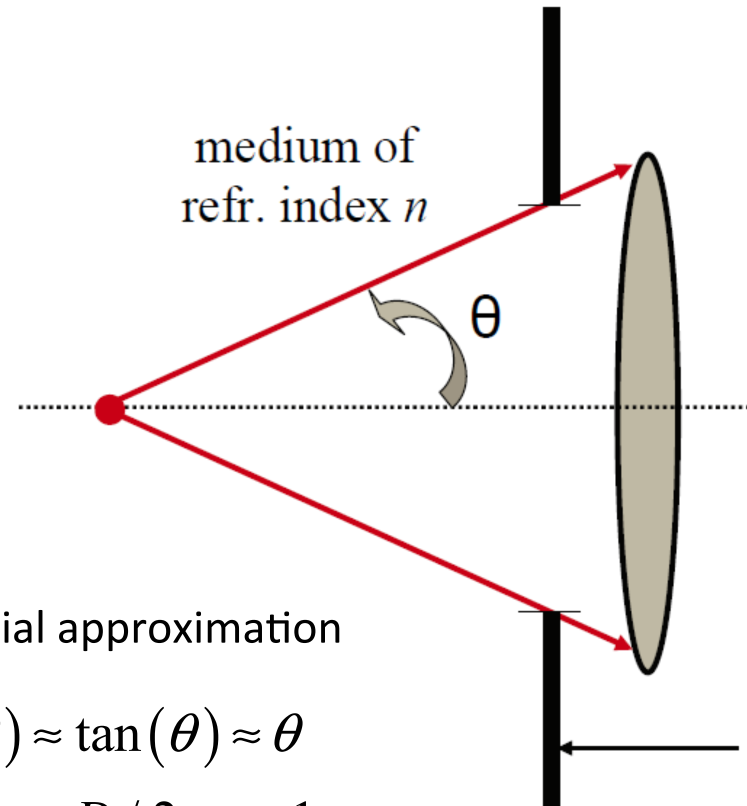
- A specimen to be observed is placed on the *stage* of a microscope, directly beneath the objective, a converging lens with a relatively short focal length.
- The objective creates a magnified real image that is further enlarged by the eyepiece.
- The lateral magnification of the objective is

$$m_{\mathrm{obj}} = -\frac{s'}{s} \approx -\frac{L}{f_{\mathrm{obj}}}$$

- Together, the objective and eyepiece produce a total angular magnification

$$M = m_{\mathrm{obj}} M_{\mathrm{eye}} = -\frac{L}{f_{\mathrm{obj}}} \frac{25\ \mathrm{cm}}{f_{\mathrm{eye}}}$$

Numerical Aperture



θ: half-angle subtended by the imaging system from an *axial* object

Numerical Aperture
(NA) = *n* sinθ

Speed (f/#)=1/2(NA)
pronounced f-number, e.g.
f/8 means (f/#)=8.

Aperture stop
the physical element which limits the angle of acceptance of the imaging system

Paraxial approximation

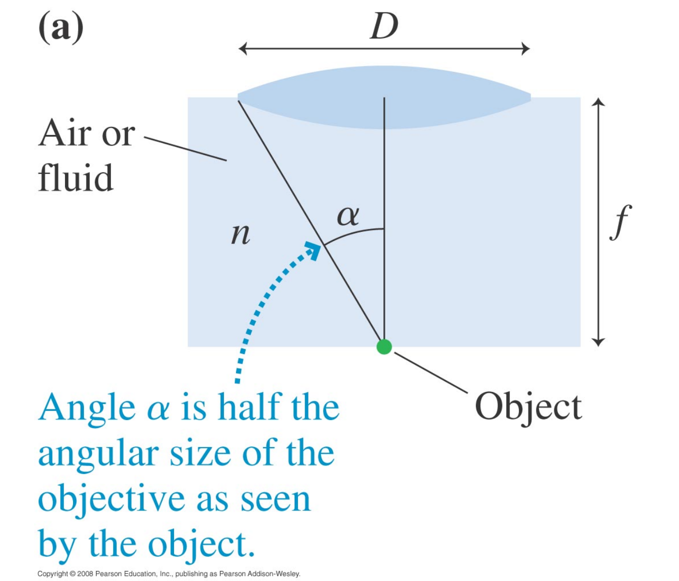
$$\sin(\theta) \approx \tan(\theta) \approx \theta$$

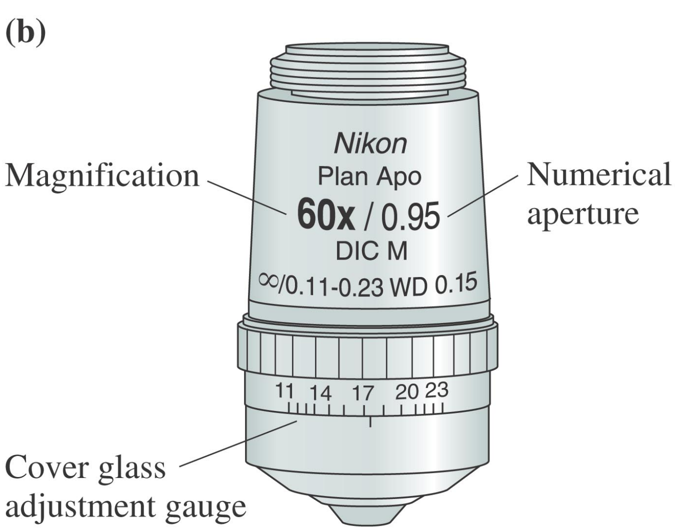
$$\rightarrow NA = \frac{D/2}{f} = \frac{1}{2f/\#}$$

The spatial resolution limit due to diffraction
≈ 1.22×f λ /D=0.61×λ/NA [Rayleigh Criterion].

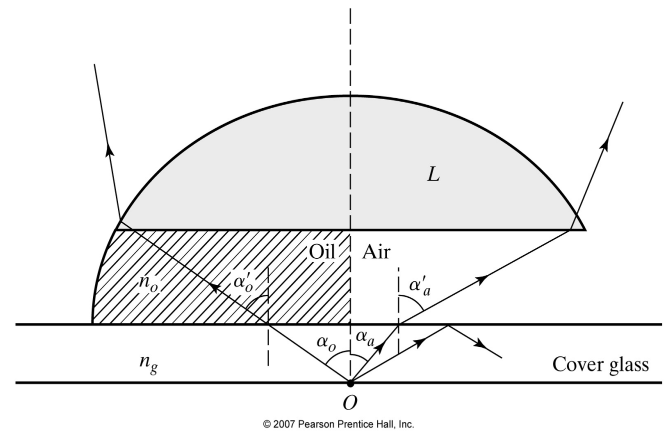
The Microscope: Objective and Numerical Aperture (NA)

$$NA = n \sin\alpha$$





Oil-immersion microscope



Microscopes

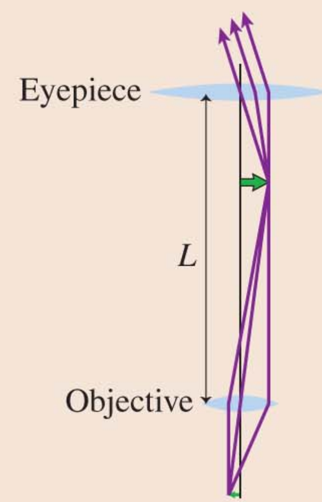
The object is very close to the focal point of the objective. The total magnification is

$$M = -\frac{L}{f_{obj}}\frac{25\text{ cm}}{f_{eye}}$$

The spatial resolution is

$$d_{min} = 0.61\lambda/\text{NA}$$

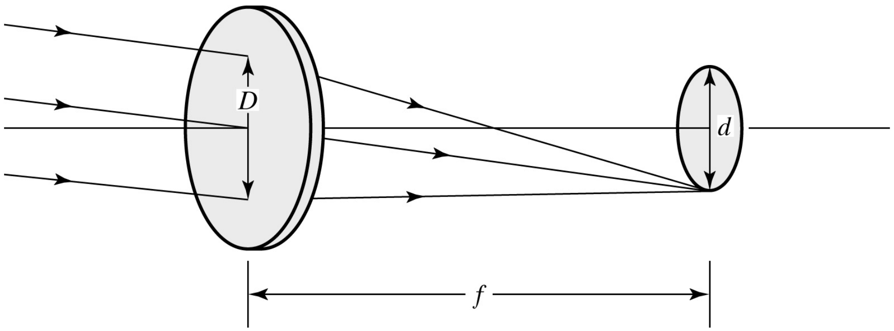
where NA is the numerical aperture of the objective lens.



f-number and irradiance/intensity [W/m²]

The light intensity on the detector is related to the lens's *f*-number by

$$I \propto \frac{D^2}{f^2} = \frac{1}{(f\text{-number})^2}$$



Cameras

Forms a real, inverted image on a detector. The lens's ***f*-number** is

$$f\text{-number} = \frac{f}{D}$$

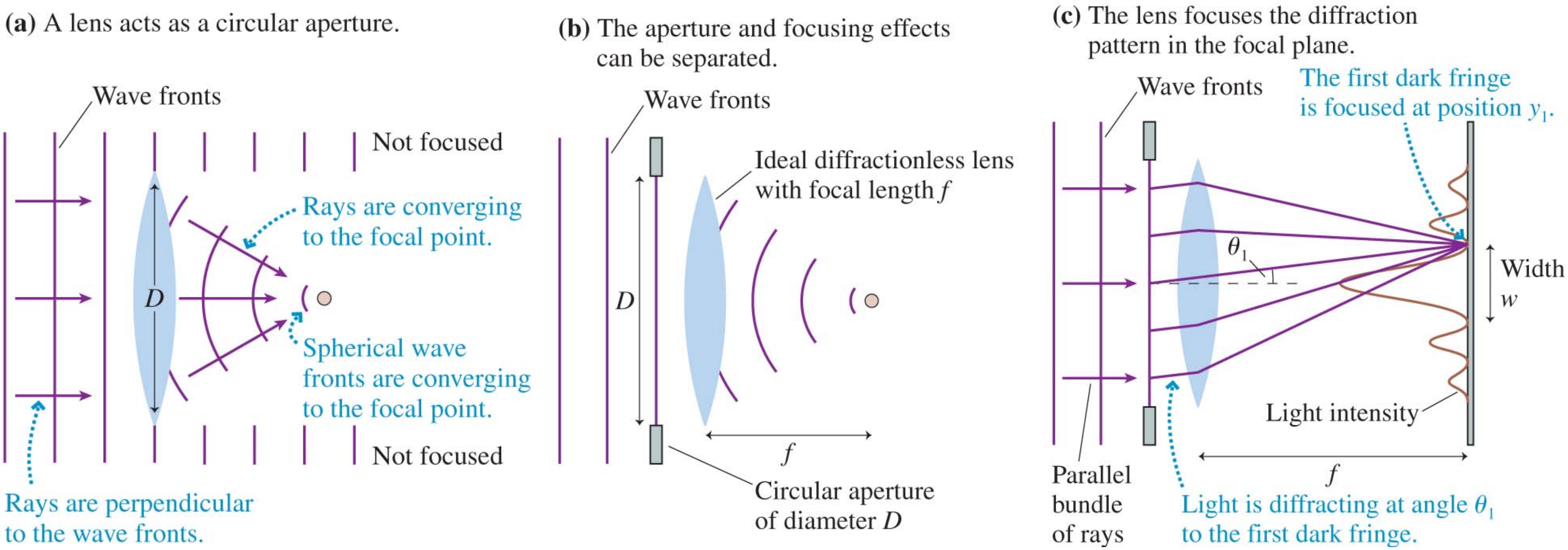
The light intensity on the detector is

$$I \propto \frac{1}{(f\text{-number})^2}$$

TABLE 3-2 STANDARD RELATIVE APERTURES AND IRRADIANCE AVAILABLE ON CAMERAS

$A = f$-number		$(A = f\text{-number})^2$	E_e
1		1	E_0
1.4	$\sqrt{2} \approx 1.4$	2	$E_0/2$
2		4	$E_0/4$
2.8		8	$E_0/8$
4		16	$E_0/16$
5.6		32	$E_0/32$
8		64	$E_0/64$
11		128	$E_0/128$
16		256	$E_0/256$
22		512	$E_0/512$

A lens both focuses and diffracts the light



The Resolution of Optical Instruments

The minimum spot size to which a lens can focus light of wavelength λ is

$$w_{\min} \approx 2f\theta_1 = \frac{2.44\lambda f}{D} \qquad \text{(minimum spot size)}$$

where D is the diameter of the circular aperture of the lens, and f is the focal length.
In order to resolve two points, their angular separation must be greater than $\theta_{\min}$, where

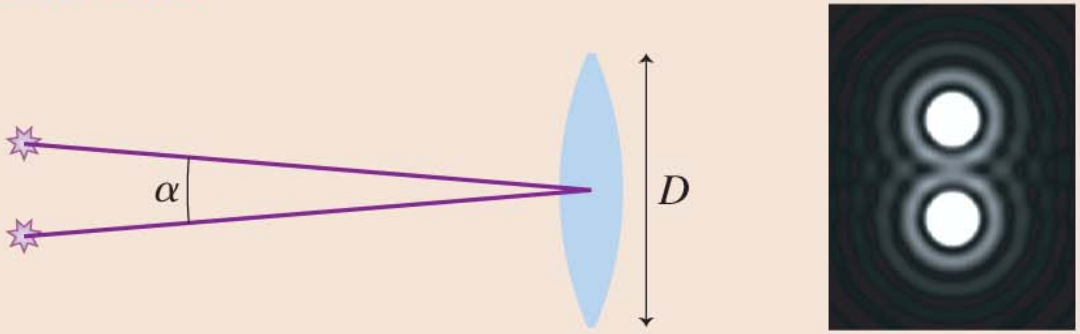
$$\theta_{\min} = \frac{1.22\lambda}{D} \qquad \text{(angular resolution of a lens)}$$

is called the angular resolution of the lens.

Important Concepts

Lens **power**: $P = \frac{1}{f}$ diopters, $1\ \text{D} = 1\ \text{m}^{-1}$

Resolution

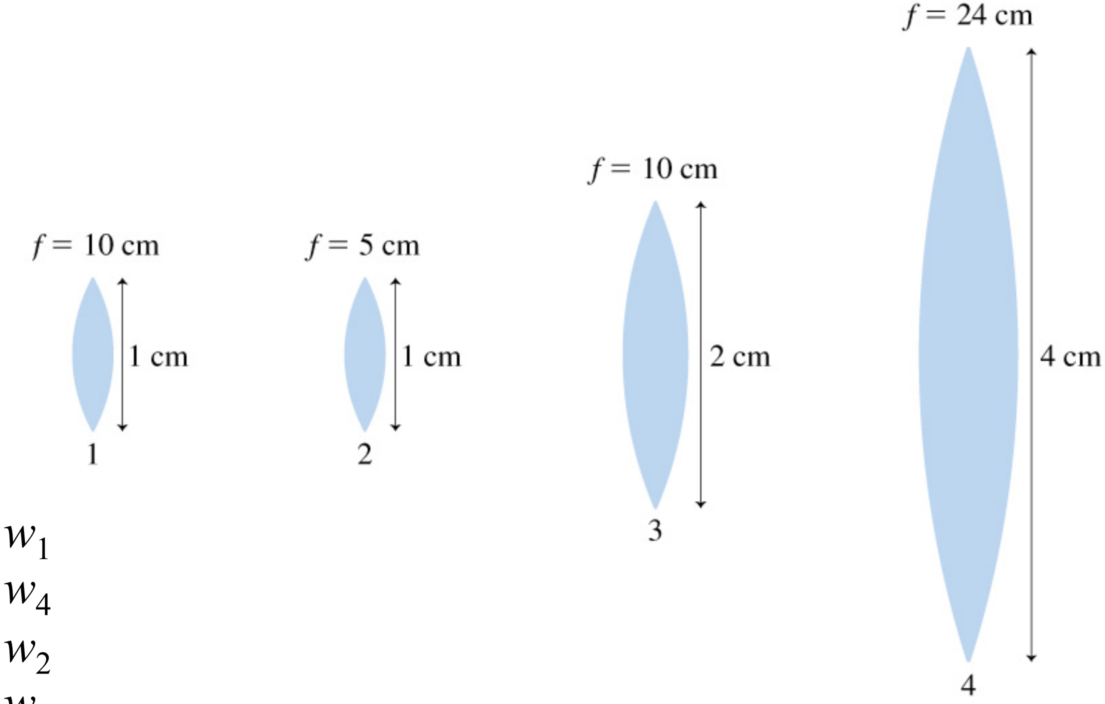


The **angular resolution** of a lens of diameter D is

$$\theta_{\text{min}} = 1.22\lambda/D$$

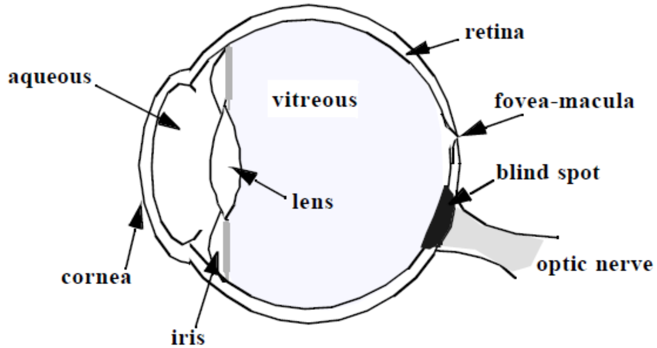
Rayleigh's criterion states that two objects separated by an angle α are marginally resolvable if $\alpha = \theta_{\text{min}}$.

Four diffraction-limited lenses focus plane waves of light with the same wavelength *l*. Rank order, from largest to smallest, the spot sizes w_1 to w_4.



A. $w_2 = w_3 > w_4 > w_1$
B. $w_1 = w_2 > w_3 > w_4$
C. $w_4 > w_3 > w_1 = w_2$
D. $w_1 > w_4 > w_2 = w_3$
E. $w_2 > w_1 = w_3 > w_4$

Eye



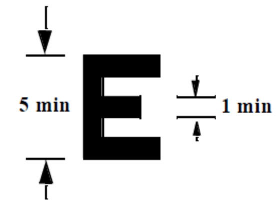
	t (mm)	n	R (mm)
cornea f	0	1.376	7.7
b	0.5		6.8
aqueous		1.336	
lens f	4.0	1.386-1.406	10.0 (relaxed), 5 (focused)
b	7.0		–6.0 (relaxed), –5 (focused)
vitreous		1.336	
retina	24.4		

Visual Acuity (VA)

The separation between cone cells in the fovea corresponds to about 1′ (0.3 mrad). At close viewing distance of 25 cm, this gives a resolution of 75 μm.

This is close to the diffraction limit imposed by NA of the eye.

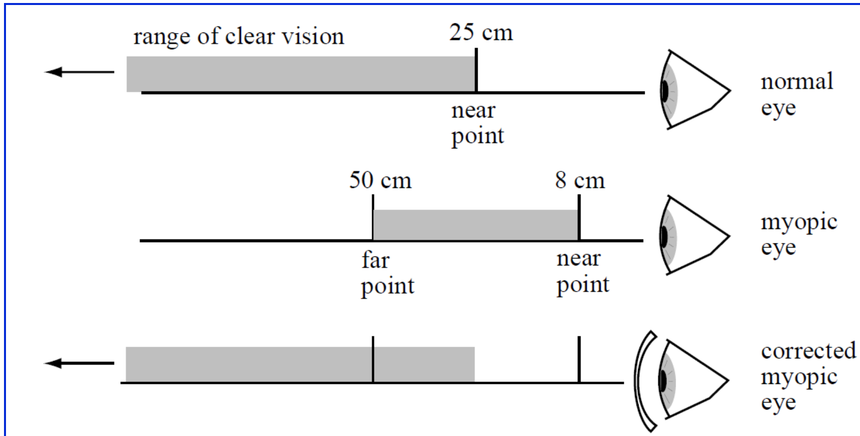
Visual acuity (VA) is defined relative to a standard of 1 minute of arc. VA = 1/(the angular size of smallest element of a letter that can be distinguished [in min])



VA is usually expressed as $\dfrac{\text{dist to target (usually 20 ft)}}{\text{dist at which target element is 1 min}}$

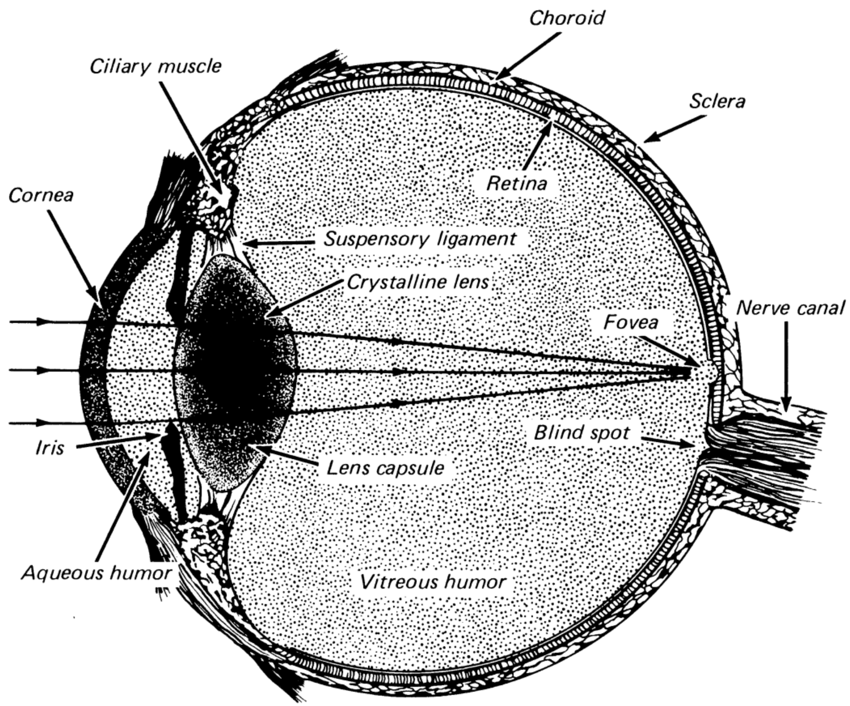
For 20/20 vision, the minimum element is 1 min at 20 ft.

The overall power of the eye is ~ 58.6 D. The lens surfaces are not spherical, and the lens index is higher at the center (on-axis). Both effects correct spherical aberration. The diameter of the iris ranges from 1.5 → 8 mm.



Topics/Keywords:
Eye model, Visual Acuity, Cones/Rods accommodation, eyeglasses, nearsightedness/myopia, farsightedness/ hyperopia

Human Eye – Gullstrand Model



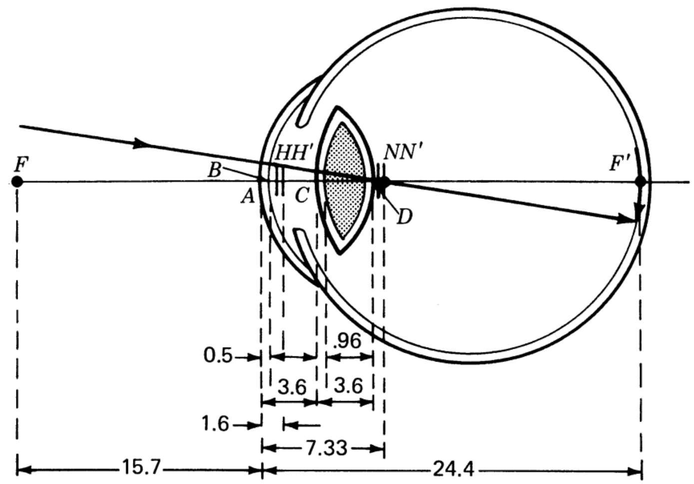


Table 10A PRINCIPAL DIMENSIONS FOR GULLSTRAND'S SCHEMATIC EYE
Overall power of eye = 58.64 D

	Refractive index	Axis position, mm	Radius curvature, mm
Cornea, anterior	1.376	0	7.7
and posterior		0.5	6.8
Aqueous humor	1.336		
Vitreous humor	1.336		
Lens:			
Cortex, anterior	1.386	3.6	10.0
and posterior		7.2	−6.0
Core, anterior	1.406	4.15	7.9
and posterior		6.57	5.8
Cardinal points:			
AH		1.348	
AH′		1.602	
AN		7.08	
AN′		7.33	
AF		−15.70	
AF′		24.38	

Retina – Cones and Rods

Rods are most sensitive to light, but do not sense color, motion

Cones are color sensitive in bright light.

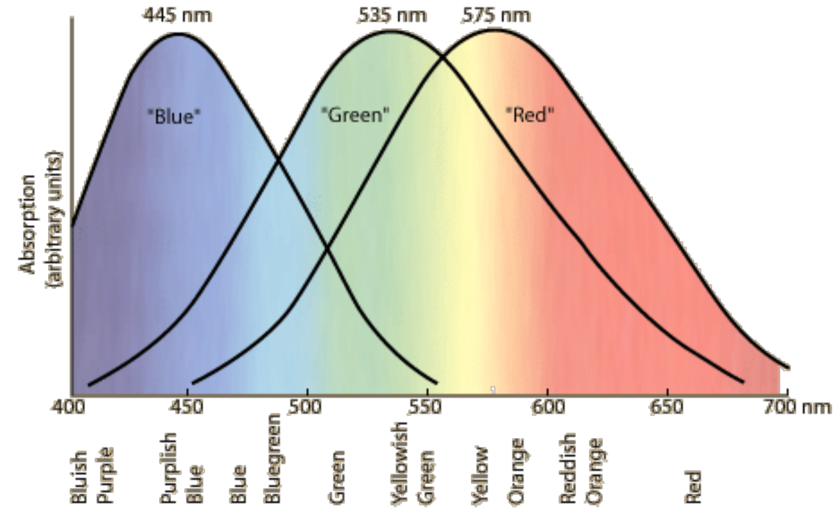
You have ~ 6 million cones, ~ 120 million rods, but only 1 million nerve fibers.

Cones are 1 -1.5 µm diameter, 2 –2.5 µm apart in the fovea.

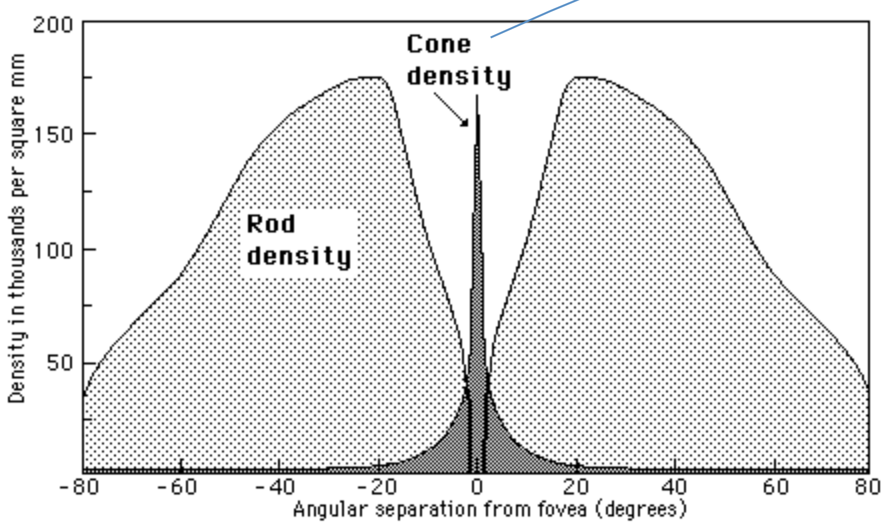
Rods are ~ 2 µm diameter

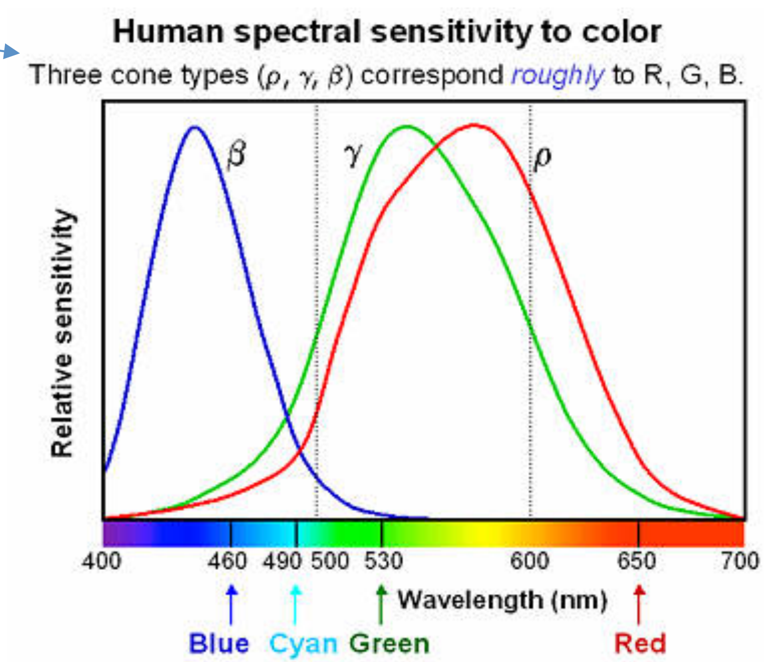
The macula is 5° to the outside of the axis.

The fovea is the central 0.3 mm of the macula. It has **only cones** and is the center of sharp vision.

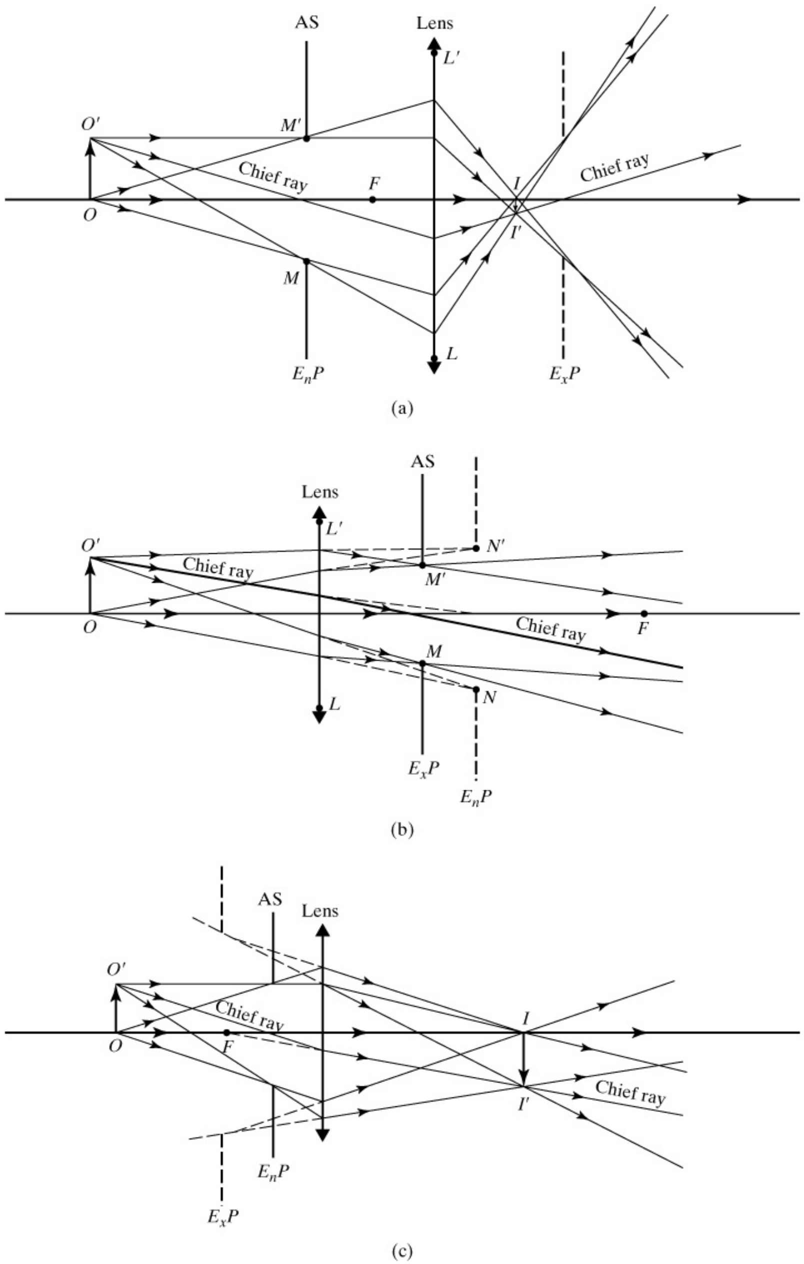


Current understanding is that the 6 to 7 million cones can be divided into "red" cones (64%), "green" cones (32%), and "blue" cones (2%) based on measured response curves.





Stops, Pupils, and Windows



<u>Summary of Terms</u>

Brightness

Aperture stop AS: The real element in an optical system that limits the size of the cone of rays accepted by the system from an axial object point.

Entrance pupil E_nP: The image of the aperture stop formed by the optical elements (if any) that precede it.

Exit pupil E_xP: The image of the aperture stop formed by the optical elements (if any) that follow it.

Field of view

Field stop ES: The real element that limits the angular field of view formed by an optical system.

Entrance window E_nW: The image of the field stop formed by the optical elements (if any) that precede it.

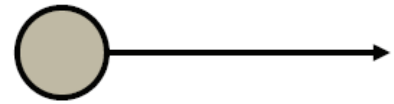
Exit window E_xW: The image of the field stop formed by the optical elements (if any) that follow it.

Chief Ray

The chief, or principal, ray is a ray from an object point that passes through the axial point, in the plane of the entrance pupil.

Particle Nature of Light

Photon=elementary light particle



Mass=0

Speed $c=3\times10^{8}$ m/sec

According to Special Relativity, a mass-less particle travelling at light speed can still carry energy (& momentum)!

Energy $E=h\nu$ → relates the dual particle & wave nature of light;

h=Planck's constant

$=6.6262\times10^{-34}$ J sec

$= 4.1357\times10^{-15}$ eV s

ν is the temporal oscillation frequency of the light waves

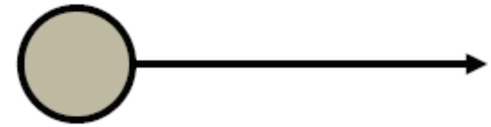
$$N\ [\#/s] = \frac{Power}{Energy\,/\,photon} = \frac{[Watt = J\,/\,s]}{[J]}$$

N is the number of photons per second.

See Example 1-2, page 10 in Pedrotti[3].

Wave-Particle Duality of Light

Photon=elementary light particle



Energy $E=h\nu$

$c=\lambda\nu$

"Dispersion relation"

(holds in vacuum only)

h=Planck's constant
$=6.6262\times10^{-34}$ J sec

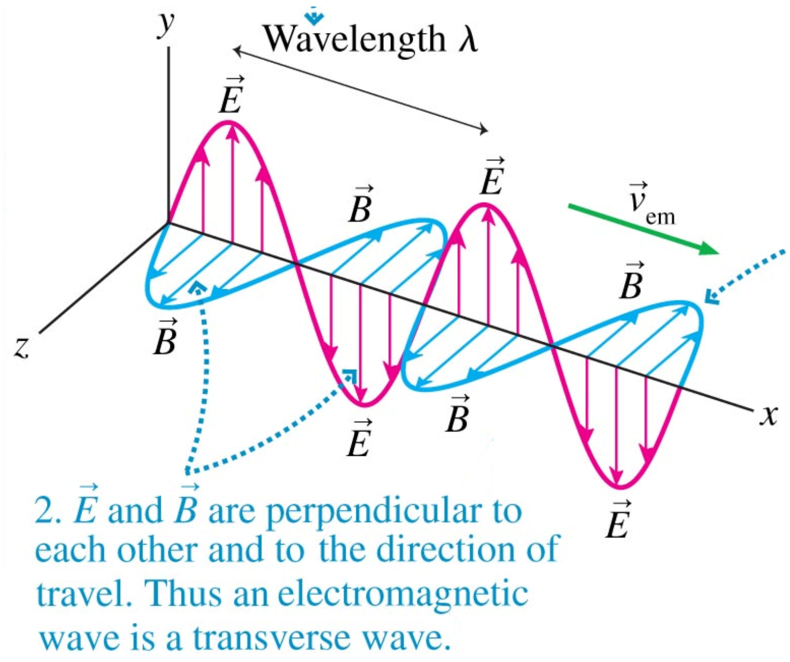
ν=frequency (sec^{-1})
λ=wavelength (m)

Maxwell's theory

- Maxwell showed that E and B fields could sustain themselves (free from charges or currents) if they took the form of an electromagnetic (EM) wave.
- Maxwell's theory predicted that an EM wave would travel with speed:

$$v_{em} = \frac{1}{\sqrt{\varepsilon_0 \mu_0}}$$

$$v_{em} = c = \text{ speed of light}$$



Light is an electromagnetic wave!

Animation http://www.walter-fendt.de/ph14e/emwave.htm

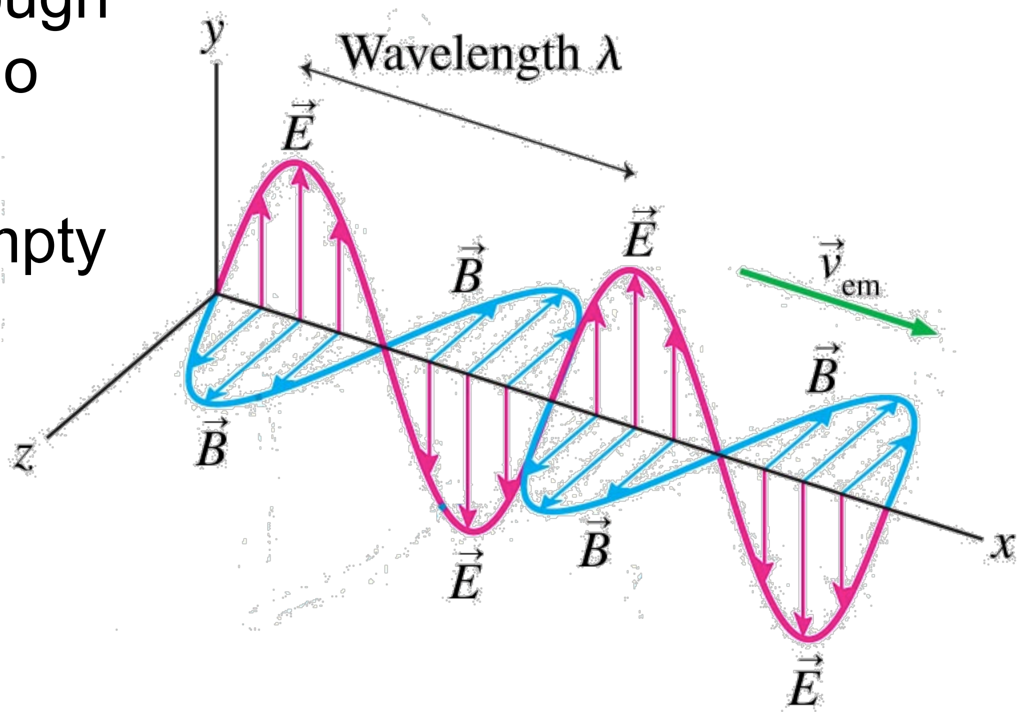
Electromagnetic (EM) Waves

- EM waves can travel through empty space (vacuum); no medium is necessary!
- The speed of light **c** in empty space is

$$c = \frac{1}{\sqrt{\varepsilon_0 \mu_0}}$$

$$= 299{,}792{,}458 \text{ m/s}$$

c = 3x10^{8} m/s



- EM waves carry energy and momentum
- The **speed is constant** so the frequency f is determined by the wavelength λ and speed of light c:

$$f = c / \lambda$$

Properties of Electromagnetic Waves

Any electromagnetic wave must satisfy four basic conditions:

1. The fields $\boldsymbol{E}$ and $\boldsymbol{B}$ and are perpendicular to the direction of propagation $\boldsymbol{v}_{\mathrm{em}}$.Thus an electromagnetic wave is a transverse wave.
2. $\boldsymbol{E}$ and $\boldsymbol{B}$ are perpendicular to each other in a manner such that $\boldsymbol{E} \times \boldsymbol{B}$ is in the direction of $\boldsymbol{v}_{\mathrm{em}}$.
3. The wave travels in vacuum at speed $v_{\mathrm{em}} = c$
4. $E = cB$ at any point on the wave.

Properties of Electromagnetic Waves

The energy flow of an electromagnetic wave is described by the **Poynting vector** defined as

$$\vec{S} \equiv \frac{1}{\mu_0} \vec{E} \times \vec{B}$$

The magnitude of the Poynting vector is

$$S = \frac{EB}{\mu_0} = \frac{E^2}{c\mu_0}$$

$$I = \frac{P}{A} = S_{avg}$$

The intensity of an electromagnetic wave whose electric field amplitude is E_0 is

$$I = \frac{P}{A} = \frac{1}{2} c \epsilon_0 E_0^2 = \frac{1}{2} \frac{c}{\mu_0} B_0^2$$

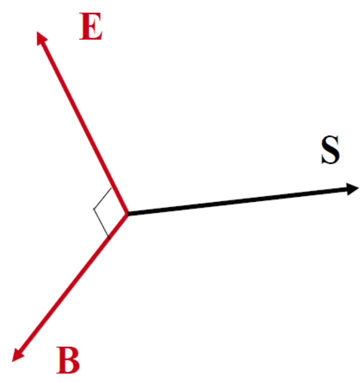
Intensity of an electromagnetic wave with field amplitudes E_0 and B_0

Energy and Intensity

Summary (free space or isotropic media)

$$\mathbf{S} = \frac{1}{\mu_0}\mathbf{E}\times\mathbf{B};\quad \|\mathbf{S}\| = c\varepsilon_0\|\mathbf{E}\|^2$$ Poynting vector

$$\langle\|\mathbf{S}\|\rangle = \frac{1}{T}\int_t^{t+T}\|\mathbf{S}\|\mathrm{d}t$$ Irradiance (or intensity)



$$\mathbf{S} = \mathbf{E}\times\mathbf{H}$$

$$\mathbf{S} = \frac{1}{\mu_0}\mathbf{E}\times\mathbf{B} = c^2\varepsilon_0\mathbf{E}\times\mathbf{B}$$

so in free space

$$\mathbf{S}\,\|\,\mathbf{k}$$

S has units of W/m^2 so it represents energy flux (energy per unit time & unit area)

- **Poynting vector** describes flows of E-M power
- Power flow is directed along this vector (usually parallel to **k)**
- Intensity is average energy transfer (i.e. the time averaged Poyning vector: I=<**S**>=P/A, where P is the power (energy transferred per second) of a wave that impinges on area A.

$$\langle\sin^2(kx-\omega t)\rangle = \langle\cos^2(kx-\omega t)\rangle = \frac{1}{2}$$

$$|\langle\mathbf{S}\rangle| = I \equiv |\langle\mathbf{E}(t)\times\mathbf{H}(t)\rangle| = \frac{c\varepsilon_0}{2}E^2 = \frac{c\varepsilon_0}{2}\left(E_x{}^2 + E_y{}^2\right)$$

$$c\varepsilon_0 \approx 2.654\times10^{-3}\,A/V$$

example $E = 1V/m$

$$I = ?\,W/m^2$$

$$\hbar\omega[eV] = \frac{1239.85}{\lambda[nm]}$$

$$\hbar = 1.05457266\times10^{-34}\,Js$$

EXAMPLE: The electric field of a laser beam

A helium-neon laser, the laser commonly used for classroom demonstrations, emits a 1.0-mm-diameter laser beam with a power of 1.0 mW. What is the amplitude of the oscillating electric field in the laser beam?

MODEL The laser beam is an electromagnetic plane wave.

SOLVE 1.0 mW, or 1.0×10^{-3} J/s, is the energy transported per second by the light wave. This energy is carried within a 1.0-mm-diameter beam, so the light intensity is

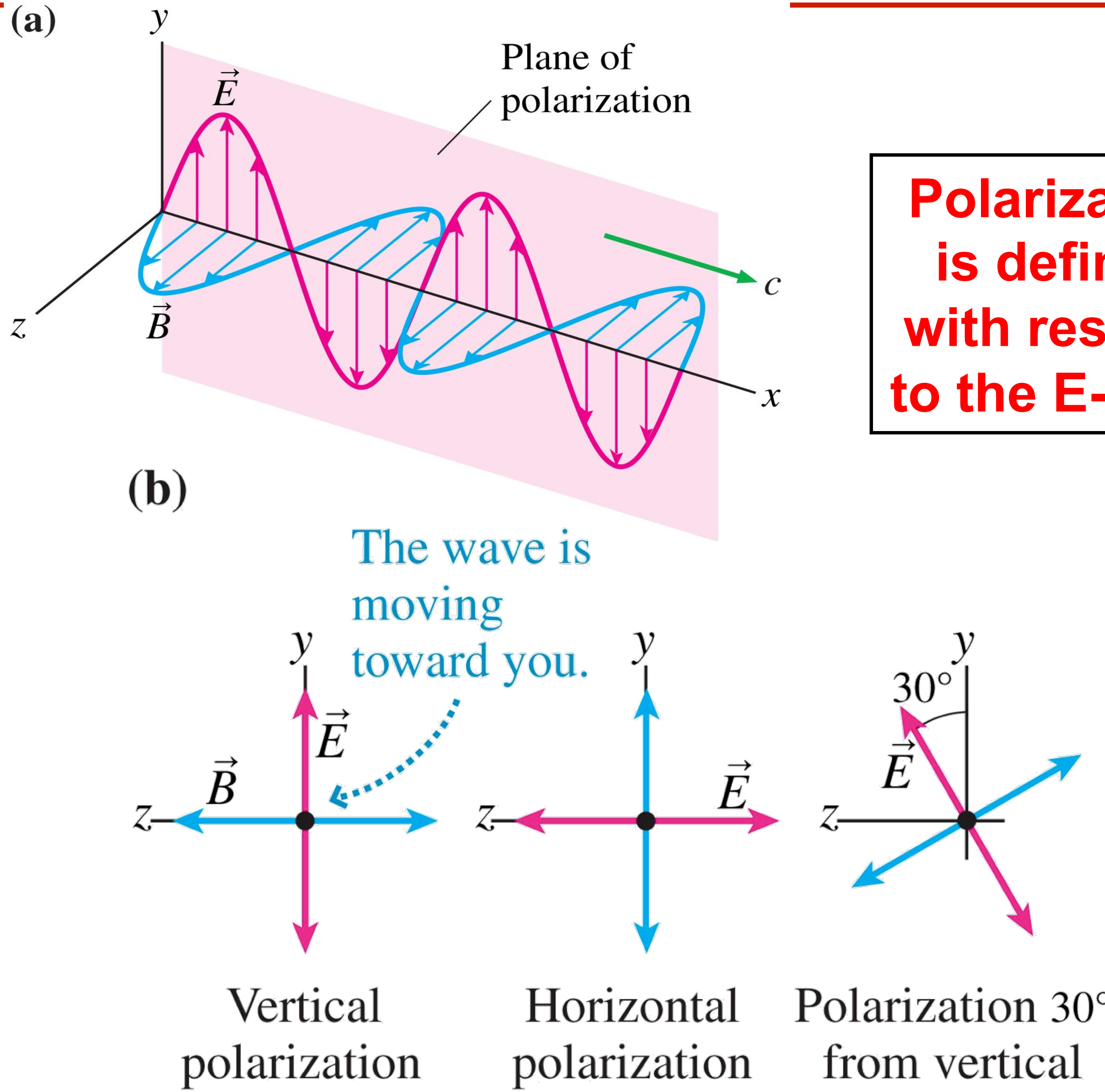
$$I = \frac{P}{A} = \frac{P}{\pi r^2} = \frac{1.0 \times 10^{-3}\ \text{W}}{\pi (0.00050\ \text{m})^2} = 1270\ \text{W/m}^2$$

We can use Equation 35.37 to relate this intensity to the electric field amplitude:

$$E_0 = \sqrt{\frac{2I}{c\epsilon_0}} = \sqrt{\frac{2(1270\ \text{W/m}^2)}{(3.00 \times 10^8\ \text{m/s})(8.85 \times 10^{-12}\ \text{C}^2/\text{N m}^2)}}$$

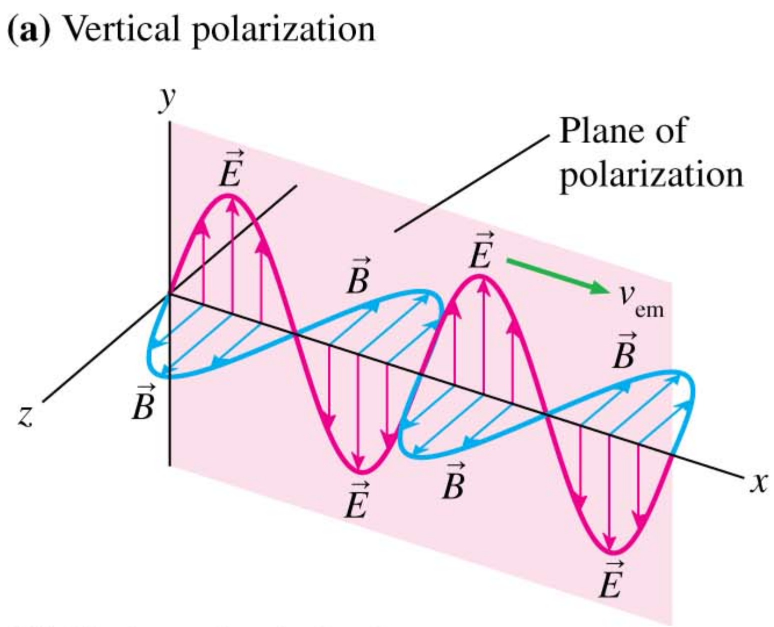
$$= 980\ \text{V/m}$$

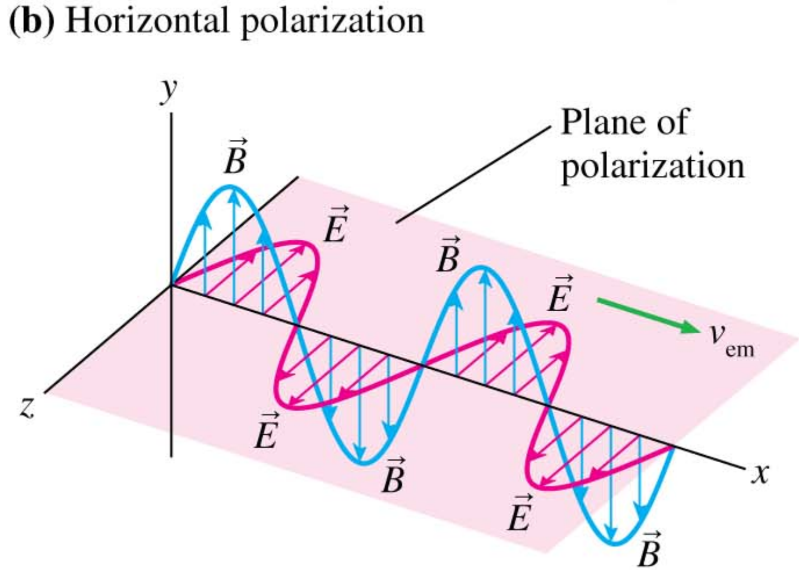
Polarization



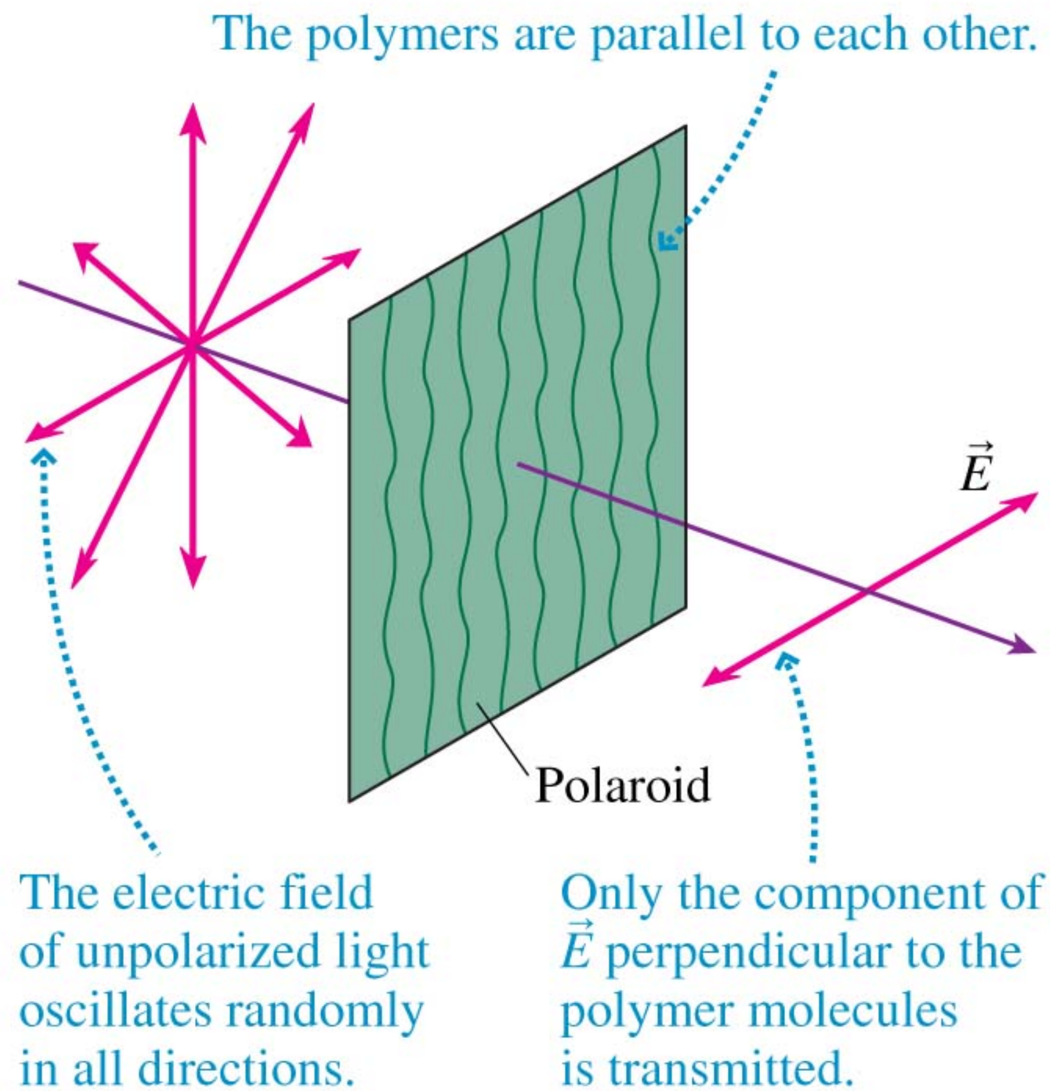
Polarization is defined with respect to the E-field.

Polarization & Plane of Polarization





A Polarizing Filter



Malus's Law

Suppose a *polarized* light wave of intensity I_0 approaches a polarizing filter. ϑ is the angle between the incident plane of polarization and the polarizer axis. The transmitted intensity is given by Malus's Law:

$$I_{\text{transmitted}} = I_0 \cos^2\theta \qquad \text{(incident light polarized)}$$

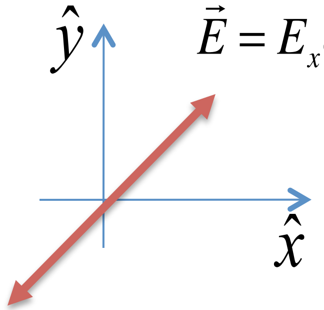
If the light incident on a polarizing filter is *unpolarized*, the transmitted intensity is

$$I_{\text{transmitted}} = \frac{1}{2} I_0 \qquad \text{(incident light unpolarized)}$$

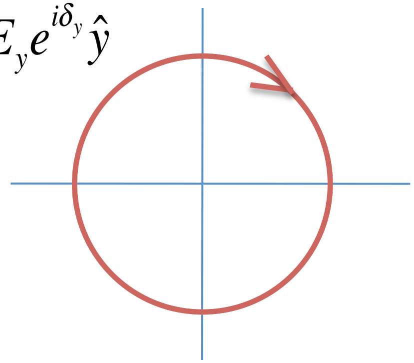
In other words, a polarizing filter passes 50% of unpolarized light and blocks 50%.

Polarization: Summary

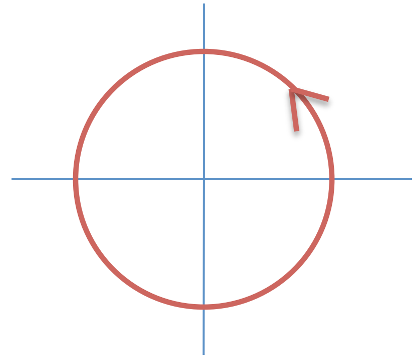
$$\vec{E} = E_x e^{i\delta_x}\hat{x} + E_y e^{i\delta_y}\hat{y}$$



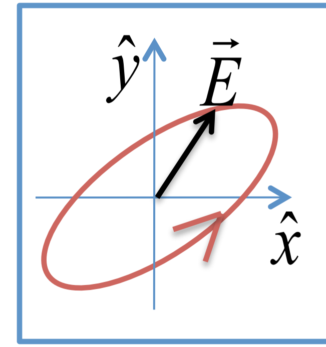
linear polarization
y-direction



right circular
polarization



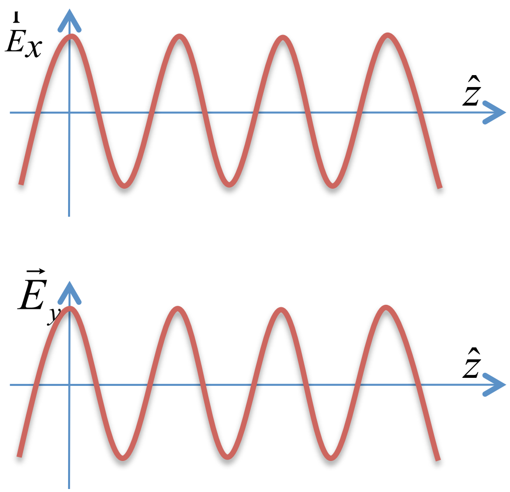
left circular
polarization



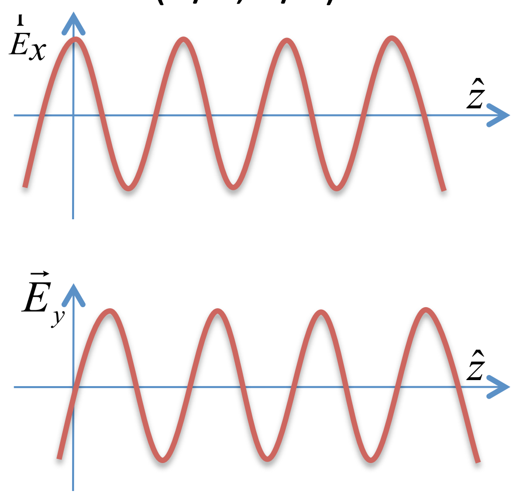
left elliptical
polarization

Phase difference

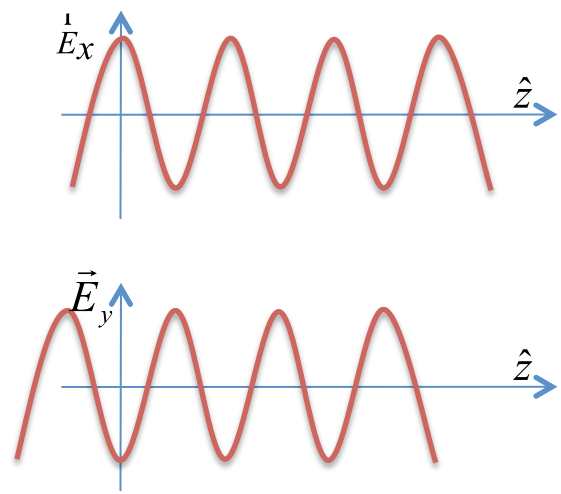
Phase difference = 0^0



Phase difference ➔
90^0 (π/2, λ/4)

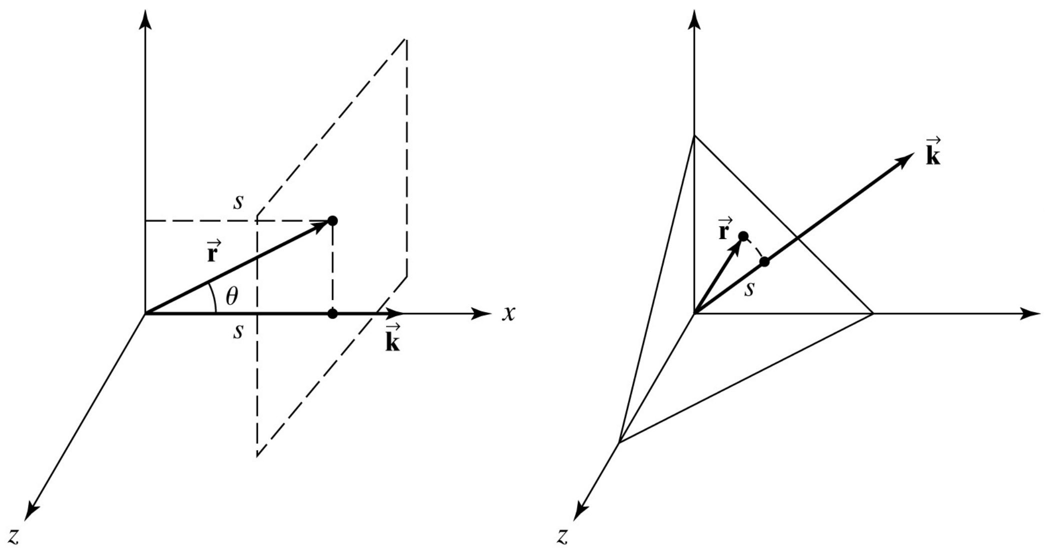


Phase difference ➔
180^0 (π, λ/2)

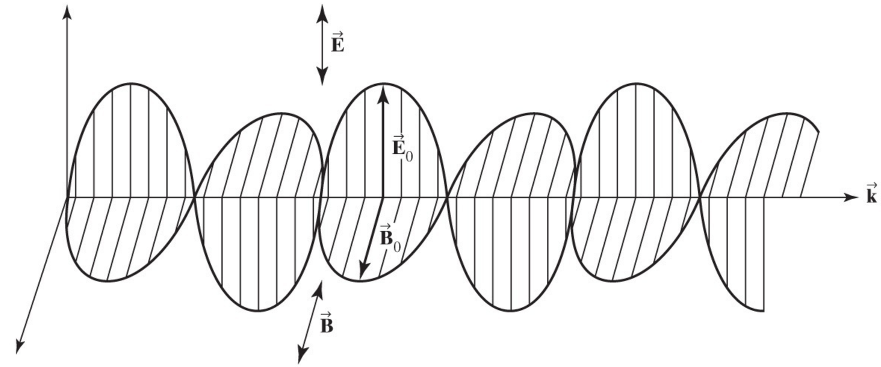


Wave Number, Wave Vector, and Momentum

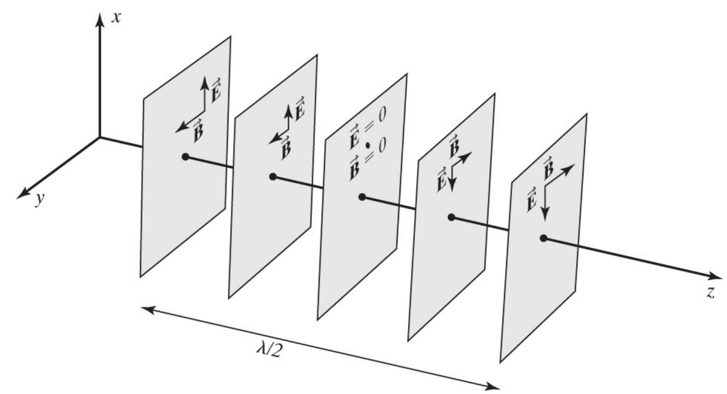
Chapter 4, Pedrotti^3)



(a) (b)



(a)



(b)

Wave equations in a medium

The induced polarization in Maxwell's Equations yields another term in the wave equation:

$$\frac{\partial^2 E}{\partial z^2} - \mu\varepsilon \frac{\partial^2 E}{\partial t^2} = 0 \qquad \frac{\partial^2 E}{\partial z^2} - \frac{1}{v^2}\frac{\partial^2 E}{\partial t^2} = 0$$

This is the **Inhomogeneous Wave Equation**.

The polarization is the driving term for a new solution to this equation.

$$\frac{\partial^2 E}{\partial z^2} - \mu_0\varepsilon_0 \frac{\partial^2 E}{\partial t^2} = 0 \qquad \frac{\partial^2 E}{\partial z^2} - \frac{1}{c^2}\frac{\partial^2 E}{\partial t^2} = 0$$

Homogeneous (Vacuum) Wave Equation

$$\begin{aligned}\mathbf{E}(z,t) &= \mathrm{Re}\{\mathbf{E}_0 e^{i(kz-\omega t)}\} \\ &= \tfrac{1}{2}\{\mathbf{E}_0 e^{i(kz-\omega t)} + \mathbf{E}_0^* e^{-i(kz-\omega t)}\} \\ &= |\mathbf{E}_0|\cos(kz-\omega t)\end{aligned}$$

$$\frac{c}{v} = n$$

Phase velocity

*Phase velocity can exceed the speed of light in a dispersive medium where the refractive index n is not necessarily >1.

Monochromatic plane waves

Plane waves have straight wave fronts

- As opposed to spherical waves, etc.
- Suppose

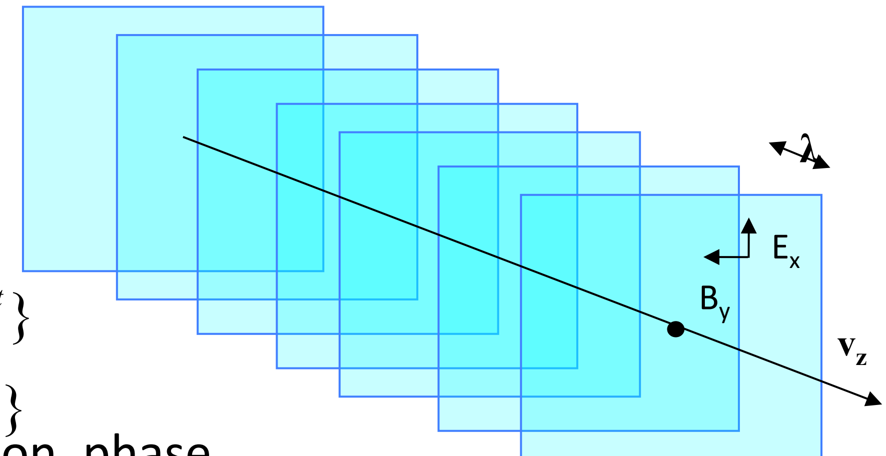
$$\mathbf{E}(\mathbf{r}) = \mathbf{E}_0 e^{i\mathbf{k}\cdot\mathbf{r}}$$

➔

$$\mathbf{E}(\mathbf{r},t) = \mathrm{Re}\{\mathbf{E}(\mathbf{r})e^{-i\omega t}\}$$

$$= \mathrm{Re}\{\mathbf{E}_0 e^{i\mathbf{k}\cdot\mathbf{r}} e^{-i\omega t}\}$$

$$= \mathrm{Re}\{\mathbf{E}_0 e^{i(\mathbf{k}\cdot\mathbf{r}-\omega t)}\}$$

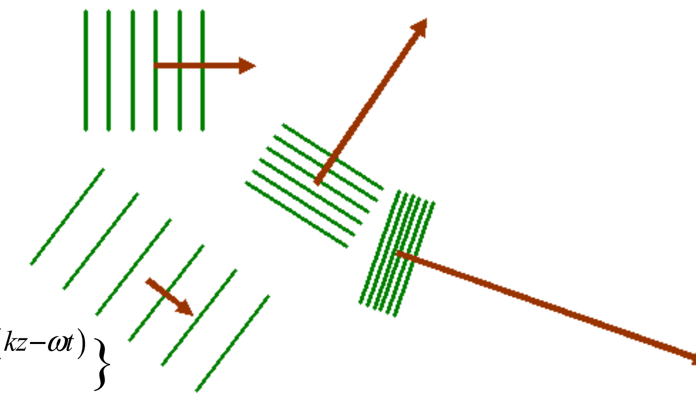


- $\mathbf{E}_0$ still contains: amplitude, polarization, phase
- Direction of propagation given by wavevector:

  $\mathbf{k} = (k_x, k_y, k_z)$ where $|\mathbf{k}| = 2\pi/\lambda = \omega/c$
- Can also define

  $\mathbf{E} = (E_x, E_y, E_z)$
- Plane wave propagating in z-direction

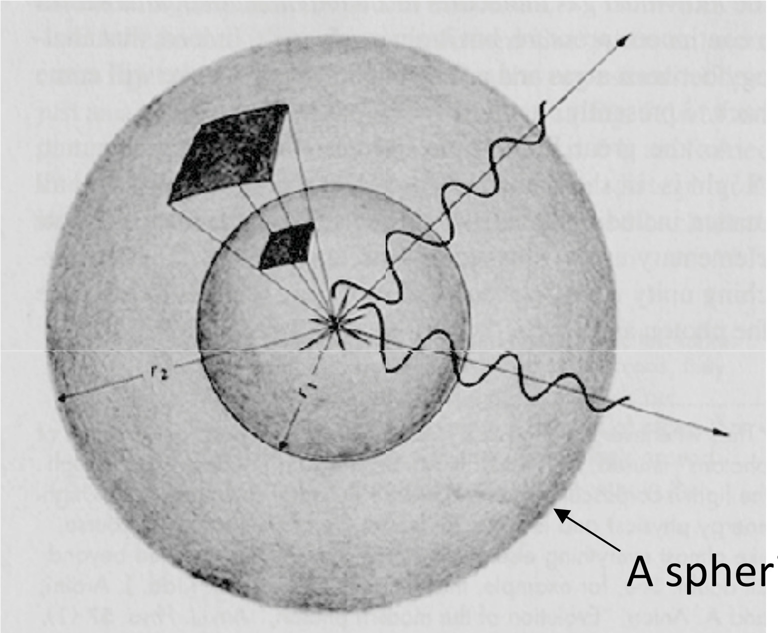
$$\mathbf{E}(z,t) = \mathrm{Re}\{\mathbf{E}_0 e^{i(kz-\omega t)}\} = \tfrac{1}{2}\{\mathbf{E}_0 e^{i(kz-\omega t)} + \mathbf{E}_0^* e^{-i(kz-\omega t)}\}$$



Key words: energy, momentum, wavelength, frequency, phase, amplitude…

Spherical waves

A spherical wave is also a solution to Maxwell's equations and is a good model for the light scattered by a molecule.



Note that k and r are **not** vectors here!

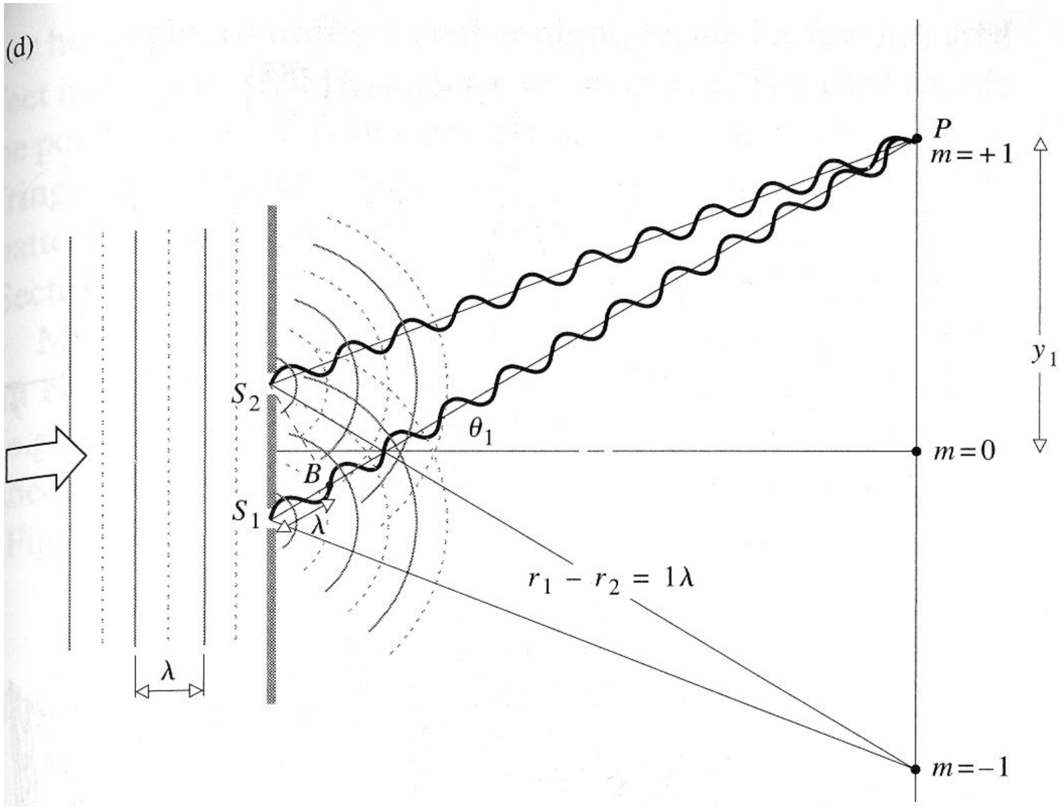
$$E(\vec{r},t) \propto \left(E_0 / r\right)\mathrm{Re}\{\exp[i(kr-\omega t)]\}$$

- where *k* is a scalar, and
- *r* is the radial magnitude.

A spherical wave has spherical wave-fronts.

Unlike a plane wave, whose amplitude remains constant as it propagates, a spherical wave weakens. Its irradiance goes as $1/r^2$.

Young's double slit interference experiment



order m maxima occur at:

$$m\lambda \approx a \sin\theta_m \approx a \frac{y_m}{s}$$

Interference

$$\mathbf{E}(\mathbf{r}) = \mathbf{E}_0 e^{i\mathbf{k}\cdot\mathbf{r}}$$

$$\mathbf{E}(\mathbf{r},t) = \mathrm{Re}\{\mathbf{E}(\mathbf{r})e^{-i\omega t}\}$$

$$= \mathrm{Re}\{\mathbf{E}_0 e^{i\mathbf{k}\cdot\mathbf{r}} e^{-i\omega t}\}$$

$$= \mathrm{Re}\{\mathbf{E}_0 e^{i(\mathbf{k}\cdot\mathbf{r}-\omega t)}\}$$

Michelson Interferometer

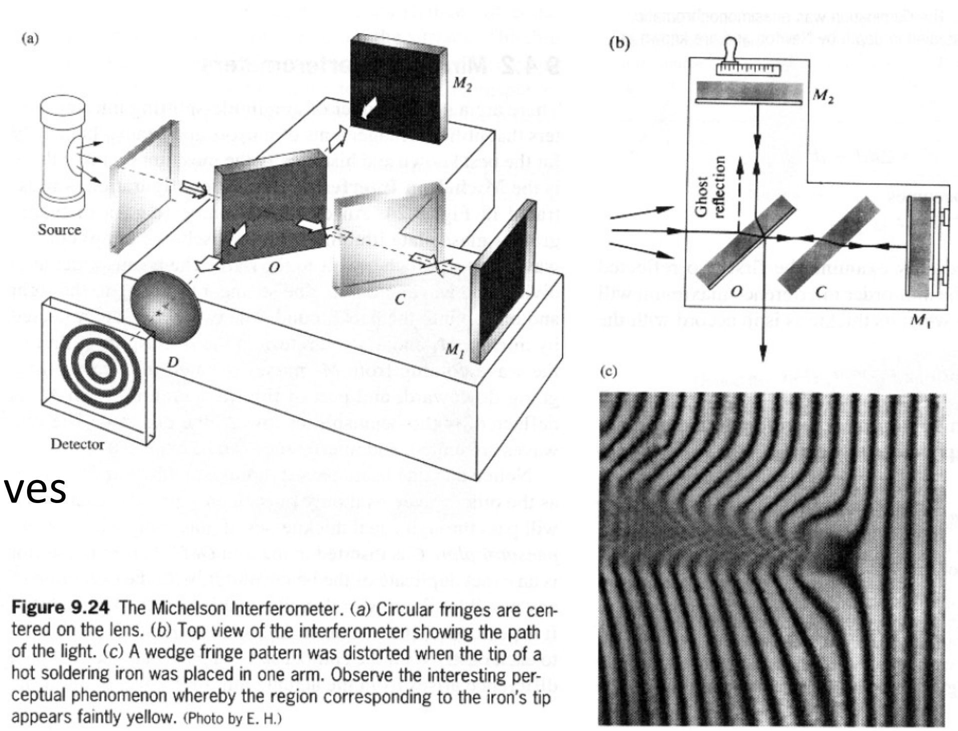


Figure 9.24 The Michelson Interferometer. (*a*) Circular fringes are centered on the lens. (*b*) Top view of the interferometer showing the path of the light. (*c*) A wedge fringe pattern was distorted when the tip of a hot soldering iron was placed in one arm. Observe the interesting perceptual phenomenon whereby the region corresponding to the iron's tip appears faintly yellow. (Photo by E. H.)

Consider the Optical Path Difference (OPD)
Or simply the superposition of two plane waves

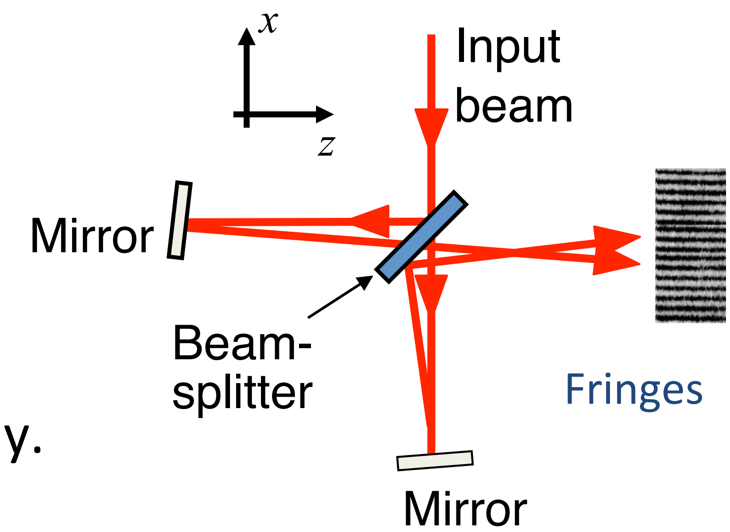
$$\mathbf{E}(\mathbf{r}) = \mathbf{E}_1 e^{i\mathbf{k}_1\bullet\mathbf{r}_1} + \mathbf{E}_2 e^{i\mathbf{k}_2\bullet\mathbf{r}_2}$$

$$I = |\mathbf{E}(\mathbf{r})|^2 = \mathbf{E}\times\mathbf{E}^*$$

Key words/Topics:
Michelson Interferometer, Dielectric thin film, Anti-reflection coating, Fringes of equal thickness, Newton rings.

The Michelson Interferometer and Spatial Fringes

- Suppose we misalign the mirrors
- so the beams cross at an angle
- when they recombine at the beam
- splitter. And we won't scan the delay.

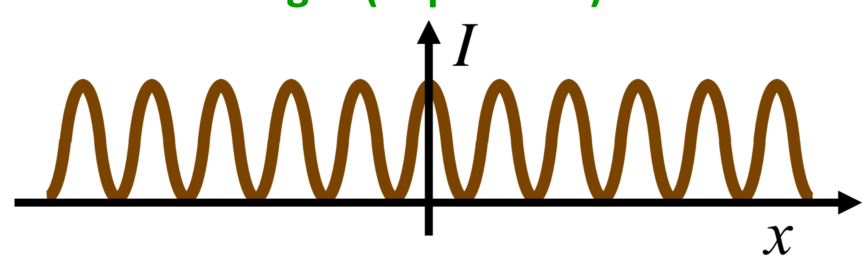


- If the input beam is a plane wave, the cross term becomes:

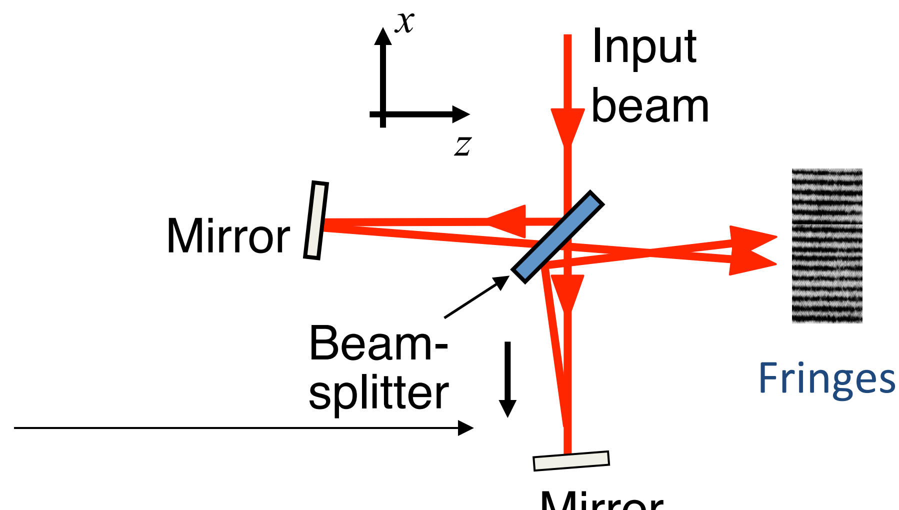
$$\mathrm{Re}\left\{E_0 \exp\left[i(\omega t - kz\cos\theta - kx\sin\theta\right] E_0^* \exp\left[-i(\omega t - kz\cos\theta + kx\sin\theta\right]\right\}$$

$$\propto \mathrm{Re}\left\{\exp\left[-2ikx\sin\theta\right]\right\}$$

$$\propto \cos(2kx\sin\theta)$$



Crossing beams maps delay onto position.



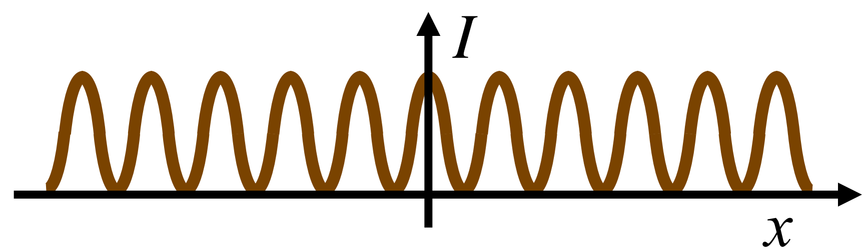
•Suppose we change one arm's path length.

$$\mathrm{Re}\left\{E_0 \exp\left[i(\omega t - kz\cos\theta - kx\sin\theta + 2kd\right] E_0^* \exp\left[-i(\omega t - kz\cos\theta + kx\sin\theta\right]\right\}$$

$$\propto \mathrm{Re}\left\{\exp\left[-2ikx\sin\theta + 2kd\right]\right\}$$

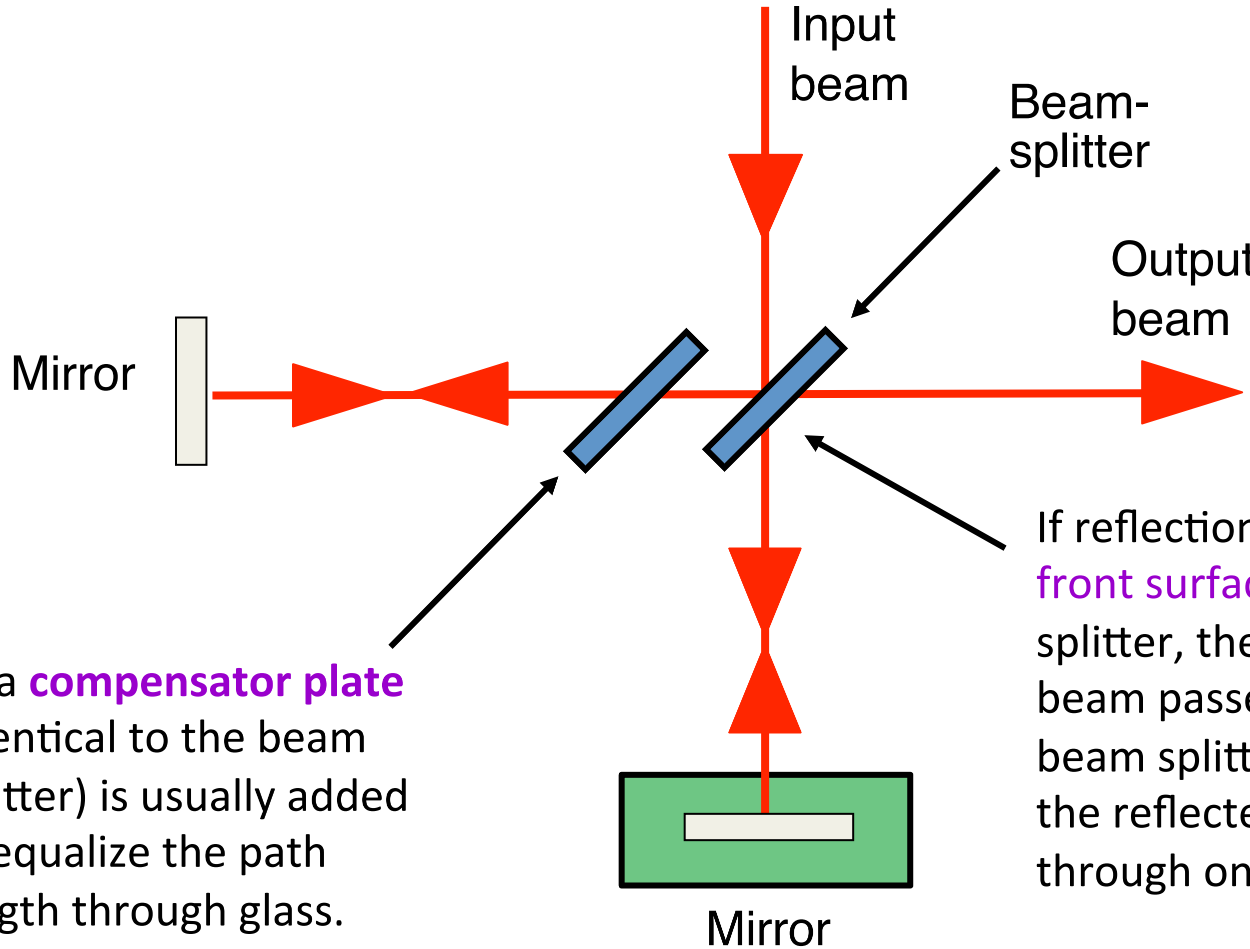
$$\propto \cos(2kx\sin\theta + 2kd)$$

Fringes (in position)



The fringes will shift in phase by $2kd$.

Michelson interferometers: the compensator plate



So a **compensator plate** (identical to the beam splitter) is usually added to equalize the path length through glass.

If reflection occurs off the front surface of beam splitter, the transmitted beam passes through beam splitter three times; the reflected beam passes through only once.

Interference Fringes and Newton Rings

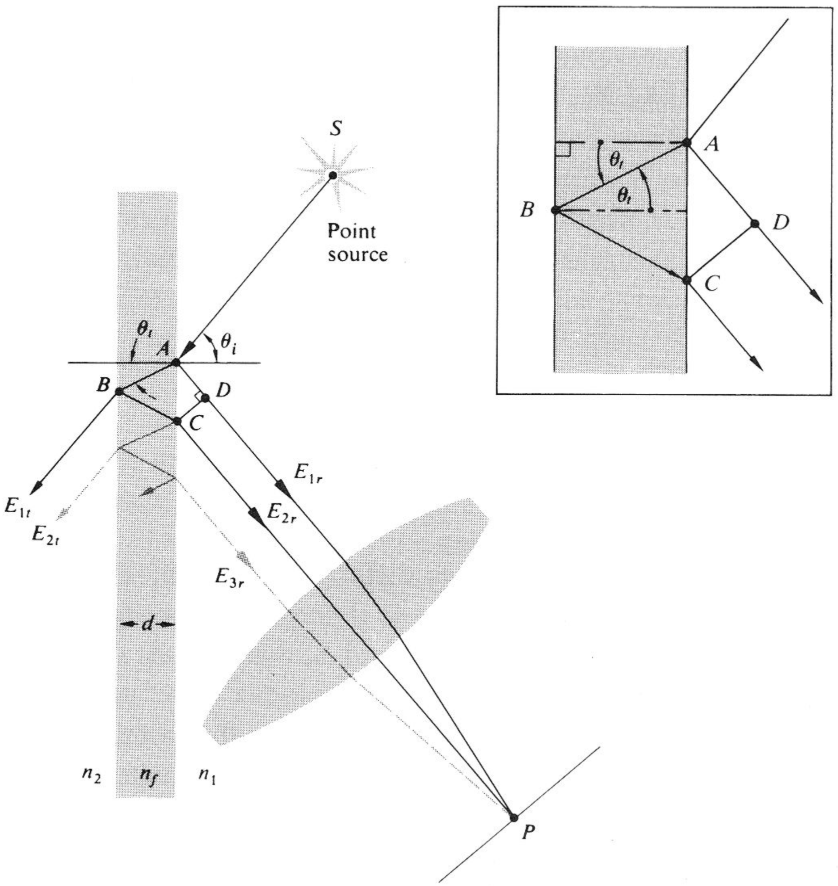
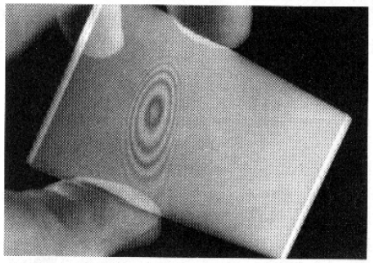


Figure 9.17 Fringes of equal inclination.



Newton's rings with two microscope slides. The thin film of air between the slides creates the interference pattern. (Photo by E. H.)

Newton's Rings

From the figure, if $R \gg d$, then

$$x^2 R^2-(R-d)^2 \Rightarrow x^2 \approx 2Rd$$

The interference maximum will occur if

$$2n_f d_m = (m+\frac{1}{2})\lambda_0$$

Thus, the radius of the bring rings are

$$x_m = \sqrt{(m+\frac{1}{2})\lambda_f R}$$

Similarly, the radius of dark rings are

$$x_m = \sqrt{m\lambda_f R}$$

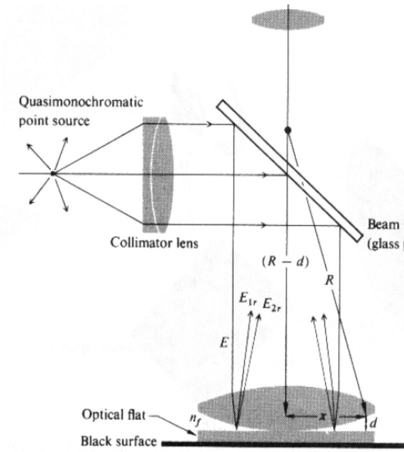
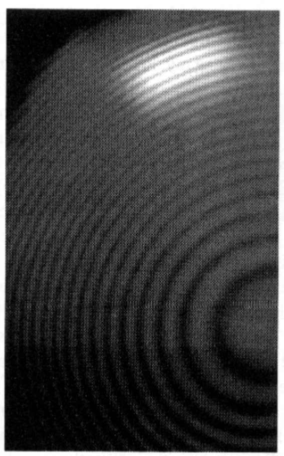


Figure 9.23 A standard setup to observe Newton's rings



Interference from the thin air film between a convex lens and the flat sheet of glass it rests on. The illumination was quasimonochromatic. These fringes were first studied in depth by Newton and are known as Newton's rings. (Photo by E.H.)

Phase shift on reflection at an interface

Near-normal incidence

p phase shift if $n_i < n_t$

0 (or 2p phase shift) if $n_i > n_t$

$$r_\perp = \left(\frac{E_{0r}}{E_{0i}}\right)_\perp = \frac{n_i \cos\theta_i - n_t \cos\theta_t}{n_i \cos\theta_i + n_t \cos\theta_t}$$

$$t_\perp = \left(\frac{E_{0t}}{E_{0i}}\right)_\perp = \frac{2n_i \cos\theta_i}{n_i \cos\theta_i + n_t \cos\theta_t}$$

$$r_\parallel = \left(\frac{E_{0r}}{E_{0i}}\right)_\parallel = \frac{n_t \cos\theta_i - n_i \cos\theta_t}{n_t \cos\theta_i + n_i \cos\theta_t}$$

$$t_\parallel = \left(\frac{E_{0t}}{E_{0i}}\right)_\parallel = \frac{2n_i \cos\theta_i}{n_t \cos\theta_i + n_i \cos\theta_t}$$

Note: independent of polarization

$\theta_i = 0$ and $\theta_t = 0$

$$r_\perp = r_\parallel = \frac{n_t - n_i}{n_t + n_i}$$

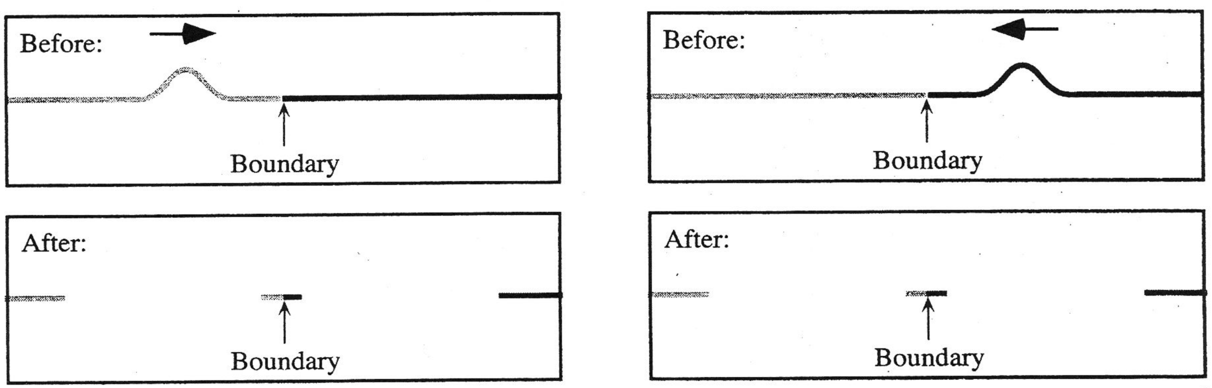
$$t_\perp = t_\parallel = \frac{2n_i}{n_t + n_i}$$

$$R_\perp = R_\parallel = \left(\frac{n_t - n_i}{n_t + n_i}\right)^2$$

$$T_\perp = T_\parallel = \frac{4n_t n_i}{(n_t + n_i)^2}$$

I. Transmission and reflection at a boundary

The sketches below show a pulse approaching a boundary between two springs. In one case, the pulse approaches the boundary from the left; in the other, from the right. The springs are the same in both cases, and the linear mass density is greater for the spring on the right than for the spring on the left.

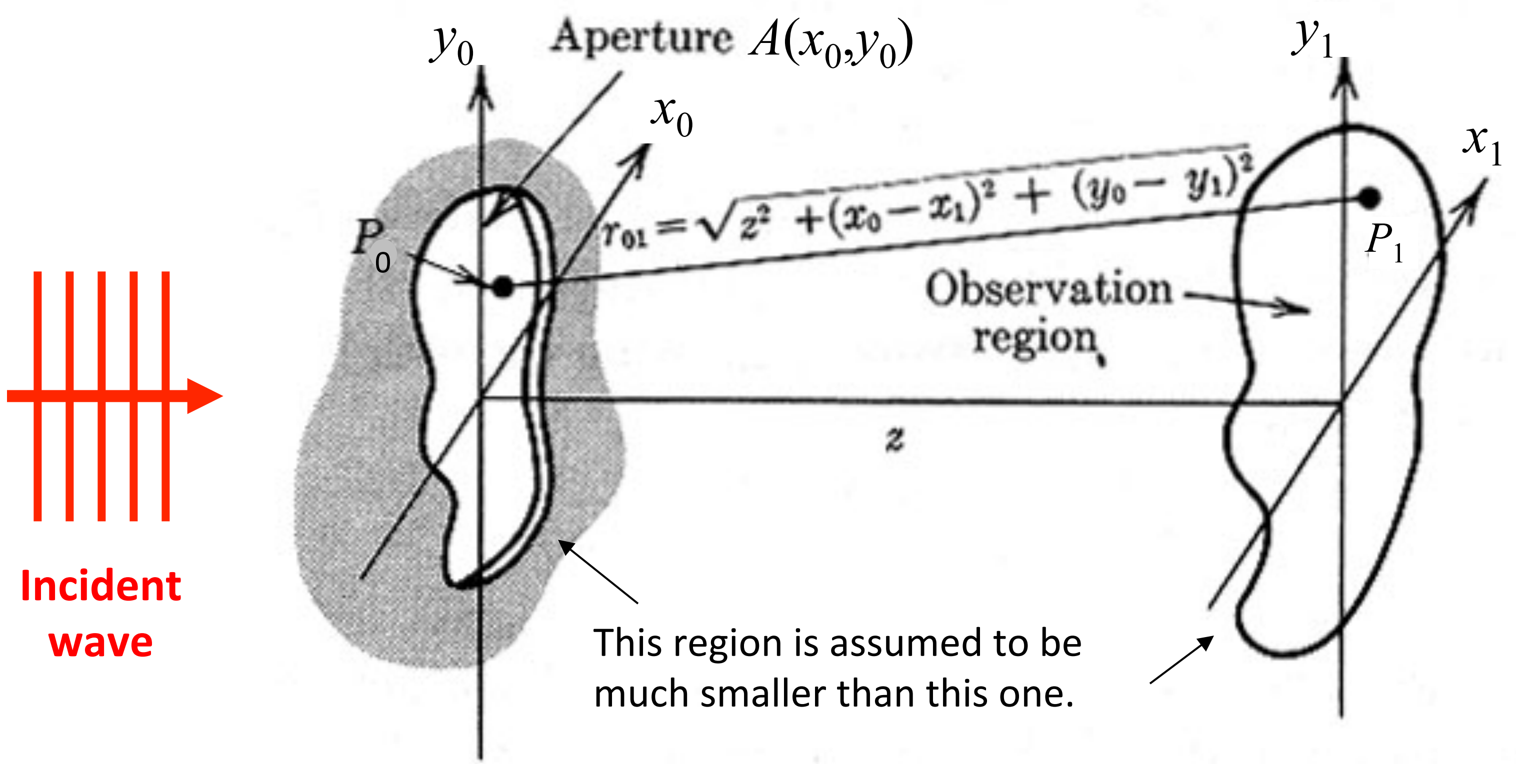


Complete the sketches to show the shape of the springs a short time after the trailing edge of the pulse shown has reached the boundary. Be sure to show correctly (1) the relative widths of the pulses and (2) which side of the spring each pulse is on. (Ignore relative amplitudes.)

Diffraction Geometry

We wish to find the light electric field after a screen with a hole in it.

This is a very general problem with far-reaching applications.



What is $E(x_1,y_1)$ at a distance z from the plane of the aperture?

Fraunhofer Diffraction: The Far Field

We can approximate r_{01} in the denominator by z, and if D is the size of the aperture, $D^2 \geq x_0^2 + y_0^2$, so when $k D^2 / 2z << 1$, the quadratic terms << 1, so we can neglect them:

$$r_{01} = \sqrt{z^2 + (x_0 - x_1)^2 + (y_0 - y_1)^2} \approx z\left[1 + (x_0 - x_1)^2 / 2z^2 + (y_0 - y_1)^2 / 2z^2\right]$$

$$kr_{01} \approx kz + k\left(x_0^2 - 2x_0 x_1 + x_1^2\right)/2z + k\left(y_0^2 - 2y_0 y_1 + y_1^2\right)/2z$$

Small, so neglect these terms.

Independent of x_0 and y_0, so factor these out.

$$E(x_1, y_1) = \frac{\exp(ikz)}{i\lambda z} \exp\left[ik\frac{x_1^2 + y_1^2}{2z}\right] \iint_{A(x_0, y_0)} \exp\left\{-\frac{ik}{z}(x_0 x_1 + y_0 y_1)\right\} E(x_0, y_0)\, dx_0\, dy_0$$

This condition means going a distance away: $z >> kD^2/2 = \pi D^2/\lambda$

If $D = 1$ mm and $\lambda = 1\ \mu m$, then $z >> 3$ m.

Diffraction Solution

The field in the observation plane, $E(x_1,y_1)$, at a distance z from the aperture plane is given by:

$$E(x_1,y_1,z) \;=\; \iint_{A(x_0,y_0)} h(x_1-x_0,y_1-y_0,z)E(x_0,y_0)\,dx_0\,dy_0$$

where :

$$h(x_1-x_0,y_1-y_0,z)=\frac{1}{i\lambda}\frac{\exp(ikr_{01})}{r_{01}}$$

Spherical wave

and :

$$r_{01}=\sqrt{z^2+\left(x_0-x_1\right)^2+\left(y_0-y_1\right)^2}$$

A very complicated result! And we cannot approximate r_{01} in the exp by z because it gets multiplied by k, which is big, so relatively small changes in r_{01} can make a big difference!

Fraunhofer Diffraction

We'll neglect the phase factors, and we'll explicitly write the aperture function in the integral:

$$E(x_1, y_1) \propto \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} \exp\left\{-\frac{ik}{z}\left(x_0 x_1 + y_0 y_1\right)\right\} A(x_0, y_0)\, E(x_0, y_0)\, dx_0\, dy_0$$

$E(x_0,y_0)$ = constant if a plane wave

This is just a Fourier Transform!

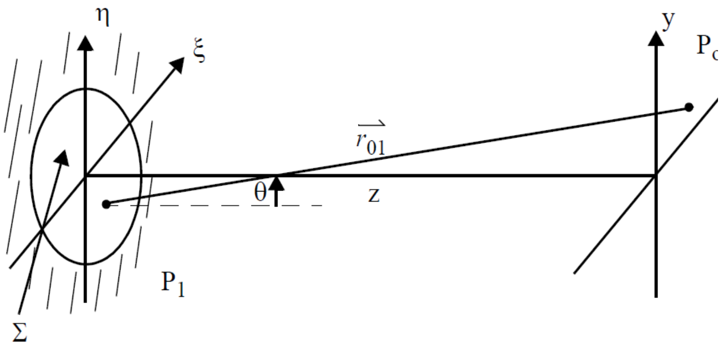
Interestingly, it's a Fourier Transform from position, x_0, to another position variable, x_1 (in another plane). Usually, the Fourier "conjugate variables" have reciprocal units (e.g., t & *w*, or x & k). The conjugate variables here are really x_0 and $k_x = kx_1/z$, which have reciprocal units.

So the far-field light field is the Fourier Transform of the apertured field!

Diffraction: Summary

<u>Fresnel approximation</u>

Huygens-Fresnel integral in rectangular coordinates:



$$r_{01} = [z^2 + (x-\xi)^2 + (y-\eta)^2]^{1/2}$$

The Fresnel approximation involves setting: $r_{01} \cong z$ in the denominator, and

$$r_{01} \cong z\left[1 + \frac{1}{2}\frac{(x-\xi)^2}{z} + \frac{1}{2}\frac{(y-\eta)^2}{z}\right] \quad \text{in exponent}$$

This is equivalent to the paraxial approximation in ray optics.

$$U(x,y) = \frac{\exp(jkz)}{j\lambda z}\int_{-\infty}^{\infty}\int d\xi d\eta U(\xi,\eta)\exp\left\{\frac{jk}{2z}[(x-\xi)^2 + (y-\eta)^2]\right\} \qquad \text{(A)}$$

Let's examine the validity of the Fresnel approximation in the Fresnel integral. The next higher order term in exponent must be small compared to 1. So the valid range of the Fresnel approximation is:

$$z^3 \gg \frac{\pi}{4\lambda}[(x-\xi)^2 + (y-\eta)^2]^2_{max}$$

For field sizes of 1 cm, $\lambda = 0.5\mu m$, we find $z \gg 25$ cm.

Actually we should look at the effect on the total integral. Upon closer analysis, it is found that the Fresnel approximation holds for a much closer z. This is referred to as the "near-field region".

Farther out in z, we can approximate the quadratic phase as flat

$$z \gg \frac{k(\xi^2 + \eta^2)_{max}}{2}$$

This region is referred to as the "far-field" or Fraunhofer region.

$$U(x,y) = \frac{e^{jkz}e^{j\frac{k}{2z}(x^2+y^2)}}{j\lambda z}\underbrace{\iint d\xi d\eta U(\xi,\eta)\exp\left[-j\frac{2\pi}{\lambda z}(x\xi + y\eta)\right]}_{\mathcal{F}\{U(\xi,\eta)\}\big|_{f_x = \frac{x}{\lambda z}, f_y = \frac{y}{\lambda z}}}$$

Now this is exactly the Fourier transform of the aperture distribution with

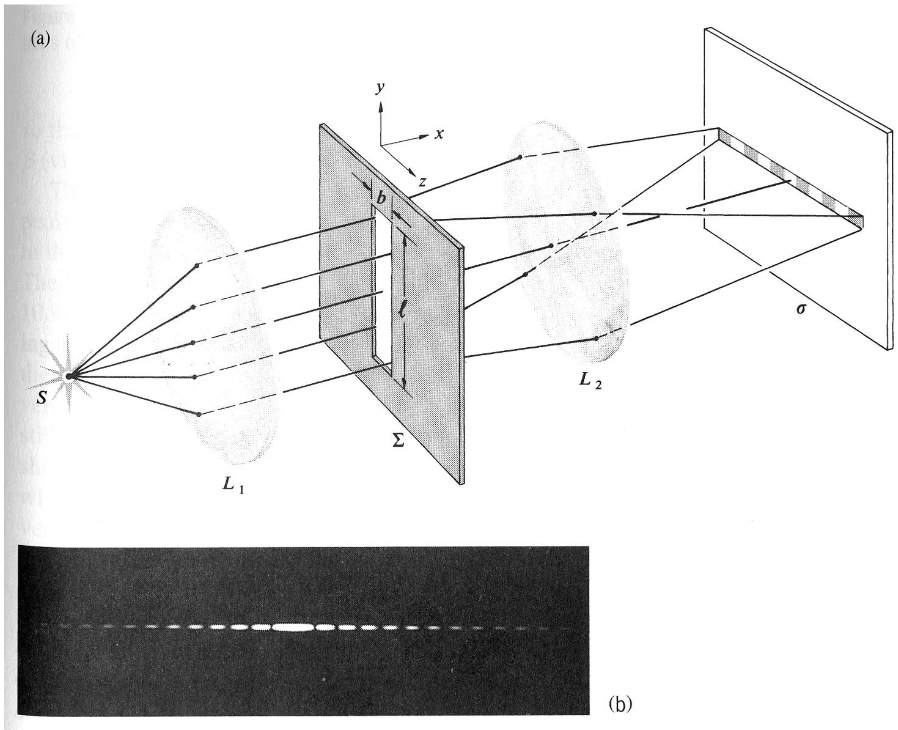
$$f_x = \frac{x}{\lambda z} \qquad f_y = \frac{y}{\lambda z}$$

The Fraunhofer region is farther out. For the field size of 1 cm, and $\lambda = 0.5\mu m$, we find the valid range of $z \gg 150$ meters!

Again, examining the full integral, Fraunhofer is actually accurate and usable to much closer distances.

Diffraction: single, double, multiple slits

Study Guide: Hecht Ch. 10.2.1-10.2.6 (detailed lengthy discussions), Fowles Ch. 5 (short but clear presentation), or Lecture Notes



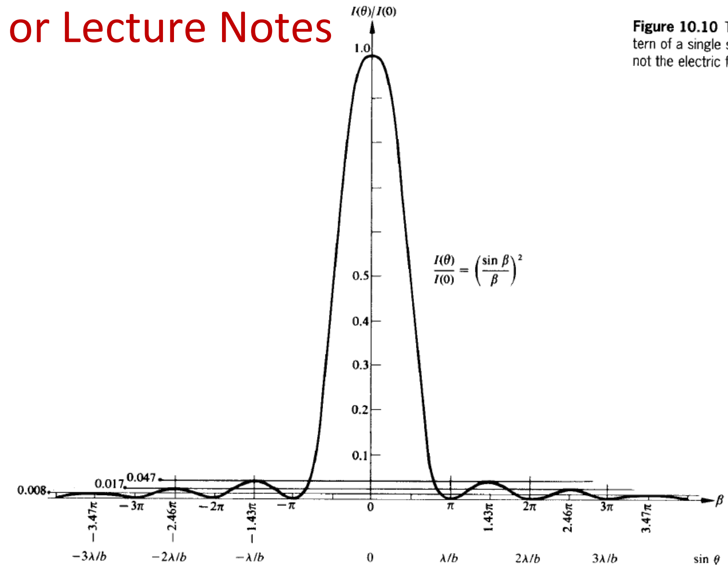


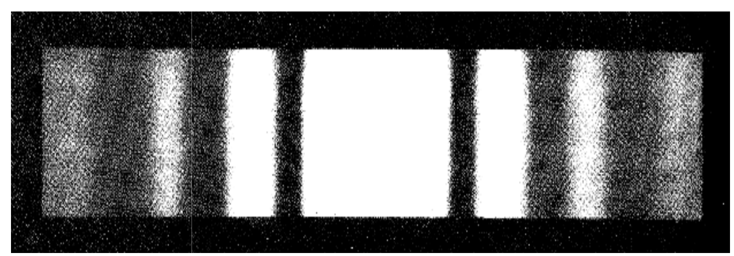
Figure 10.10 The Fraunhofer diffraction pattern of a single slit. This is the irradiance (and not the electric field) distribution

$$I(\beta) = I(0)\left(\frac{\sin\beta}{\beta}\right)^2$$

$$\beta = \frac{kb}{2}\sin\theta = \pi\frac{b}{\lambda}\sin\theta$$

Single Slit ($\Delta x \ll \Delta y \Rightarrow \beta_x \ll \beta_y$)

$\mathrm{sinc}(\beta_y)$ changes much faster than $\mathrm{sinc}(\beta_x)$



Java applet – Single Slit Diffraction
http://www.walter-fendt.de/ph14e/singleslit.htm

Diffraction: Double and Multiple Slits

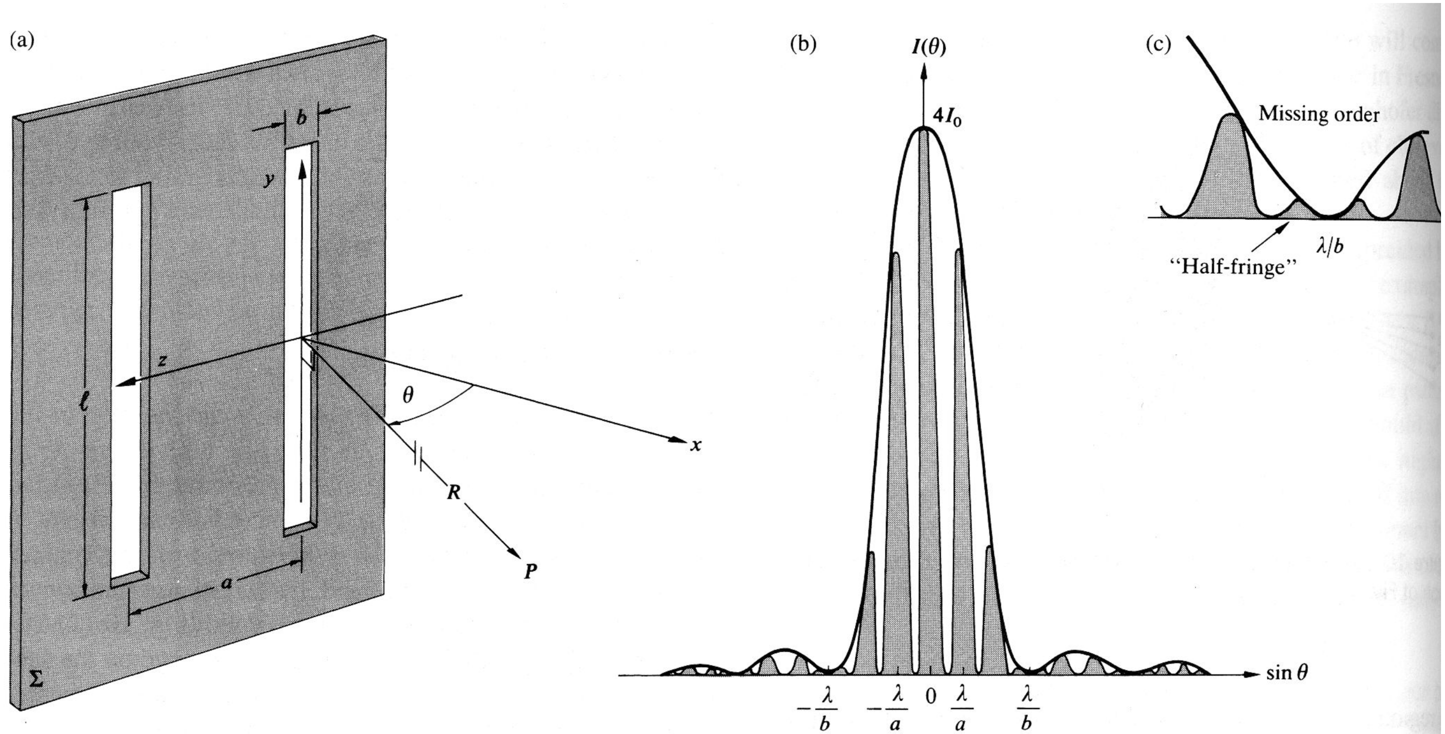
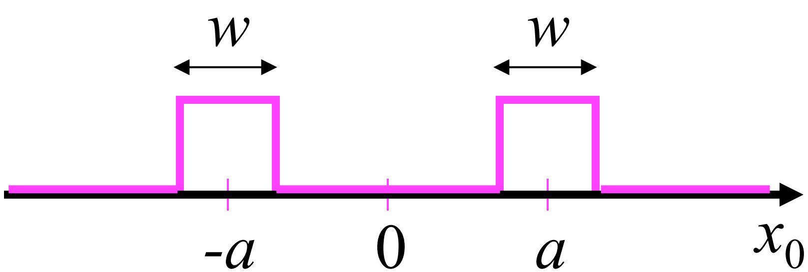


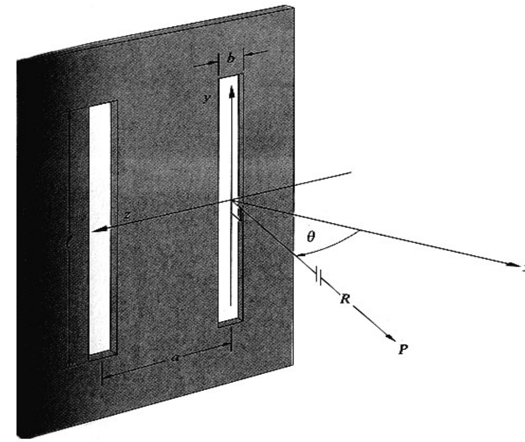
Figure 10.13 (*a*) Double-slit geometry. Point *P* on σ is essentially infinitely far away. (*b*) A double-slit pattern ($a = 3b$).

$$I(\theta) = I(0)\left(\frac{\sin\beta}{\beta}\right)^2\left(\frac{\sin N\gamma}{N\sin\gamma}\right)^2 \quad \beta = \frac{1}{2}kb\sin\theta; \quad \gamma = \frac{1}{2}ka\sin\theta$$

See also
http://demonstrations.wolfram.com/MultipleSlitDiffractionPattern/ and
http://wyant.optics.arizona.edu/multipleSlits/multipleSlits.htm

Fraunhofer diffraction from two slits (Fourier Transform)



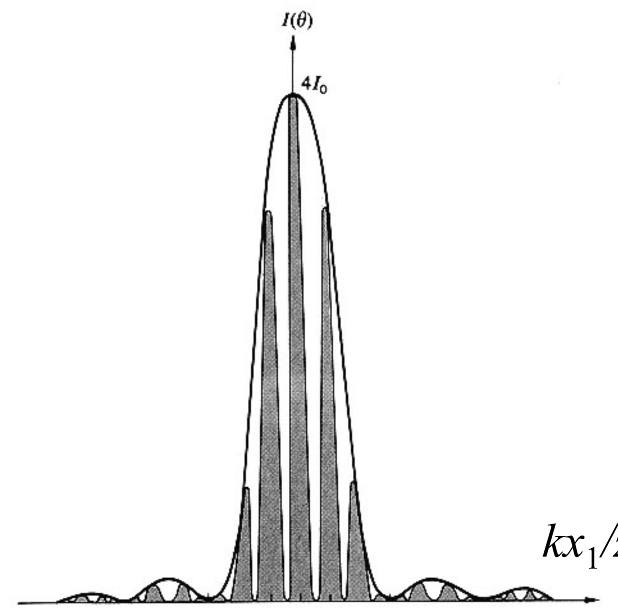


$$A(x_0) = \mathrm{rect}[(x_0+a)/w] + \mathrm{rect}[(x_0-a)/w]$$

$$E(x_1) \propto \mathscr{F}\{A(x_0)\}$$

$$\propto \mathrm{sinc}[w(kx_1/z)/2]\exp[+ia(kx_1/z)] + \mathrm{sinc}[w(kx_1/z)/2]\exp[-ia(kx_1/z)]$$

$$\boxed{E(x_1) \propto \mathrm{sinc}(wkx_1/2z)\cos(akx_1/z)}$$

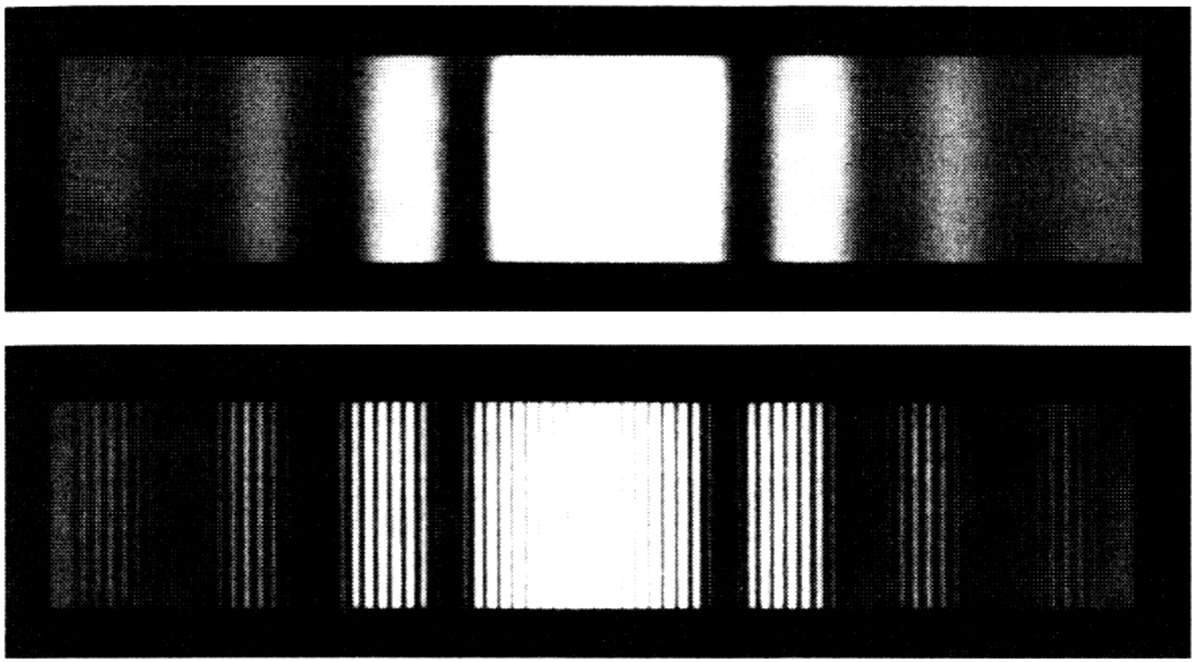


Diffraction from one- and two-slit screens

Fraunhofer diffraction patterns

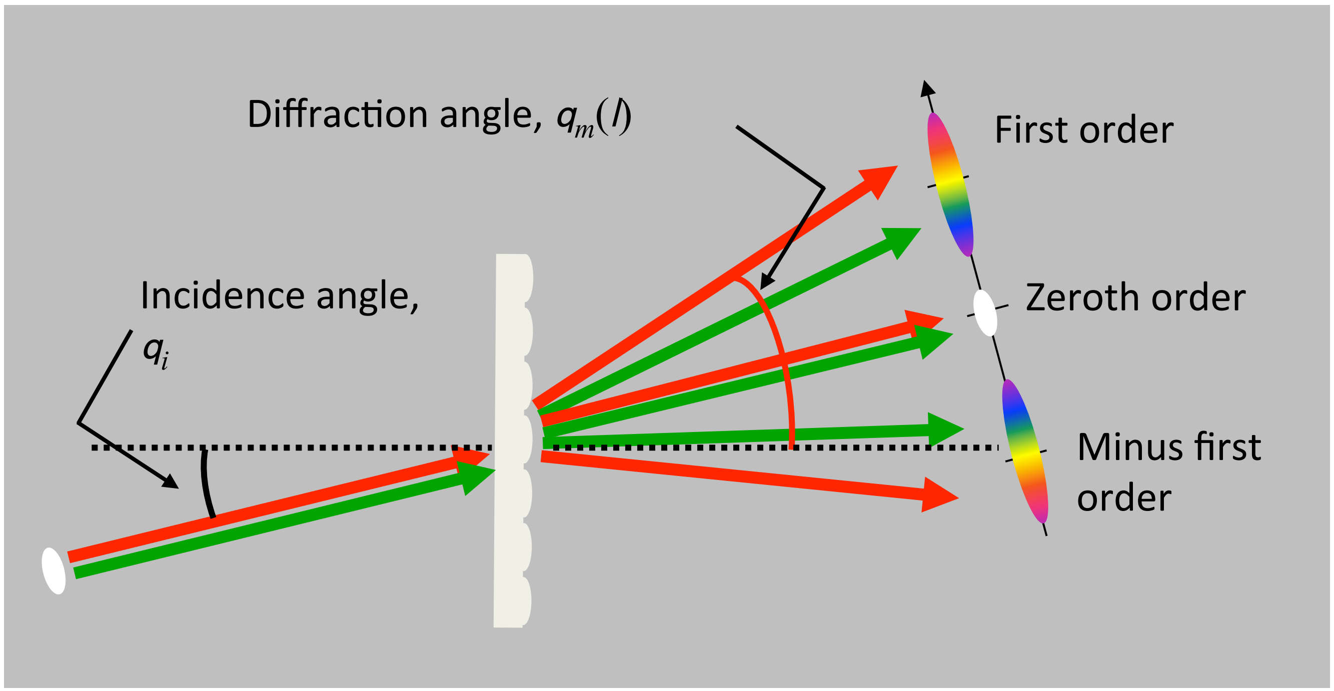
One slit

Two slits



Diffraction orders

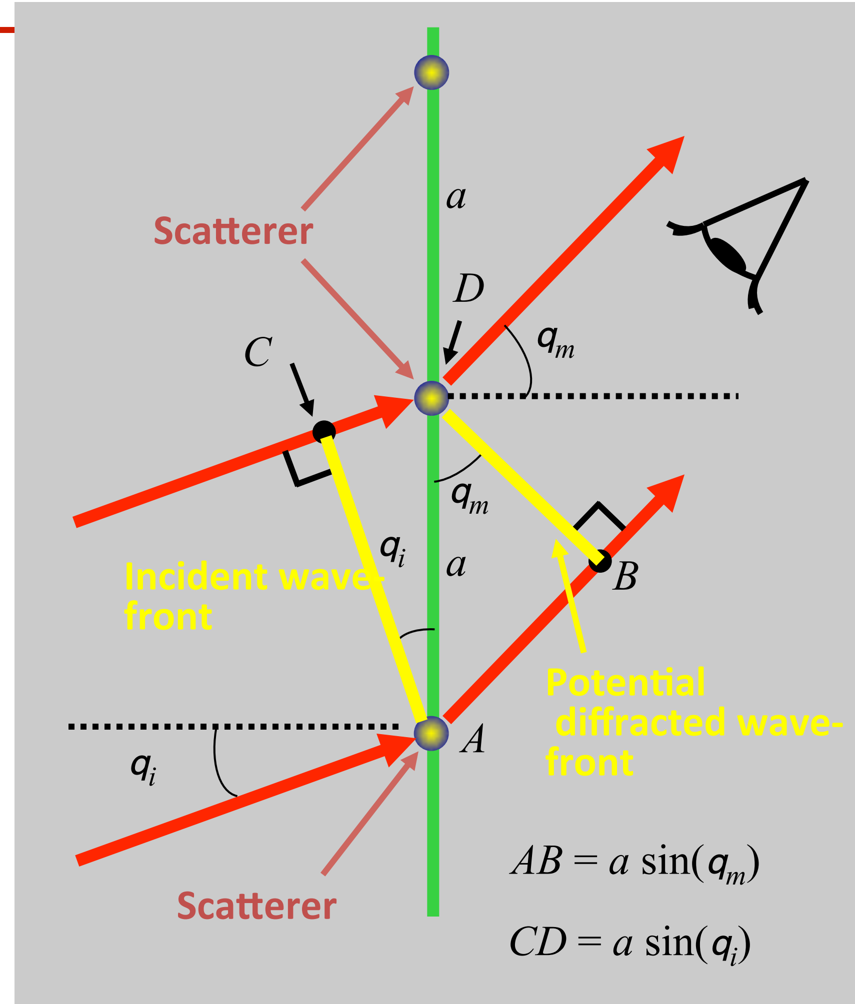
Because the diffraction angle depends on *l*, different wavelengths are separated in the nonzero orders.



No wavelength dependence occurs in zero order.

The longer the wavelength, the larger its deflection in each nonzero order.

Diffraction Gratings



•Scattering ideas explain what happens when light impinges on a periodic array of grooves. Constructive interference occurs if the delay between adjacent beamlets is an integral number, m, of wavelengths.

Path difference: $AB - CD = ml$

$$a[\sin(\theta_m) - \sin(\theta_i)] = m\lambda$$

where m is any integer.

A grating has solutions of zero, one, or many values of m, or **orders**.

Remember that m and q_m can be negative, too.

The Diffraction Grating

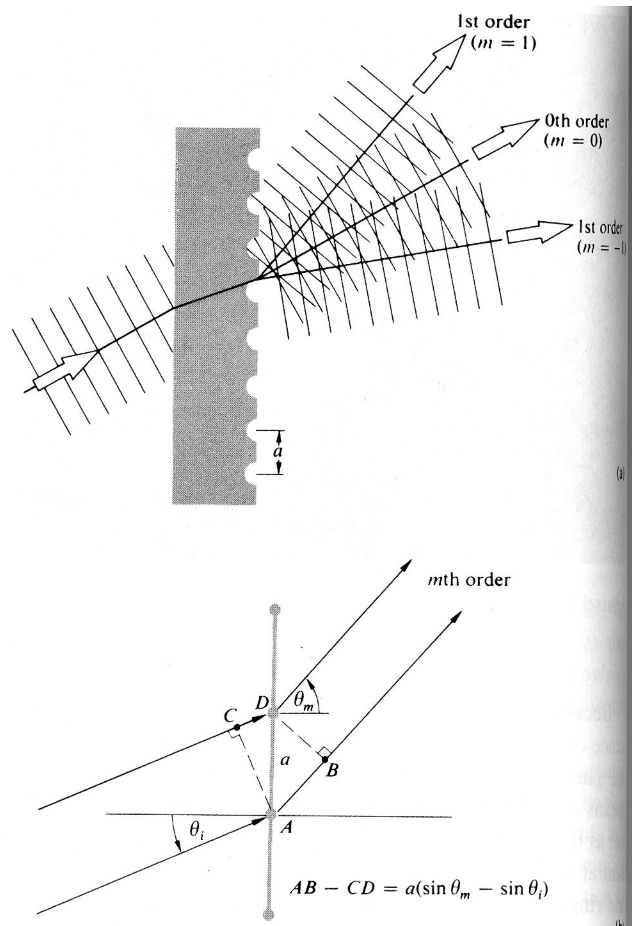


Figure 10.28 A transmission grating.

Grating Equation

(Optical Path Difference OPD= m λ)

$$a\left(\sin\theta_m - \sin\theta_i\right) = m\lambda$$

$$a\sin\theta_m = m\lambda$$ **Normal incidence θ_i =0**

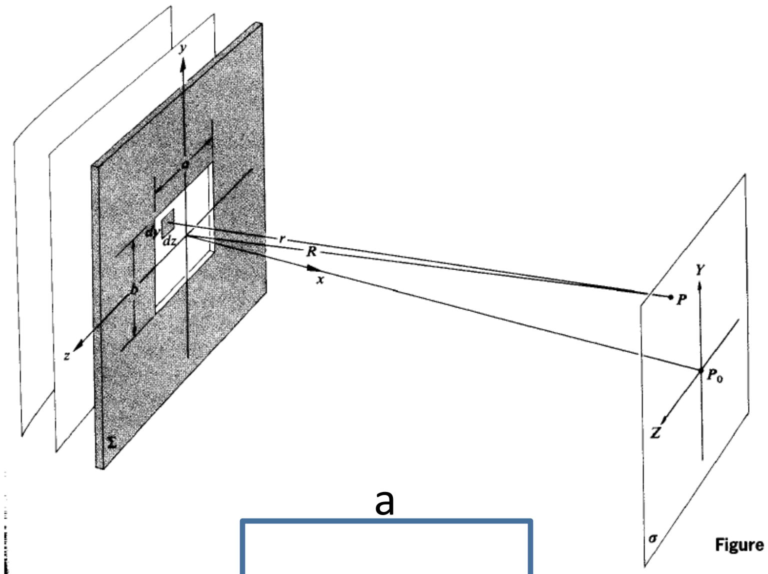
The chromatic/spectral resolving power of a grating

$$R \equiv \frac{\lambda}{\Delta\lambda} = mN$$

m is the order number, and
N is the total number of gratings.

Uniform Rectangular Aperture

Uniform Rectangular Aperture



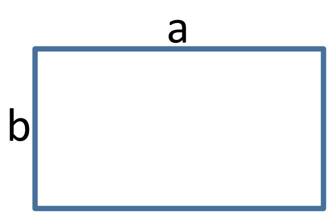
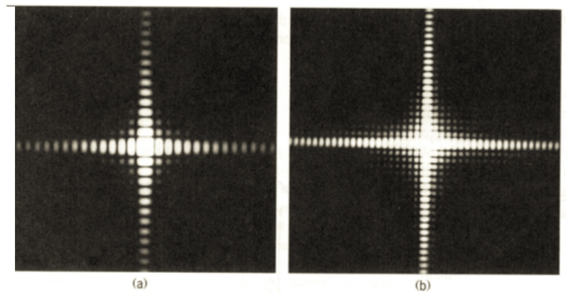
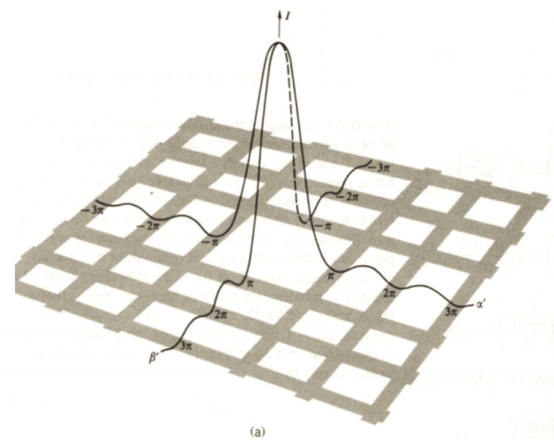


Figure 10.19 A rectangular aperture.



(a) Fraunhofer pattern of a square aperture. (b) The same pattern further exposed to bring out some of the faint terms. (Photos by E. H.)



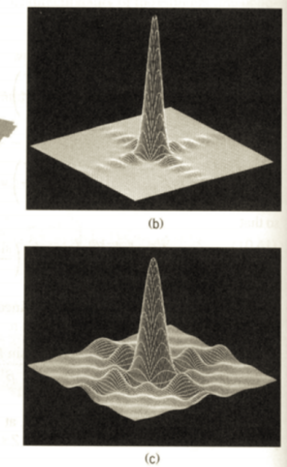
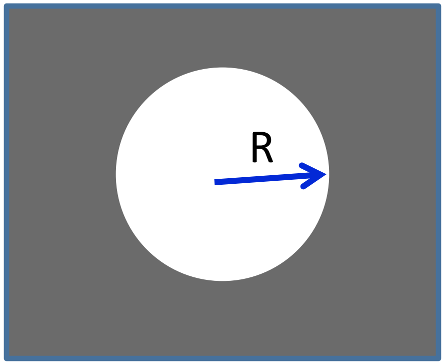
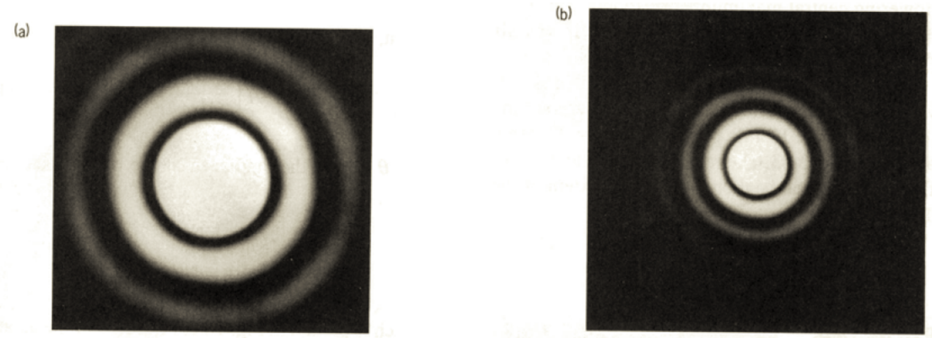


Figure 10.20 (a) The irradiance distribution for a square aperture. (b) The irradiance produced by Fraunhofer diffraction at a square aperture. (c) The electric-field distribution produced by Fraunhofer diffraction via a square aperture. (Photos courtesy R. G. Wilson, Illinois Wesleyan University.)

$$I(\theta) = I(0)\left(\frac{\sin\alpha}{\alpha}\right)^2\left(\frac{\sin\beta}{\beta}\right)^2 \quad \alpha = \frac{1}{2}ka\sin\theta; \quad \beta = \frac{1}{2}kb\sin\theta$$

Uniform Circular Aperture





Airy rings using (a) a 0.5-mm hole diameter and (b) a 1.0-mm hole diameter. (Photo by E. H.)

$$I(\theta) = I(0)\left(\frac{2J_1(\rho)}{\rho}\right)^2$$

$$\rho = kR\sin\theta; \quad k = \frac{2\pi}{\lambda}$$

A circular aperture yields a diffracted "Airy Pattern," which involves a Bessel function.

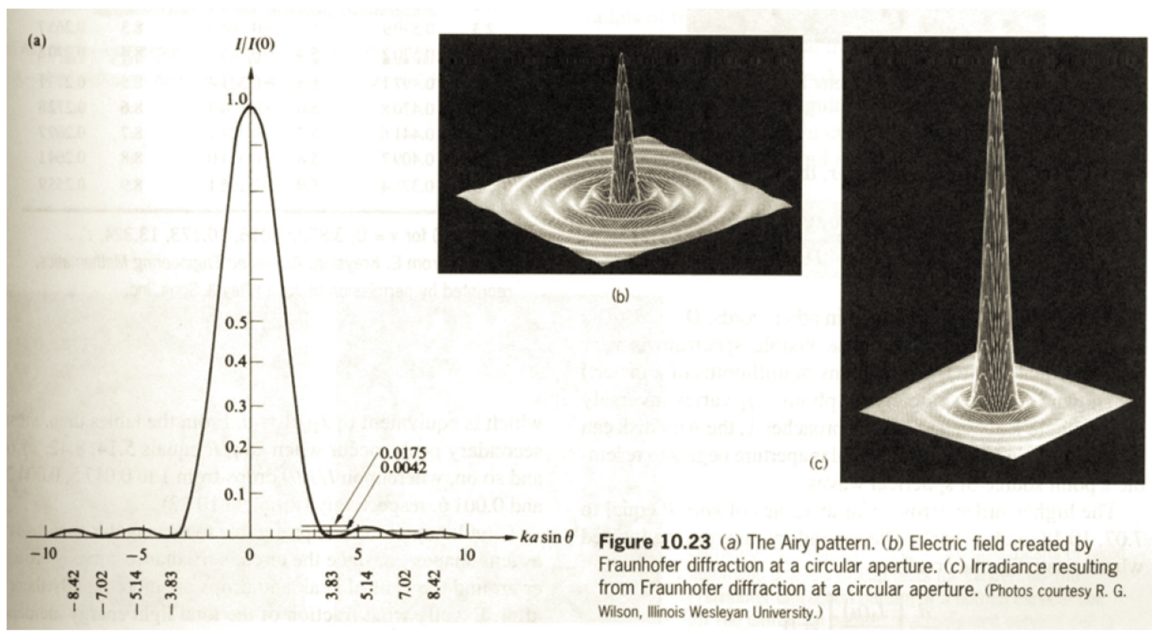
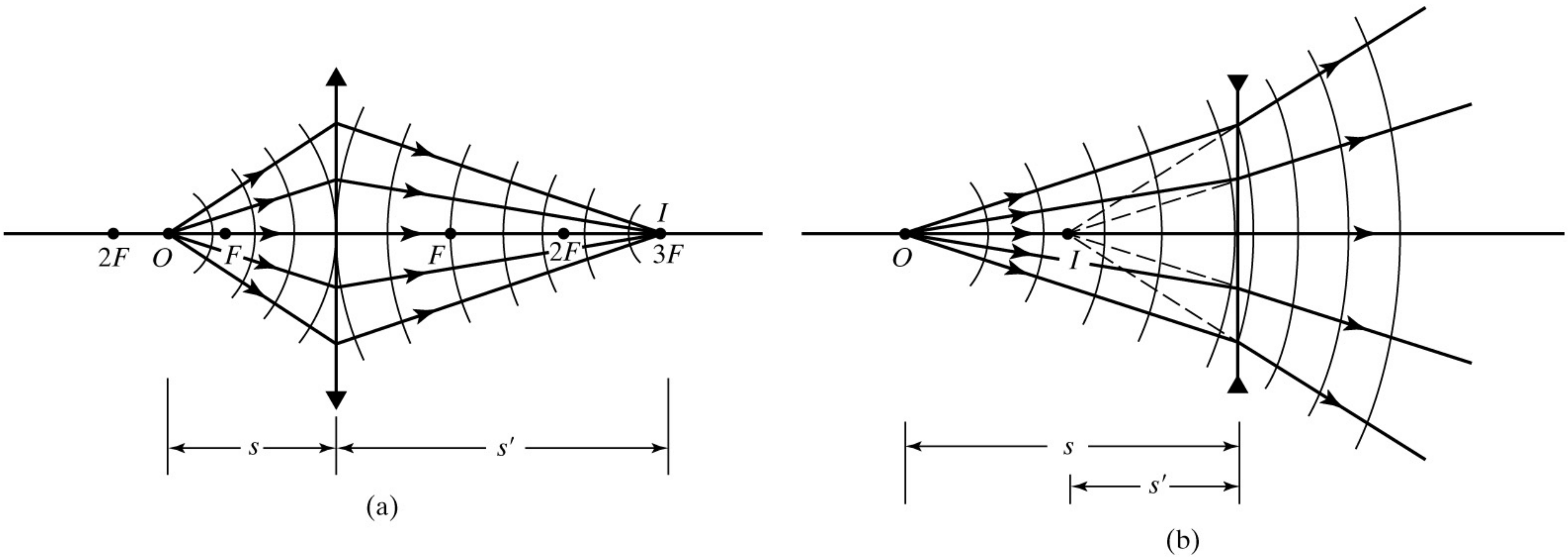


Figure 10.23 (a) The Airy pattern. (b) Electric field created by Fraunhofer diffraction at a circular aperture. (c) Irradiance resulting from Fraunhofer diffraction at a circular aperture. (Photos courtesy R. G. Wilson, Illinois Wesleyan University.)

Change in curvature of wavefronts by a thin lens



Wave optics of a lens

We have previously seen that light passing through a lens experiences a phase delay given by:

$$\varphi(x,y) = \exp\left[-jk(n-1)\left(\frac{x^2+y^2}{2}\right)\left(\frac{1}{R_1}-\frac{1}{R_2}\right)\right] \quad \text{(neglecting the constant phase)}$$

The focal length, f is given by:

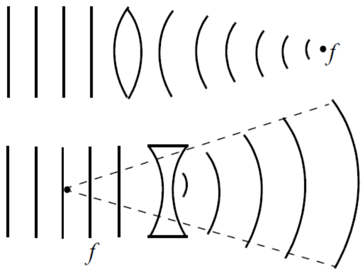
$$\boxed{\frac{1}{f} \equiv (n-1)\left(\frac{1}{R_1}-\frac{1}{R_2}\right)} \quad \text{The "lens makers formula"}$$

The transmission function is now:

$$\boxed{\varphi(x,y) = \exp\left[-j\frac{k}{2f}(x^2+y^2)\right]}$$

This is the paraxial approximation to the spherical phase

Note: the incident plane-wave is converted to a spherical wave converging to a point at f behind t lens (f positive) or diverging from the point at f in front of lens (f negative).



Diffraction from the lens pupil

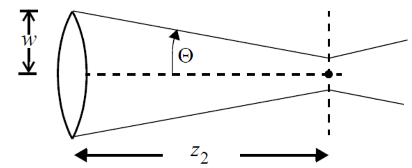
Suppose the lens is illuminated by a plane wave, amplitude A. The lens "pupil function" is $P(x,y)$.

The full effect of the lens is $\quad U_l'(x,y) = \varphi(x,y)P(x,y)$

The focal plane amplitude distribution is a Fourier transform of the lens pupil function $P(x,y)$, multiplied by a quadratic phase term. However, the intensity distribution is exactly

$$I_f(u,v) = \frac{A^2}{\lambda^2 f^2}|\mathcal{F}[P(x,y)]|^2 \qquad f_x = \frac{u}{\lambda f} \quad f_y = \frac{v}{\lambda f}$$

Example: a circular lens, with radius w



$$P = \mathrm{circ}\left(\frac{q}{w}\right) \qquad (q^2 = x^2+y^2)$$

$$\text{let} \quad h(r) = \mathcal{F}[P(\lambda z_2 q)] = \mathcal{F}\left[\mathrm{circ}\left(\frac{\lambda z_2 q}{w}\right)\right] \qquad (r^2 = u^2+v^2)$$

$$= \frac{A}{\lambda z_2}\left[2\frac{J_1(2\pi wr/\lambda z_2)}{2\pi wr/\lambda z_2}\right]$$

$$|h(r)|^2 = \frac{A^2}{\lambda^2 z_2^2}\left[2\frac{J_1(2\pi wr/\lambda z_2)}{2\pi wr/\lambda z_2}\right]^2$$

The spot diameter is (for an aperture with a radius w, diameter D).

$$d = 1.22\frac{\lambda f}{w} = 1.22\frac{\lambda}{\theta} = 2.44\frac{\lambda f}{D}$$

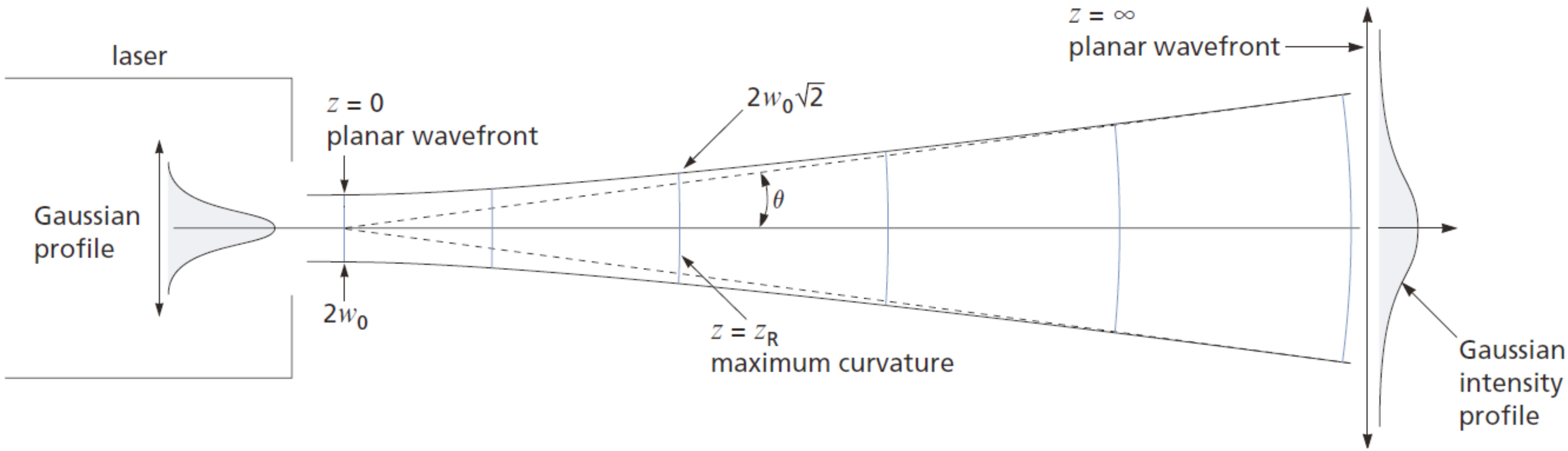
The resolution of the lens as defined by the "Rayleigh" criterion is

$$d/2 = 0.61\lambda/\theta$$

For a small angle θ,

$$d/2 = 0.61\lambda/\sin\theta = 0.61\frac{\lambda}{NA}$$

Gaussian Beam Optics



$$I(r) = I_0 e^{-2r^2/w^2} = \frac{2P}{\pi w^2} e^{-2r^2/w^2}$$

$$R(z) = z\left[1 + \left(\frac{\pi w_0^2}{\lambda z}\right)^2\right]$$

and

$$w(z) = w_0\left[1 + \left(\frac{\lambda z}{\pi w_0^2}\right)^2\right]^{1/2}$$

$$w(z) = w_o\left[1 + \left(\frac{\lambda z}{\pi w_o^2}\right)^2\right]^{1/2} = w_o\left[1 + \left(\frac{z}{z_R}\right)^2\right]^{1/2} \qquad (2)$$

where we have defined a new parameter, called the Rayleigh range,

$$z_R = \pi w_o^2 / \lambda, \qquad (3)$$

which combines the wavelength and waist radius into a single parameter and completely describes the divergence of the Gaussian beam. Note that the Rayleigh range is the distance from the beam waist to the point at which the beam radius has increased to $\sqrt{2} w_o$. For a 633 nm red He-Ne laser with a waist of 0.4 mm, $z_R \approx 0.8$ m.

When $z >> z_R$, Eq. (2) simplifies to $w = w_o z / z_R$ and the laser beam diverges at a constant angle

$$\theta = \frac{w}{z} = \frac{w_o}{z_R} = \frac{\lambda}{\pi w_o} \qquad (4)$$

Review Lab #2

Note that the smaller the Rayleigh range, the more rapidly the beam diverges.

Basic Fourier Optics
(Optional)
You may solve some problems 'faster' knowing Fourier transform. But knowledge of Fourier transform and Fourier optics is not required for the final exam.

Fourier Transform Notation

There are several ways to denote the Fourier transform of a function.

If the function is labeled by a lower-case letter, such as f,
we can write:

$$f(t) \implies F(w)$$

If the function is already labeled by an upper-case letter, such as E, we can write:

$$E(t) \to \mathscr{F}\{E(t)\}$$

or:

$$E(t) \to \tilde{E}(\omega)$$

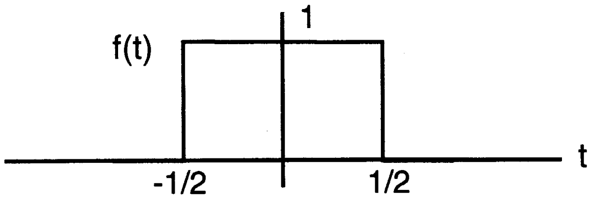
Example: the Fourier Transform of a rectangle function: rect(t)

$$F(\omega) = \int_{-1/2}^{1/2} \exp(-i\omega t)\,dt = \frac{1}{-i\omega}\left[\exp(-i\omega t)\right]_{-1/2}^{1/2}$$

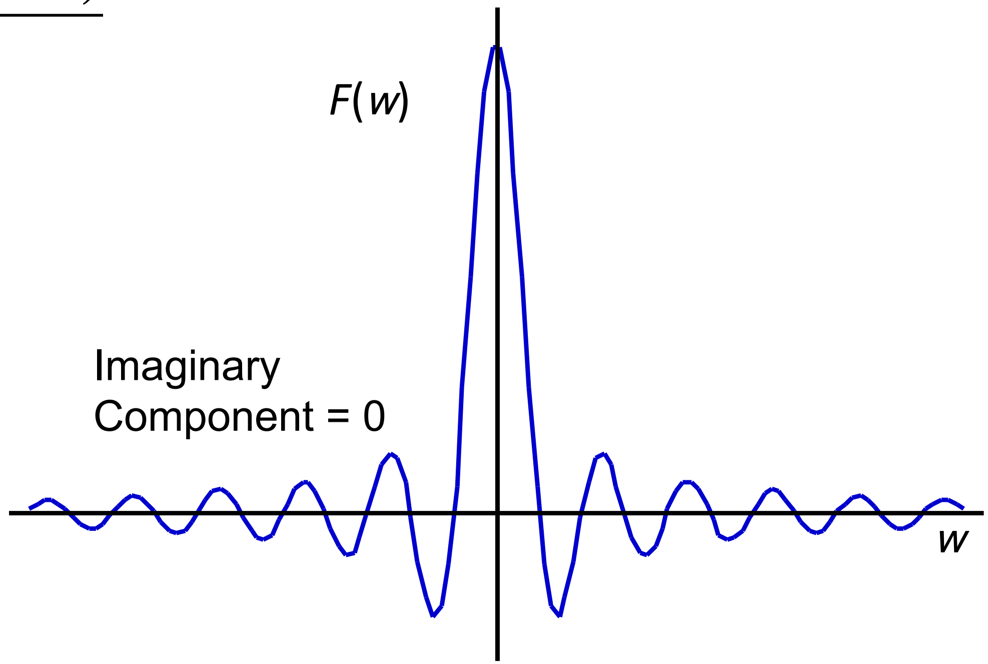
$$= \frac{1}{-i\omega}\left[\exp(-i\omega/2) - \exp(i\omega/2)\right]$$

$$= \frac{1}{(\omega/2)} \frac{\exp(i\omega/2) - \exp(-i\omega/2)}{2i}$$

$$= \frac{\sin(\omega/2)}{(\omega/2)}$$

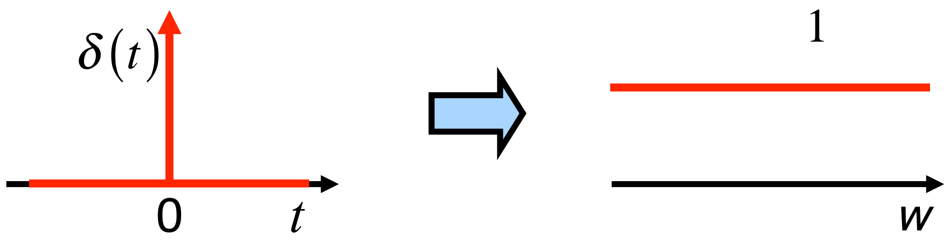


$$\boxed{F(\omega) = \mathrm{sinc}(\omega/2)}$$



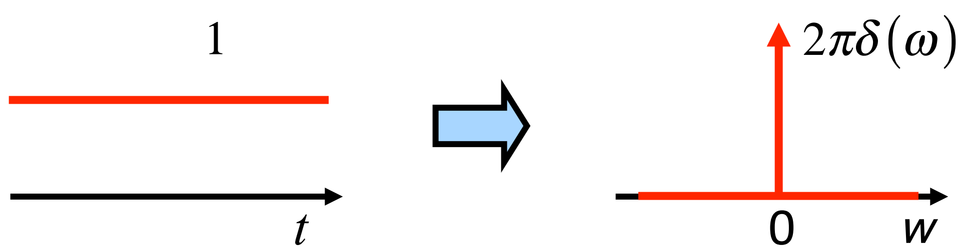
The Fourier Transform of a *delta function* is 1.

$$\int_{-\infty}^{\infty} \delta(t) \exp(-i\omega t)\, dt = \exp(-i\omega[0]) = 1$$



And the Fourier Transform of 1 is 2p*d*(*w*):

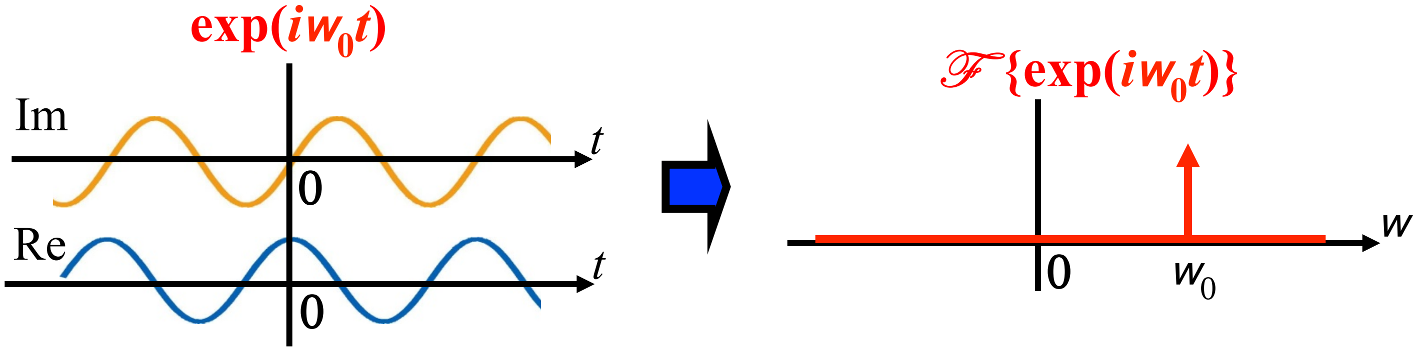
$$\int_{-\infty}^{\infty} 1 \exp(-i\omega t)\, dt = 2\pi\, \delta(\omega)$$



The Fourier transform of $\exp(iw_0 t)$

$$\mathcal{F}\{\exp(i\omega_0 t)\} = \int_{-\infty}^{\infty} \exp(i\omega_0 t)\exp(-i\omega t)\,dt$$

$$= \int_{-\infty}^{\infty} \exp(-i[\omega-\omega_0]t)\,dt = 2\pi\,\delta(\omega-\omega_0)$$



The function $\exp(iw_0t)$ is the essential component of Fourier analysis.

It is a pure frequency.

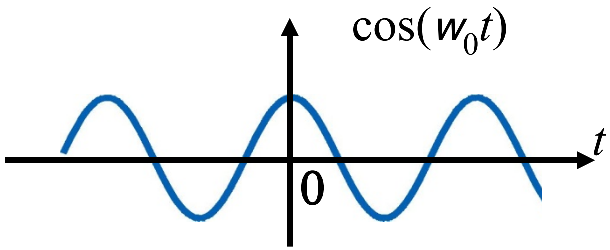
The Fourier transform of cos($w_0 t$)

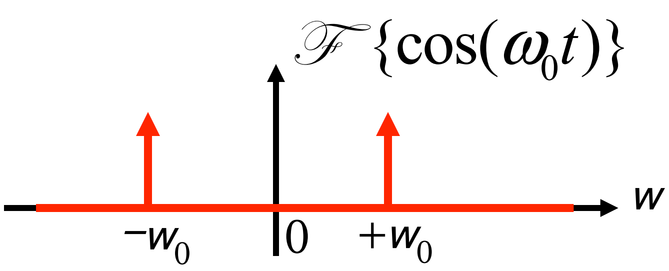
$$\mathcal{F}\{\cos(\omega_0 t)\} = \int_{-\infty}^{\infty} \cos(\omega_0 t)\exp(-i\,\omega t)\,dt$$

$$= \frac{1}{2}\int_{-\infty}^{\infty} \left[\exp(i\,\omega_0\,t) + \exp(-i\,\omega_0\,t)\right]\exp(-i\,\omega t)\,dt$$

$$= \frac{1}{2}\int_{-\infty}^{\infty} \exp(-i[\omega - \omega_0]t)\,dt \quad + \quad \frac{1}{2}\int_{-\infty}^{\infty} \exp(-i[\omega + \omega_0]t)\,dt$$

$$= \pi\,\delta(\omega - \omega_0) \quad + \quad \pi\,\delta(\omega + \omega_0)$$





Scale Theorem

The Fourier transform
of a scaled function, $f(at)$:

$$\mathcal{F}\{f(at)\} = F(\omega/a) \,/\, |a|$$

Proof:

$$\mathcal{F}\{f(at)\} = \int_{-\infty}^{\infty} f(at) \exp(-i\omega t)\, dt$$

Assuming $a > 0$, change variables: $u = at$

$$\mathcal{F}\{f(at)\} = \int_{-\infty}^{\infty} f(u) \exp(-i\omega[u/a])\, du \,/\, a$$

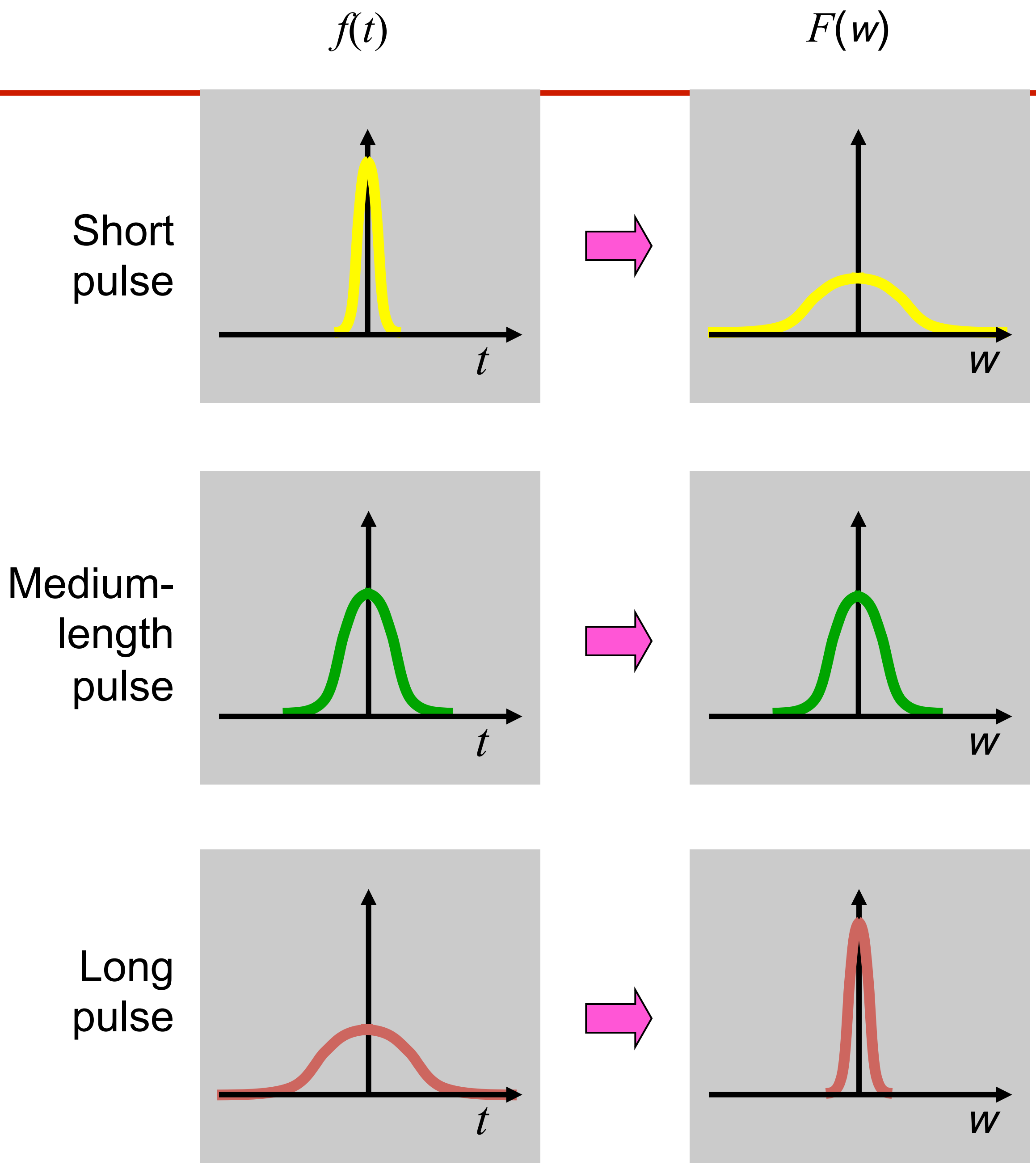
$$= \int_{-\infty}^{\infty} f(u) \exp(-i\,[\omega/a]\, u)\, du \;/\; a$$

$$= F(\omega/a) \,/\, a$$

If $a < 0$, the limits flip when we change variables, introducing a minus sign, hence the absolute value.

The Scale Theorem in action

The shorter the pulse, the broader the spectrum!

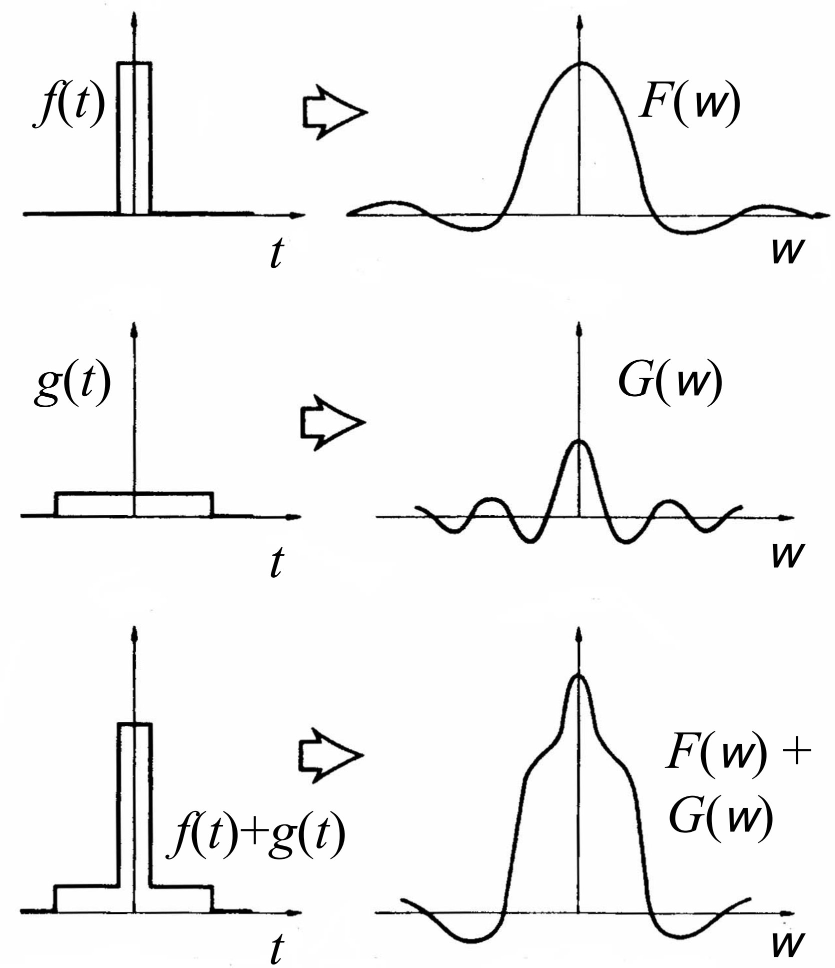


This is the essence of the Uncertainty Principle!

The Fourier Transform of a sum of two functions

$$\mathscr{F}\{a\,f(t) + b\,g(t)\} = a\mathscr{F}\{f(t)\} + b\mathscr{F}\{g(t)\}$$

Also, constants factor out.



Shift Theorem

The Fourier transform of a shifted function, $f(t-a)$:

$$\mathcal{F}\{f(t-a)\} = \exp(-i\omega a)F(\omega)$$

Proof :

$$\mathcal{F}\{f(t-a)\} = \int_{-\infty}^{\infty} f(t-a)\exp(-i\omega t)\,dt$$

Change variables : $u = t - a$

$$\int_{-\infty}^{\infty} f(u)\exp(-i\omega[u+a])\,du$$

$$= \exp(-i\omega a)\int_{-\infty}^{\infty} f(u)\exp(-i\omega u)\,du$$

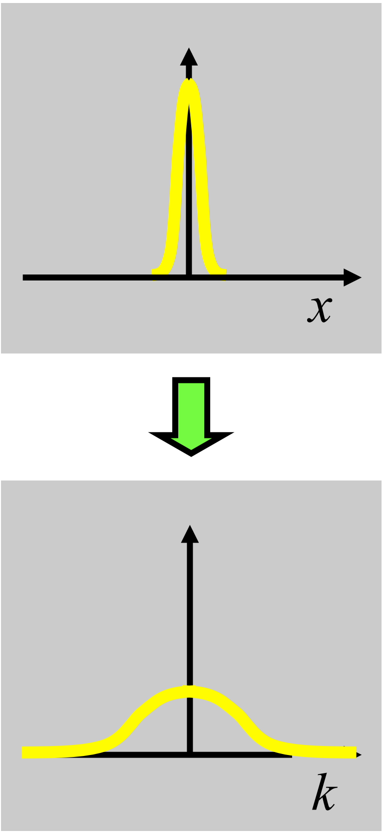
$$= \exp(-i\omega a)F(\omega)$$

Fourier Transform with respect to space

If $f(x)$ is a function of position,

$$F(k) = \int_{-\infty}^{\infty} f(x) \exp(-ikx)\, dx$$

$$\mathcal{F}\{f(x)\} = F(k)$$



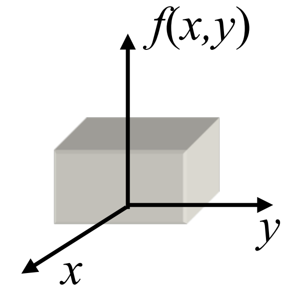
We refer to k as the **spatial frequency**.

Everything we've said about Fourier transforms between the t and w domains also applies to the x and k domains.

The 2D Fourier Transform

$$\mathcal{F}^{(2)}\{f(x,y)\} = F(k_x,k_y)$$

$$= \iint f(x,y)\ \exp[-i(k_x x + k_y y)]\ dx\ dy$$



If $f(x,y) = f_x(x)\, f_y(y)$,

then the 2D FT splits into two 1D FT's.

But this doesn’t always happen.

$\mathcal{F}^{(2)}\{f(x,y)\}$