Fourier Optics and Spatial Light Modulators
PHY 431 Fall 2011

Sources:

- Physics 39a/169b, Brandeis University
- Holoeye OptiXplore Manual

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Things to learn in this laboratory:

Optics: The experiment is mostly optics, including learning about the laser, polarizers, and lenses, and developing the techniques for using these effectively in an optical system. You should learn how and why the experiment is put together the way it is, including the choice and location of all the elements. You should learn the practical aspects of handling optical components, and how to accurately assemble them into a system so it functions as well as possible.

Liquid Crystal Display optics: The filter in this system is a computer controlled liquid crystal display device. It functions with polarized light, changing both the polarization and phase of the transmitted light, in each programmable pixel of the display. In combination with polarizers, you can use it to generate patterns of intensity variation, or even pure phase variation.

Image analysis: Phase modulation can be analyzed in the optical system and the resulting image recorded by a CCD camera. This is accomplished readily using the free software ImageJ.

Fourier Analysis: One of the central themes of this course is Fourier analysis, and this laboratory is the embodiment of that theme. You should become familiar with several of the important results of Fourier Optics such as the Convolution Theorem and the Array Theorem and how they apply to the study of diffraction. The Fourier analysis of the thin lens will also be investigated.

Safety:

Laser Safety: NEVER look into a laser beam, or allow a beam to reflect into your eyes of the eyes of someone else in the lab. Before working with any optics in the laser beam, THINK about what you are doing and where the beam might be aimed by what you are doing.

Handling Optics: Never touch the optical surface of any optical component, lens, LCD, etc. Only handle things by their outer edges or by their mountings. Never lay lenses or any optical components on the table with the optical surface touching the table. If you see dirt or a fingerprint on an optical component, DO NOT try to rub it off with a tissue. As a final note, make sure you have carefully read over the operation and safety section of the Holoeye manual. If at any point clarification is needed, please do not hesitate to ask. Enjoy the lab!
Introduction:

The technology of *Spatial Light Modulators* (SLM) grew from the need to quickly convert data in electronic form into spatially modulated coherent optical signals. This allows for the ability to introduce information into an optical system, since the information can be carried directly by the optical amplitude. To achieve this, one needs to manipulate the complex optical fields transmitted through the optical system. Originally photographic films were used for wavefront modulation but much more powerful optical information processing systems can be realized if the film is replaced by a spatial light modulator capable of changing transited light in real time in response to optical or electronic control signals (Goodman).

Over the history of optical information processing, a great many different SLM technologies have emerged along with the proliferation of practical application. Such SLMs technologies include: Liquid Crystal SLMs, magneto-optic SLMs, deformable mirror SLMs, and multiple-quantum-well (MQW) SLMs. Liquid Crystal SLMs are the most present in everyday technology and will be the type you will have the opportunity to work with. (Goodman).

In this lab you will be investigating the properties and capabilities of a *twisted-nematic liquid crystal SLM*. The SLM is composed of carefully chosen dimensions of spatially separated *liquid crystal* cells. *Liquid Crystals* are considered a phase of matter in which the molecule order is between the crystalline solid state and the liquid state. Each LC cell is composed of long, cigar-shaped molecules sandwiched between two alignment layers, which set the angle of the molecule’s long axis alignment at the interface. The two layers, however, do not share the same angle so the molecules form a “helix” structure as they traverse the cell, thus giving rise to a *twisted* appearance—See figure 1 (much like putting a deck of cards between your two hands and fanning it around—Hecht pg. 372). *Nematic* liquid crystals are ones where the molecules tend to be parallel but their position is randomly distributed across the cell. A wonderful set of pictures that do a much greater justice at conveying what a TN-LCD is can be found in Hecht on pages 370-373.

The helix structure of the twisted nematic crystal can be used to change the polarization status of incident light. When the polarization of the light is parallel to the molecules of the cell at the entrance facet, the polarization follows the twist of the molecule axis. You can think of the cell as being a series of thin wave-plates each with a minute gradation of their optical axis. Therefore the light leaves the cell with a polarization that is perpendicular to the incident polarization. In order to realize a dynamic optical element, a voltage is applied to the LC cell. This voltage causes changes of the

![Figure 1: Twisted-nematic liquid crystal cell](image-url)
molecular orientation, as is illustrated in figure 2 for three voltages $V_A$, $V_B$, $V_c$. Additionally to the twist caused by the alignment layers, the molecules experience a voltage-dependent tilt if the voltage is higher than a certain threshold voltage. With increasing voltage, only some molecules close to the cell surface are still influenced by the alignment layers, but the majority of molecules in the center of the cell will be aligned parallel to the electric field direction.

![Tilt of twisted LC molecules with voltage increasing from (A) to (C)](image)

Figure 2.

The liquid crystal is birefringent and thus can be defined by two indices of refraction similar to a waveplate. The unique feature about the liquid crystal display in the SLM is the extraordinary refractive index is dependant on the angle the molecules make with the normal of the entrance and exit layer, typically defined as the $z$-axis or direction of propagation. The explicit formula is given by

$$\frac{1}{n_{eo}^2(\theta)} = \frac{\cos^2(\theta)}{n_o^2} + \frac{\sin^2(\theta)}{n_{eo}^2}$$

Eq. 1

The angle $\theta$ is a function of the applied voltage, so the value of $n_{eo}$ is also dependant on voltage. As $\theta \to 0$ we can see the value of $n_{eo} \to n_o$ and the liquid crystal becomes isotropic i.e. the optical properties of the material are the same in all directions. Hence, light passing through will only experience one refractive index. This idea is of importance to understanding how the polarization of light is changed by the SLM.
The following is a short list of accessible texts, with germane chapters indicated, which you should reference to your assimilation of some of the more difficult concepts.

**Further Readings:**

**Diffraction and Fourier Optics:**
1) Hecht Ch 7.3-7.4.4, Ch 10-11, Ch 13.2-13.2.3
2) Fowles Ch 5
3) Introduction to Fourier Optics Ch 2-4
4) Pedrotti’s Intro. To Optics Ch 11, 13, 21

**Spatial Light Modulators:**
1) Intro. To Fourier Optics Ch 7.2
2) Hecht Ch 8.12
3) Pedrotti’s Intro. To Optics Ch 17-5
4) OptiXplorer Manual

* Additionally it would be beneficial to review polarization, interference and some geometrical optics such as microscopes.
Section 1: Amplitude modulation and projection

Objectives:
To understand the following experiments and the functionality of the LC display, the polarization of the light plays a crucial role. Therefore the polarization characteristics of the LC display will be determined first. Then a projector setup will be assembled using the light modulator as the image source. During the experiment contrast and pixel size will be determined.

Measurement of the polarization properties of the SLM.

In this experiment, the polarization properties of the light modulator will be determined. Place the unaddressed LC display between two polarizers, or alternatively a linearly polarized light source and a polarizer. The two polarizers each have a polarization axis, which are not indicated on the polarizer. You will have to use a polarizer with a known fixed axis and the power-meter, to determine the polarization axis of the two variable polarizers. Mark the polarization angle with a small piece of tape and align this mark with the 0 degree mark. Next ensure that the beam incident onto the LC display is both collimated and completely illuminating the display. This is done using, what is commonly referred to as, a cage system, which consists of two lenses of focal lengths \( f = -30 \text{mm} \) and \( f = 200 \text{mm} \) attached to a rail guide. Adjust the lens separation in order expand and collimate the beam. Using the second polarizer as an analyzer, measure the angular distribution of the intensity by setting the polarizer at zero degrees and varying the analyzer angle; 10-degree increments will suffice. Record the data and graph normalized intensity vs. analyzer angle. Without removing the SLM, comment on the intensity curve produced if the SLM were removed from the setup.

Q1) Is the light leaving the SLM linear, elliptical or circular polarized? How can you tell?

Projector Setup

Using a LED/White light source, place an \( f = 75 \text{mm} \) lens in such a way that the beam is collimated and large enough to illuminate the entire LC display. Make sure that no scattered light bypasses the display using a rectangular aperture, which can be constructed from a piece of cardboard. Place the polarizer, analyzer and SLM behind the collimating lens similar to figure 1. The SLM is positioned as close to the collimating lens as possible, so that the aperture of the display is fully illuminated. Place an objective lens \( (f = 100 \text{mm}) \) behind the display and adjust it in such a way that a focused and enlarged image appears on the screen. Try using an iris diaphragm with different apertures.

Q2) What is the influence of the aperture regarding the depth of focus, brightness, and influence of lens aberrations.

All lenses used are Plano-convex/concave. Indicate which side should face the light source in order to reduce aberration and then implement your decision in order to optimize your projector.
Open the OptiXplorer software by going to Start → All Programs → Holoeye App. Software 2.8 for the OptiXplorer (x2). The window that will open should say OptiXplorer 2.8 in the upper, left corner. If this is not the case or the program will not open, re-read the instructions, try again and if there is still a problem, ask for assistance.

Use the OptiXplorer software to address the SLM with a white screen by clicking on Elementary Optical Functions → Show Blank Screen. A homogeneous gray level screen should appear. Enlarge the blank screen so that it fills the entire computer display. Position the mouse pointer on the right edge of the window until a toolbar appears. Familiarize yourself with the name of each button in the toolbar and its corresponding function; play around with the software for a few minutes.

Once you are comfortable with the names and general functions of the toolbar adjust the gray value of the blank screen until it is entirely white. Press the “Inverting” button to toggle between white and black. Next, rotate the polarizers, which are perpendicular to each other, in adequate step sizes—10 degrees works fine—and measure the changing contrast quantitatively. Remember contrast is given by,

$$\frac{I_{\text{max}} - I_{\text{min}}}{I_{\text{max}} + I_{\text{min}}}$$

where $I_{\text{max}}$ and $I_{\text{min}}$ correspond to the white and black addressed screen respectively. Find and record what polarizer/analyzer combination achieved the maximum contrast. Again by clicking “Elementary Optical Functions” scroll down to “Circular Aperture” and address a circle of reasonable radius onto the SLM. Note what happens when one or both polarizers are removed. Keep the first polarizers fixed at the optimized position and rotate only the second one.

Q3) What do you observe in terms of contrast when you rotate the second polarizer by 45/90/180 degrees?

Pixel size of the LC display

Address the LC display with a rectangular object of known dimensions. Using the projector setup from the previous experiment, find the pixel size of the LC display; include error.
Relation between pixel voltage and modification of the polarization state

Every gray level corresponds to a specific voltage a single LCD element is addressed with. The different voltage leads to a different tilt in the liquid crystal molecules and therefore a different polarization state. Determine for six gray levels (250, 200, …0) the rotation angle of the analyzer for the smallest and largest measured power values.

The state of polarization can always be defined as an ellipse. In parametric form, the semi-major axis \( a \) corresponds to maximum power, and accordingly the semi-minor axis \( b \) to the minimum power (measured with the analyzer rotated 90 degrees to the maximum). The angle of the analyzer for the maximum power corresponds to the angle \( \Delta \), which denotes the rotation of the semi-major axis to the \( x \)-axis.

\[
\mathbf{r}(\phi) = \begin{pmatrix}
    \cos \Delta & -\sin \Delta \\
    \sin \Delta & \cos \Delta
\end{pmatrix}
\begin{pmatrix}
    a \cdot \cos \phi \\
    b \cdot \sin \phi
\end{pmatrix}
\]

\[\text{Eq. 3}\]

Using your favorite or most readily available graphing software (Kaleidagraph), plot the six different ellipses for each gray value. The results should be similar to figure 4.

![Figure 4: Rotated polarization ellipses for different grey levels (GL’s)](image-url)
Section 2: Linear and separable binary beam slit grating

Objectives:

Illuminating a spatial light modulator with a coherent light source generates diffraction patterns behind the display similar to those that appear behind a conventional optical grating. One can consider the non-addressed display as an optical grating. The reason for this lies in the structure of a single pixel. It consists of the transparent part of the liquid crystal cell and the non-transparent part of the control electronics. Assuming zero transmission for this part of the cell, the display can be seen as a two-dimensional separable grating with a structure as shown in Fig. 5. With a so-called “Fourier Lens” the diffraction pattern allows conclusions to be drawn about the characteristics of the display.

![Simplified model of a pixel with its transmission](image)

*Figure 5: Simplified model of a pixel with its transmission*

Generation of diffraction pattern

Collimate and expand the laser beam so the entire LC display is illuminated. Place a lens \((f = 250\text{mm})\) in front of the SLM such that the position of the SLM and the front focal length of the lens coincide. Move the projector screen in the rear focal plane of the lens. You should see a focused array of bright spots.

Due to the aperture of the lens, the light that can contribute to the diffraction pattern is restricted. That means, for a particular diffraction order, only partial waves from a certain part of the illuminated area on the modulator can contribute. Placing the modulator directly in front of the lens minimizes this effect. If the screen is not positioned in the rear focal plane of the lens, the size of the diffraction pattern depends on the position of the light modulator. Try placing the lens behind the SLM. Observe what happens when the SLM’s position is varied within the lens’s rear focal length. You should note the lens still acts a Fourier Transform, which generates a far-field diffraction pattern at the lens’s focal plane!
Figure 6: A Fourier lens generates the far-field diffraction pattern in its rear focal plane

Adjust the setup so you can clearly see the array of bright spots. You may need to place a diverging lens behind the setup so the diffraction pattern is enlarged for better measurements. Recall the pixel array can be thought of as a two-dimensional grating able to be separated into two perpendicular N-slit gratings. With this and Fig. 5 in mind, determine the slit width and separation for a N-slit grating in the horizontal direction. Repeat this procedure along the vertical direction.

Q4) From you measurements, what is the pixel size of the display? Compare you answer with previous measurement. Also, what is the fill factor, or what percent of each pixel is the non-transparent part of the liquid crystal cell? Include error.
Section 3: Diffractive Optics

Objectives:

Diffractive Optical Elements (DOE’s) differ from the classical optical elements like lenses and mirrors because they are based on their diffractive properties rather than reflection and refraction. When dealing with the traditional optical elements, diffraction phenomena are considered as undesirable features, which influences the performance of an optical system and hence should be minimized. DOE’s, on the other hand, make use of diffraction to manipulate the waveform of an incoming beam of light. Because of the nature of diffraction mostly highly monochromatic and coherent light is used with DOE’s. The first application of wavefront manipulation was holography. Holography inspired people to wavefront processing in which the surface of a substrate was processed to change an input wavefront into another form. In principle lenses and other classical optical elements are all wavefront processors, but their functionality is limited to relative simple actions. DOE’s allow for more complex wavefront manipulations and resulted in modern optical applications as ‘holographic head up displays’ in fighter jets and recently in automobiles.

Fresnel Zone Lens (FZL)

The optical setup for this experiment is illuminating the LCD with a expanded and collimated laser beam. Open the software OptiXplorer and go to Elementary Optical Functions → Fresnel Zone Lenses. Set the inner radius to 35 pixels. Measure the distance between the lens plane and the nth focus; this is the focal distance f. From your measurements and the formula for the innermost radius of a binary zone lens,

\[ r = \sqrt{n \cdot \lambda \cdot f} \quad \text{Eq. 4} \]

determine once again the pixel size of the LC display. Compare with your previous results.

Q5) Where does equation 3 come from? Just a brief description is needed. {Pedrotti 13-6}

Focal length of the diffractive lens

Using the OptiXplorer software address a blank screen on the SLM. With the toolbar at the right window edge a lens phase will be added. Find the focus created by this DOE and the corresponding focal length. Forewarning, the focal length can grow to a distance of 3 meters at a phase of 25, so a reasonable phase would be greater than 100.

Creating and optimizing a DOE

Add a \( f = 250 \, \text{mm} \) lens behind the SLM followed by a \( f = -30 \, \text{mm} \) lens. Download an image—less than 200x200 pixels—from the Internet or upload one from your camera/phone into a folder. In the OptiXplorer software go to File → Open Image File and upload your image. Next, click the “Compute DOE” button in the upper right corner of the image. Adjust your setup until you see your image as a diffraction pattern on the screen.
Usually, when reconstructing a DOE there is a disturbing bright point in the middle, the zero order. Using a diverging refractive lens and a diffractive converging lens added to the DOE, one could spatially separate the reconstruction planes of the zero and first order. Begin by removing the $f = 250 \text{ mm}$ lens and by replacing it with a $f = 150 \text{ mm}$ lens in front of the SLM. Create a DOE from the examples included with the OptiXplorer software a.k.a. “grid.bmp” or “cords.bmp”. For different lens phases, the focus plane and the Fourier plane is focused on the screen by moving the diverging lens. For several different lens phases, find the difference between the focus of the diffracted light and the focus of the undiffracted light.

![Diagram](image)

*Figure 7. An added lens phase causes a focusing of the diffracted light (red). The undiffracted light is focused behind the screen (black).*

By adding a prism phase to the addressed DOE is also possible to separate the orders. The diffraction pattern will be shifted in the $x$ and $y$ directions (see figure 6). Remove the positive lens in front of the SLM and replace the diverging one with a $f = -100 \text{ mm}$ lens. Adjust your setup in such a way as to clearly see the image on the projector screen. For different prism phase values record the shift of the diffraction pattern.

![Diffraction Patterns](image)

*Figure 8: Top: focused zero order; Left: zero order defocused; Right: diffraction pattern shifted*
Section 4: Interferometric measurement of the phase modulation

Objectives:

The phase modulation that can be achieved with a spatial light modulator for a coherent light source can be measured with a two-beam interference setup. Two coherent and collimated laser beams created by a double-hole mask illuminate the display. Both beams are separately guided to an appropriate half of the LCD. The left one will be addressed with a constant grey level whereas the other half will be addressed with grey levels varying from 0 to 255. A lens behind the display lets both beams interfere with one another and a microscope objective images the expanded interference pattern onto a CCD camera. A phase shift as a function of the addressed grey level will appear as a shift in the interference pattern perpendicular to the optical axis.

Fig. 9: Phase patterns used to determine the phase modulation.

Phase Measurement

The optical setup is shown in figure 7. The laser module emits a collimated beam. Construct a double-hole mask with hole diameter ~3 mm and aperture separation ~7 mm. Place this mask into your optical system so two beams are created and each passes through one half of the display. A linear polarizer in front of the SLM sets the incoming polarization state. The laser emits elliptical polarized light so the intensity is dependent on both the SLM and the first polarizer’s state. To remove this ambiguity we must set the intensity of the light leaving the first polarizer to be independent of the angle.

Q6) How can you do this? [Hint: Use a polarizer and a wave-plate]

Using an \( f = 250 \text{ mm} \) lens and an objective lens (20x/0.4) image the interference pattern onto the CCD camera. Open the software PhaseCam and view the interference pattern.

Figure 10: Two-beam interferometer to detect the phase shift
The PhaseCam software offers an automated measurement of the shift in the interference pattern. Begin by selecting the “Preview” button to make sure the interference pattern has good contrast and the detector is not saturated. Once everything is properly adjusted push “Test Image”. A stable image of the interference pattern will be displayed. By clicking the right mouse button inside of the image a particular intensity line is selected. Press “Readout Lines” to show a sinusoidal intensity profile. If the profile seems jittery and uneven, increase the “Averaging” number until the profile is smooth. Open the gray level window using the corresponding button and make sure that is occupies the full screen of the LCD. Before starting the measurements, the increments have to be chosen. This option changes the time and the resolution of one measurement. This is always a compromise since fast measurements have a low resolution and slow measurements have a high resolution.

The “Start” button starts the measurement and gray levels varying from 0 to 255 are addressed onto the active half of the grey level window in succession. Once the software is done, minimize the grey value window. You should see something similar to Figure 10 below. By pushing “Show Measurement Points” after the image is shown, the measurements points will appear as red dots. Save the data and open

![Figure 11: Phase Cam Software](image)

it in Excel. Some measurement points will have “jumped” to the next minimum. This is simply fixed by adding or subtracting the period from these measurements. Find the value of ∆y or the distance the minimum shifted from the initial minimum (gray value of 0). From this measurement and the period, plot the phase shift, ∆ϕ, as a function of grey level. Repeat this procedure for different polarizer and analyzer configurations. You will need the following equation to determine phase shift.

\[ \Delta \phi = \frac{2\pi}{g} \cdot \Delta y \]

*Eq. 5*
Reference: