Introduction

Even though several thousand different optical components are listed in this catalog, performing a few simple calculations will usually determine the appropriate optics for an application or, at the very least, narrow the list of choices.

The process of solving virtually any optical engineering problem can be broken down into two main steps. First, paraxial calculations (first order) are made to determine critical parameters such as magnification, focal length(s), clear aperture (diameter), and object and image position. These paraxial calculations are covered in the next section of this chapter.

Second, actual components are chosen based on these paraxial values, and their actual performance is evaluated with special attention paid to the effects of aberrations. A truly rigorous performance analysis for all but the simplest optical systems generally requires computer ray tracing, but simple generalizations can be used, especially when the lens selection process is confined to a limited range of component shapes.

In practice, the second step may reveal conflicts with design constraints, such as component size, cost, or product availability. System parameters may therefore require modification.
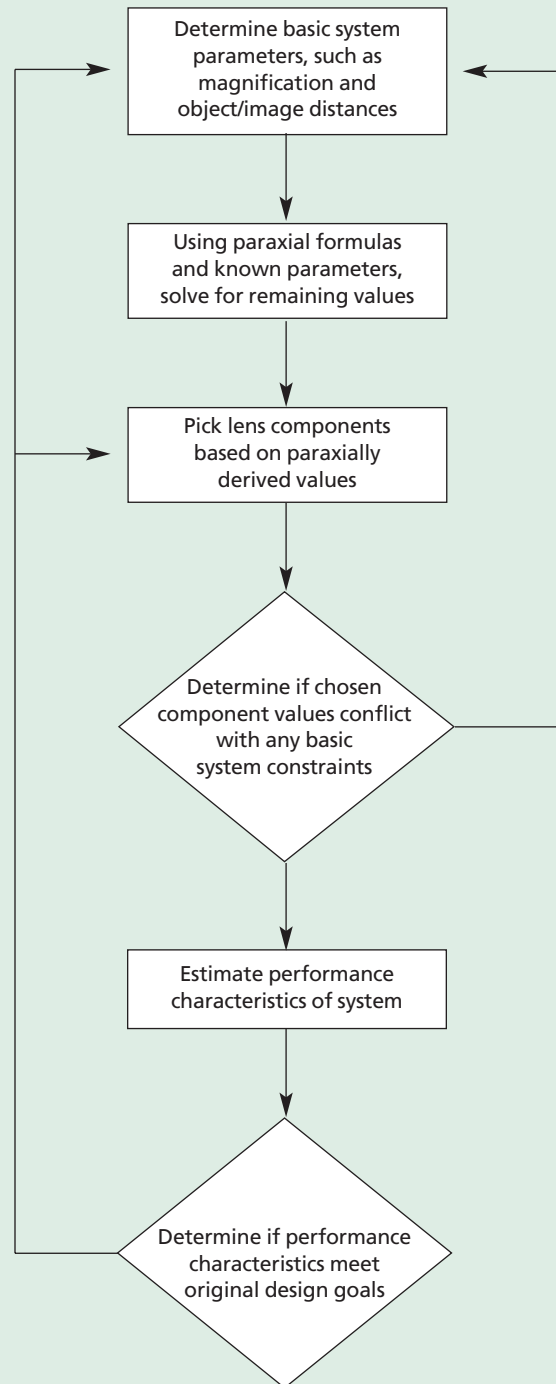
Because some of the terms used in this chapter may not be familiar to all readers, a glossary of terms is provided beginning on page 1.29.

Finally, it should be noted that the discussion in this chapter relates only to systems with uniform illumination; optical systems for Gaussian beams are covered in Chapter 2, *Gaussian Beam Optics*.

ENGINEERING SUPPORT

Melles Griot maintains a staff of knowledgeable, experienced applications engineers at each of our facilities worldwide. The information given in this chapter is sufficient to enable the user to select the most appropriate catalog lenses for the most commonly encountered applications. However, when additional optical engineering support is required, our applications engineers are available to provide assistance. Do not hesitate to contact us for help in product selection or to obtain more detailed specifications on Melles Griot products.

THE OPTICAL ENGINEERING PROCESS

Determine basic system parameters, such as magnification and object/image distances

Using paraxial formulas and known parameters, solve for remaining values

Pick lens components based on paraxially derived values

Determine if chosen component values conflict with any basic system constraints

Estimate performance characteristics of system

Determine if performance characteristics meet original design goals

Paraxial Formulas

SIGN CONVENTIONS

The validity of the paraxial lens formulas is dependent on adherence to the following sign conventions:

For lenses: (refer to figure 1.1)

s   is + for object to left of H
    (the first principal point)

s   is − for object to right of H

s″ is + for image to right of H″
    (the second principal point)

s″ is − for image to left of H″

m is + for an inverted image

m is − for an upright image

For mirrors:

f   is + for convex (diverging) mirrors

f   is − for concave (converging) mirrors

s   is + for object to left of H

s   is − for object to right of H

s″ is − for image to right of H″

s″ is + for image to left of H″

m is + for an inverted image

m is − for an upright image

When using the thin-lens approximation, simply refer to the left and right of the lens.
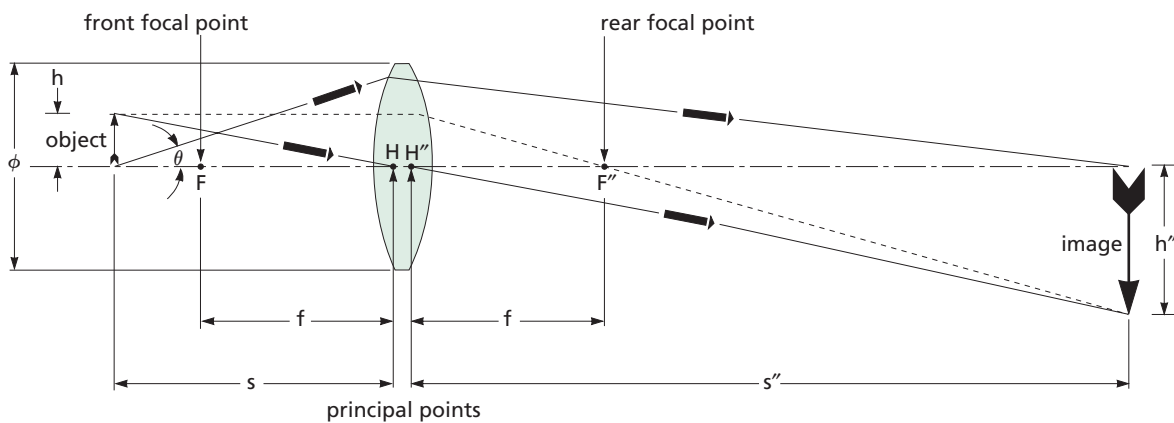
Note location of object and image relative to front and rear focal points.

ϕ = lens diameter

m = $s''/s = h''/h$ = magnification or conjugate ratio, said to be infinite if either s'' or s is infinite

θ = arcsin ($\phi/2s$)

h = object height

h'' = image height

s = object distance, positive for object (whether real or virtual) to the left of principal point H

s'' = image distance (s and s'' are collectively called conjugate distances, with object and image in conjugate planes), positive for image (whether real or virtual) to the right of the principal point H″

f = effective focal length (EFL) which may be positive (as shown) or negative. f represents both FH and H″F″, assuming lens to be surrounded by medium of index 1.0

Figure 1.1   **Sign conventions**

Typically, the first step in optical problem solving is to select a system focal length based on constraints such as magnification or conjugate distances (object and image distance). The relationship among focal length, object position, and image position is given by

$$\frac{1}{f} = \frac{1}{s} + \frac{1}{s''} .$$ (1.1)

This formula is referenced to figure 1.1 and the sign conventions given on page 1.3.

By definition, magnification is the ratio of image size to object size or

$$m = \frac{s''}{s} = \frac{h''}{h}.$$ (1.2)

This relationship can be used to recast the first formula into the following forms:

$$f = m\frac{(s + s'')}{(m + 1)^2}$$ (1.3)

$$f = \frac{sm}{m + 1}$$ (1.4)

$$f = \frac{s + s''}{m + 2 + \dfrac{1}{m}}$$ (1.5)

$$s\,(m + 1) = s + s''$$ (1.6)

where (s + s″) is the approximate object-to-image distance.

With a real lens of finite thickness, the image distance, object distance, and focal length are all referenced to the principal points, not to the physical center of the lens. By neglecting the distance between the lens' principal points, known as the hiatus, s + s″ becomes the object-to-image distance. This simplification, called the thin-lens approximation, can speed up calculation when dealing with simple optical systems.

Example 1: Object outside Focal Point

A 1-mm-high object is placed on the optical axis, 200 mm left of the left principal point of a 01 LDX 103 (f = 50 mm). Where is the image formed, and what is the magnification? (See figure 1.2.)

$$\frac{1}{s''} = \frac{1}{f} - \frac{1}{s}$$

$$\frac{1}{s''} = \frac{1}{50} - \frac{1}{200}$$

$$s'' = 66.7 \text{ mm}$$

$$m = \frac{s''}{s} = \frac{66.7}{200} = 0.33$$
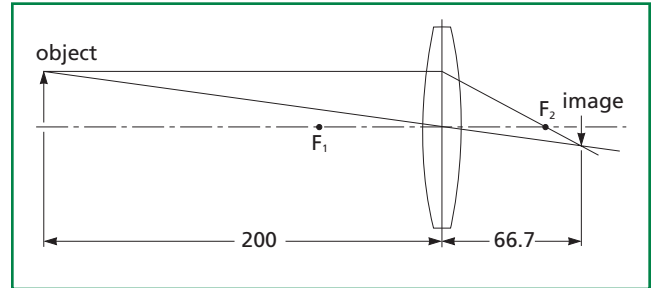
(or real image is 0.33 mm high and inverted).

Figure 1.2   **Example 1** (f = 50 mm, s = 200 mm, s″ = 66.7 mm)

Example 2: Object inside Focal Point

The same object is placed 30 mm left of the left principal point of the same lens. Where is the image formed, and what is the magnification? (See figure 1.3.)

$$\frac{1}{s''} = \frac{1}{50} - \frac{1}{30}$$

$$s'' = -75 \text{ mm}$$

$$m = \frac{s''}{s} = \frac{-75}{30} = -2.5$$

(or virtual image is 2.5 mm high and upright).

In this case, the lens is being used as a magnifier, and the image can be viewed only back through the lens.
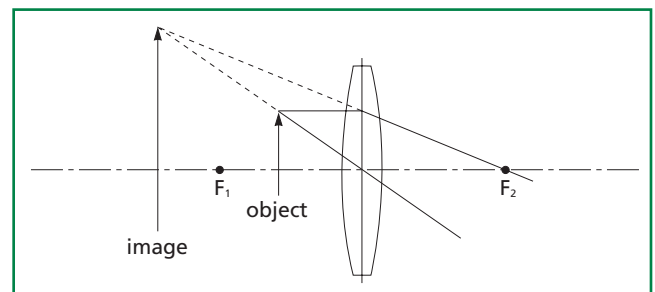

Figure 1.3   **Example 2** (f = 50 mm, s = 30 mm, s″ = −75 mm)

Example 3: Object at Focal Point

A 1-mm-high object is placed on the optical axis, 50 mm left of the first principal point of an 01 LDK 019 (f = 50 mm). Where is the image formed, and what is the magnification? (See figure 1.4.)

$$\frac{1}{s''} = \frac{1}{-50} - \frac{1}{50}$$

$$s'' = -25 \text{ mm}$$

$$m = \frac{s''}{s} = \frac{-25}{50} = -0.5$$

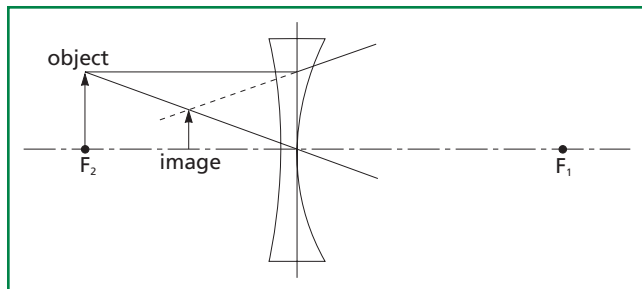(or virtual image is 0.5 mm high and upright).

Figure 1.4   **Example 3** (f = − 50 mm, s = 50 mm, s″ = − 25 mm)

A simple graphical method can also be used to determine paraxial image location and magnification. This graphical approach relies on two simple properties of an optical system. First, a ray that enters the system parallel to the optical axis crosses the optical axis at the focal point. Second, a ray that enters the first principal point of the system exits the system from the second principal point parallel to its original direction (i.e., its exit angle with the optical axis is the same as its entrance angle). This method has been applied to the three previous examples illustrated in figures 1.2 through 1.4. Note that by using the thin-lens approximation, this second property reduces to the statement that a ray passing through the center of the lens is undeviated.

F-NUMBER AND NUMERICAL APERTURE

The paraxial calculations used to determine necessary element diameter are based on the concepts of focal ratio (f-number or f/#) and numerical aperture (NA). The f-number is the ratio of the focal length of the lens to its clear aperture (effective diameter).

$$\text{f-number} = \frac{f}{\phi}. \tag{1.7}$$

To visualize the f-number, consider a lens with a positive focal length illuminated uniformly with collimated light. The f-number defines the angle of the cone of light leaving the lens which ultimately forms the image. This is an important concept when the throughput or light-gathering power of an optical system is critical, such as when focusing light into a monochromator or projecting a high-power image.

The other term used commonly in defining this cone angle is numerical aperture. Numerical aperture is the sine of the angle made by the marginal ray with the optical axis. By referring to figure 1.5 and using simple trigonometry, it can be seen that

$$NA = \sin\theta = \frac{\phi}{2f} \tag{1.8}$$

or

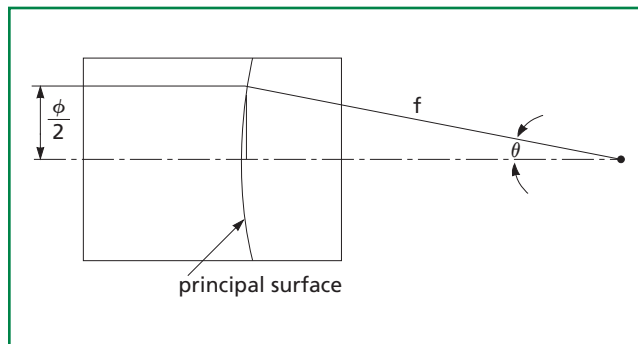$$NA = \frac{1}{2(\text{f-number})}. \tag{1.9}$$

Figure 1.5   **F-number and numerical aperture**

Ray f-numbers can also be defined for any arbitrary ray if its conjugate distance and the diameter at which it intersects the principal surface of the optical system are known.

NOTE

Because the sign convention given previously is not used universally in all optics texts, the reader may notice differences in the paraxial formulas. However, results will be correct as long as a consistent set of formulas and sign conventions is used.

Imaging Properties of Lens Systems

THE OPTICAL INVARIANT

To understand the importance of the numerical aperture, consider its relation to magnification. Referring to figure 1.6,

$$\text{NA (object side)} = \sin\theta = \frac{\phi}{2s} \tag{1.10}$$

$$\text{NA'' (image side)} = \sin\theta'' = \frac{\phi}{2s''} \tag{1.11}$$

which can be rearranged to show

$$\phi = 2s \sin\theta \tag{1.12}$$

and

$$\phi = 2s'' \sin\theta'' \tag{1.13}$$

leading to

$$\frac{s''}{s} = \frac{\sin\theta}{\sin\theta''} = \frac{\text{NA}}{\text{NA''}}. \tag{1.14}$$

Since $\dfrac{s''}{s}$ is simply the magnification of the system, we arrive at

$$m = \frac{\text{NA}}{\text{NA''}}. \tag{1.15}$$

The magnification of the system is therefore equal to the ratio of the numerical apertures on the object and image sides of the system. This powerful and useful result is completely independent of the specifics of the optical system, and it can often be used to determine the optimum lens diameter in situations involving aperture constraints.

When a lens or optical system is used to create an image of a source, it is natural to assume that, by increasing the diameter (f) of the lens, we will be able to collect more light and thereby produce a brighter image. However, because of the relationship between magnification and numerical aperture, there can be a theoretical limit beyond which increasing the diameter has no effect on light-collection efficiency or image brightness.

Since the numerical aperture of a ray is given by f/2s, once a focal length and magnification have been selected, the value of NA sets the value of f. Thus, if one is dealing with a system in which the numerical aperture is constrained on either the object or image side, increasing the lens diameter beyond this value will increase system size and cost but will not improve performance (i.e., throughput or image brightness). This concept is sometimes referred to as the optical invariant.

SAMPLE CALCULATION

To understand how to use this relationship between magnification and numerical aperture, consider the following example.

Example: System with Fixed Input NA

Two very common applications of simple optics involve coupling light into an optical fiber or into the entrance slit of a monochromator. Although these problems appear to be quite different, they both have the same limitation — they have a fixed numerical aperture. For monochromators, this limit is usually expressed in terms of the f-number. In addition to the fixed numerical aperture, they both have a fixed entrance pupil (image) size.

Suppose it is necessary, using a singlet lens from this catalog, to couple the output of an incandescent bulb with a filament 1 mm in diameter into an optical fiber as shown in figure 1.7. Assume that the fiber has a core diameter of 100 μm and a numerical aperture of 0.25, and that the design requires that the total distance from the source to the fiber be 110 mm. Which lenses are appropriate?

By definition, the magnification must be 0.1. Letting s + s'' total 110 mm (using the thin-lens approximation), we can use equation 1.3,

$$f = m \frac{(s + s'')}{(m + 1)^2}$$

to determine that the focal length is 9.1 mm. To determine the conjugate distances, s and s'', we utilize equation 1.6,

$$s (m + 1) = s + s'',$$

and find that s = 100 mm and s'' = 10 mm.

We can now use the relationship NA = ϕ/2s or NA'' = ϕ/2s'' to derive ϕ, the optimum clear aperture (effective diameter) of the lens.

With an image numerical aperture of 0.25 and an image distance (s'') of 10 mm,

$$0.25 = \frac{\phi}{20}$$

$$\phi = 5 \text{ mm}.$$

Accomplishing this imaging task with a single lens therefore requires an optic with a 9.1-mm focal length and a 5-mm diameter. Using a larger diameter lens will not result in any greater system throughput because of the limited input numerical aperture of the optical fiber. The singlet lenses in this catalog that meet these criteria are 01 LPX 003, which is plano-convex, and 01 LDX 003 and 01 LDX 005, which are biconvex.

Making some simple calculations has reduced our choice of lenses to just three. Chapter 2, *Gaussian Beam Optics*, discusses how to make a final choice of lenses based on various performance criteria.
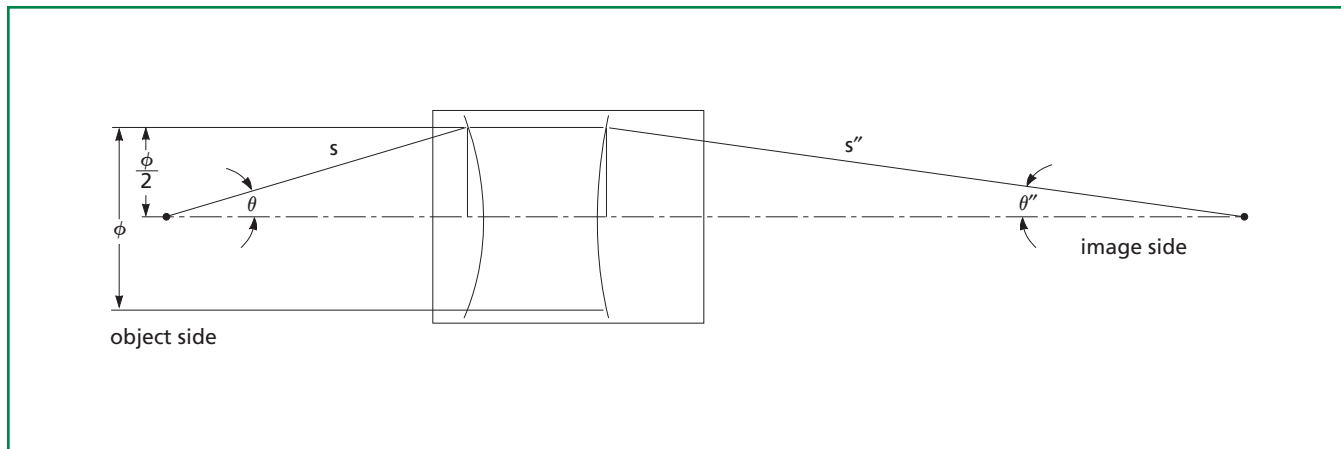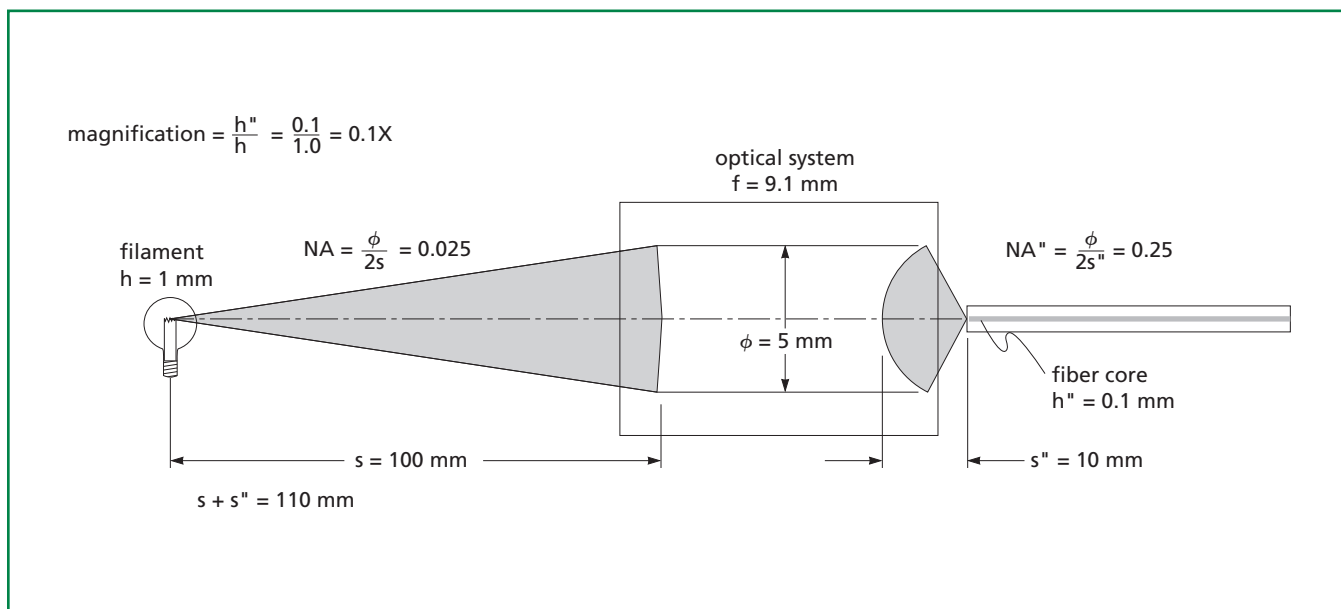
Figure 1.6　**Numerical aperture and magnification**

magnification $= \dfrac{h''}{h} = \dfrac{0.1}{1.0} = 0.1X$

optical system
f = 9.1 mm

filament
h = 1 mm

$NA = \dfrac{\phi}{2s} = 0.025$

$NA'' = \dfrac{\phi}{2s''} = 0.25$

ϕ = 5 mm

fiber core
h" = 0.1 mm

s = 100 mm

s" = 10 mm

s + s" = 110 mm

Figure 1.7　**Optical system geometry for focusing the output of an incandescent bulb into an optical fiber**

Lens Combination Formulas

PARAXIAL LENS COMBINATION FORMULAS

Many optical tasks require several lenses in order to achieve an acceptable level of performance. One possible approach to lens combinations is to consider each image formed by each lens as the object for the next lens and so on. This is a valid approach, but it is time consuming and unnecessary.

It is much simpler to calculate the effective (combined) focal length and principal-point locations and then use these results in any subsequent paraxial calculations (see figure 1.8). They can even be used in the optical invariant calculations described in the preceding section.

EFFECTIVE FOCAL LENGTH

The following formulas show how to calculate the effective focal length and principal-point locations for a combination of any two arbitrary components. The approach for more than two lenses is very simple: calculate the values for the first two elements, then perform the same calculation for this combination with the next lens. This is continued until all lenses in the system are accounted for.

The expression for the combination focal length is the same whether lens separation distances are large or small and whether f_1 and f_2 are positive or negative:

$$f = \frac{f_1 f_2}{f_1 + f_2 - d}. \tag{1.16}$$

This may be more familiar in the form

$$\frac{1}{f} = \frac{1}{f_1} + \frac{1}{f_2} - \frac{d}{f_1 f_2}. \tag{1.17}$$

Notice that the formula is symmetric with respect to interchange of the lenses (end-for-end rotation of the combination) at constant d. The next two formulas are not.

COMBINATION FOCAL-POINT LOCATION

For all cases,

$$s_2'' = \frac{f_2 (f_1 - d)}{f_1 + f_2 - d}. \tag{1.18}$$

COMBINATION SECONDARY PRINCIPAL-POINT LOCATION

Because the thin-lens approximation is obviously highly invalid for most combinations, the ability to determine the location of the secondary principal point is vital for accurate determination of d when another element is added. The simplest formula for this calculates how far the secondary principal point of the final (second) element is moved by being part of the combination:

$$z = s_2'' - f. \tag{1.19}$$

COMBINATION EXAMPLES

It is possible for a lens combination or system to exhibit principal planes that are far removed from the system. When such systems are themselves combined, negative values of d may occur. Probably the simplest example of a negative d-value situation is shown in figure 1.9. Meniscus lenses with steep surfaces have external principal planes. When two of these lenses are brought into contact, a negative value of d can occur. Other combined-lens examples are shown in figures 1.10 through 1.13.

SYMBOLS

f = combination focal length (EFL), positive if combination final focal point falls to right of combination secondary principal point, negative otherwise.

f_1 = focal length (EFL) of first element.

f_2 = focal length (EFL) of second element.

d = distance from secondary principal point of first element to primary principal point of second element (positive if primary principal point is to right of the secondary principal point, negative otherwise).

s_2'' = distance from secondary principal point of second element to final combination focal point (location of final image for object at infinity to left), positive if the focal point is to right of second element secondary principal point.

z = distance to combination secondary principal point measured from secondary principal point of second element, positive if combination secondary principal point is to right of secondary principal point of second element.

*Note: These paraxial formulas apply to **coaxial** combinations of both thick and thin lenses immersed in any fluid with refractive index independent of position. They assume that light propagates from left to right through an optical system.*
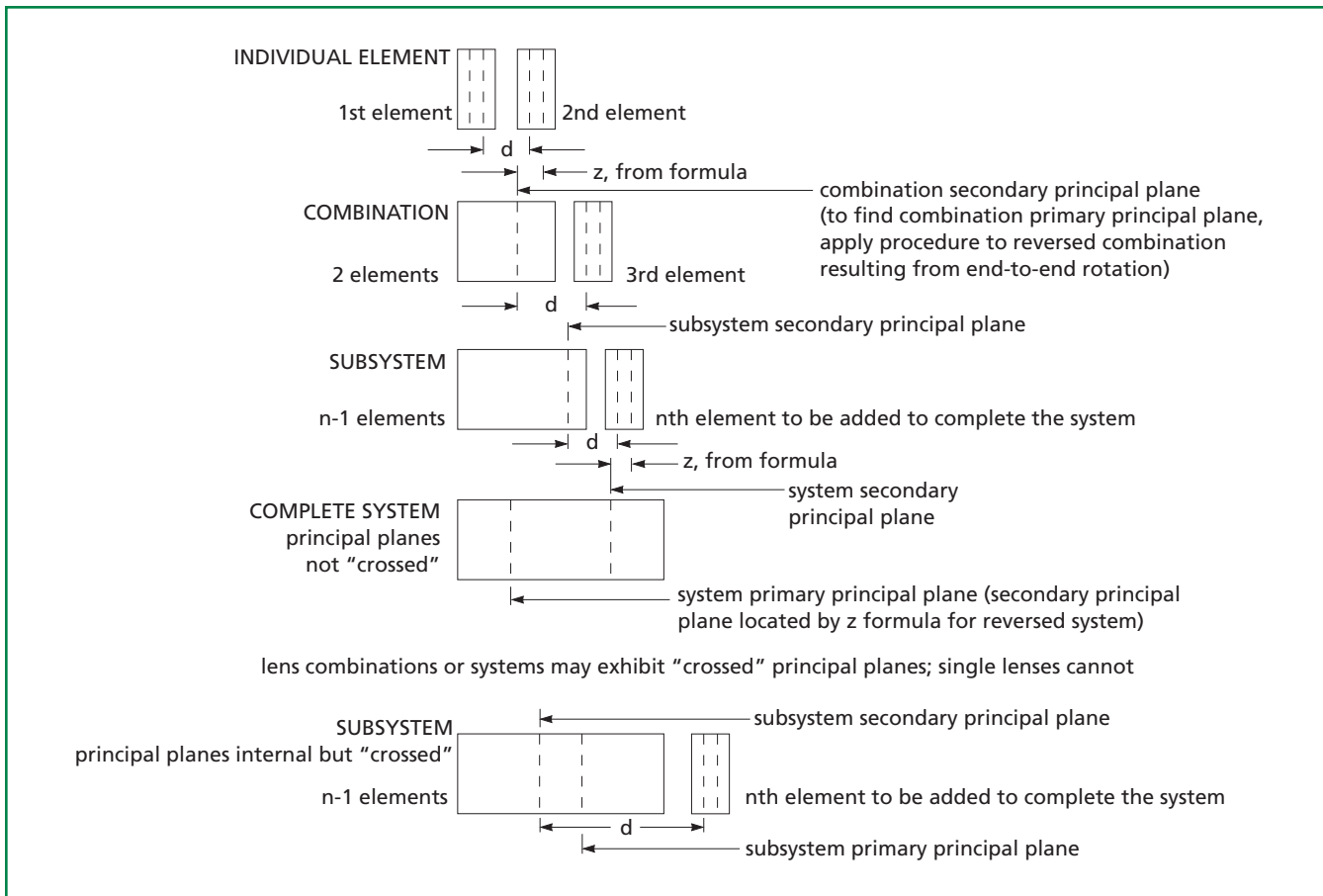
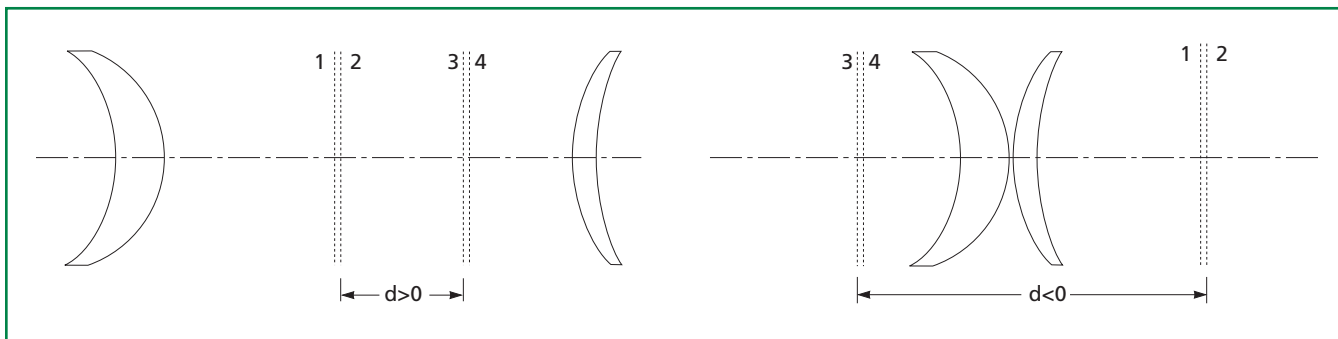Figure 1.8 **Generalization from combinations to systems**

Figure 1.9 **"Extreme" meniscus-form lenses with external principal planes (**drawing not to scale**)**
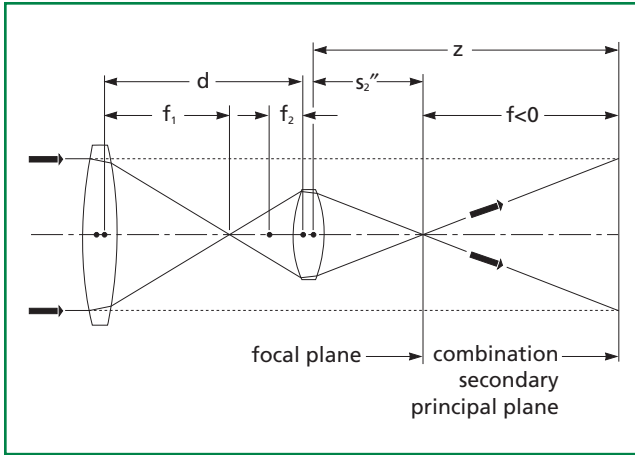
Figure 1.10   Positive lenses separated by distance greater than $f_1 + f_2$: f is negative, while both s_2'' and z are positive. Lens symmetry is not required.

Figure 1.11   Achromatic combinations: Air-spaced lens combinations can be made nearly achromatic, even though both elements are made of the same material. Achieving achromatism requires that, in the thin-lens approximation,

$$ d = \frac{(f_1 + f_2)}{2} . $$

This is the basis for Huygens and Ramsden eyepieces.

This approximation is adequate for most thick-lens situations. The signs of f_1, f_2, and d are unrestricted, but d must have a value that guarantees the existence of an air space. Element shapes are unrestricted and can be chosen to compensate for other aberrations.
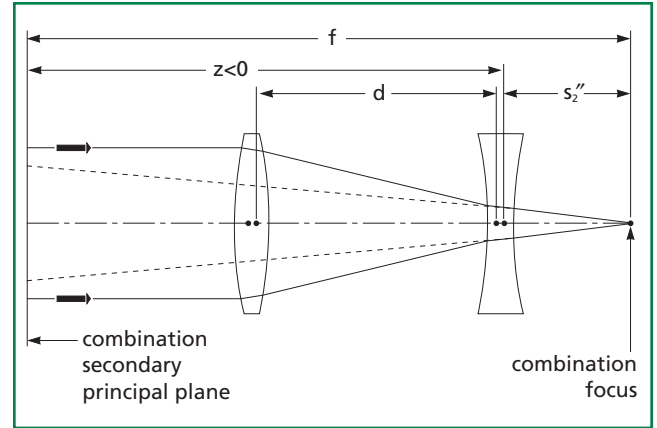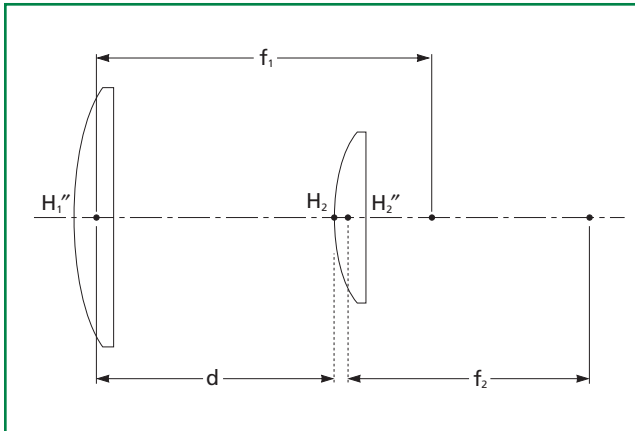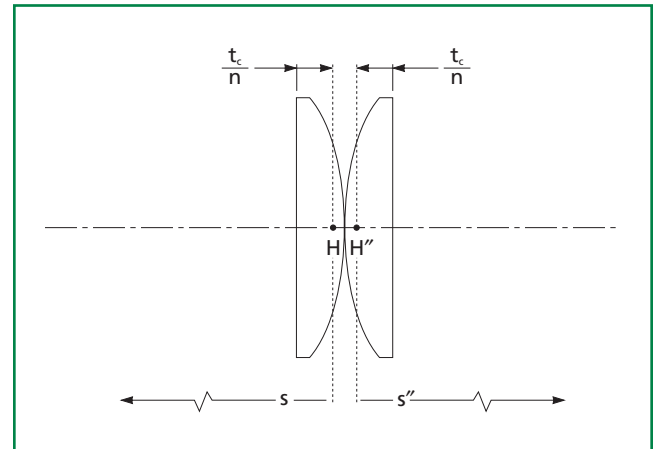

Figure 1.12   Telephoto combination: The most important characteristic of the telephoto is that the EFL, and hence the image size, can be made much larger than the distance from the first lens surface to the image would suggest by using a positive lens followed by a negative lens *(but not necessarily the lens shapes shown in the figure)*. For example, f_1 is positive and $f_2 = 4f_1/2$. Then f is negative for d less than $f_1/2$, infinite for $d = f_1/2$ (Galilean telescope or beam expander), and positive for d larger than $f_1/2$. To make the example even more specific, catalog lenses 01 LDX 189 and 01 LDK 021, with d = 78.2 mm, will yield $s_2'' = 2.0$ m, f = 5.2 m, and z = 43.2 m.

Figure 1.13   Condenser configuration: A pair of identical plano-convex lenses have their convex vertices in contact. (The lenses could also be plano aspheres.) Because d = 0, $f = f_1/2 = f_2/2$, $f_1/2 = s_2''$, and z = 0. The secondary principal point of the second element and the secondary principal point of the combination coincide at H'', at depth t_c/n beneath the vertex of the plano surface of the second element, where t_c is the element center thickness and n is the refractive index of the element. By symmetry, the primary principal point of the combination is similarly located in the first element. Combination conjugate distances must be measured from these points.

Performance Factors

After paraxial formulas have been used to select values for component focal length(s) and diameter(s), the final step is to select actual lenses. As in any engineering problem, this selection process involves a number of tradeoffs, including performance, cost, weight, and environmental factors.

The performance of real optical systems is limited by several factors, including lens aberrations and light diffraction. The magnitude of these effects can be calculated with relative ease.

Numerous other factors, such as lens manufacturing tolerances and component alignment, impact the performance of an optical system. Although these are not considered explicitly in the following discussion, it should be kept in mind that if calculations indicate that a lens system only just meets the desired performance criteria, in practice it may fall short of this performance as a result of other factors. In critical applications, it is generally better to select a lens whose calculated performance is significantly better than needed.

DIFFRACTION

Diffraction, a natural property of light arising from its wave nature, poses a fundamental limitation on any optical system. Diffraction is always present, although its effects may be masked if the system has significant aberrations. When an optical system is essentially free from aberrations, its performance is limited solely by diffraction, and it is referred to as diffraction limited.

In calculating diffraction, we simply need to know the focal length(s) and aperture diameter(s); we do not consider other lens-related factors such as shape or index of refraction.

Since diffraction increases with increasing f-number, and aberrations decrease with increasing f-number, determining optimum system performance often involves finding a point where the combination of these factors has a minimum effect.

ABERRATIONS

To determine the precise performance of a lens system, we can trace the path of light rays through it, using Snell's law at each optical interface to determine the subsequent ray direction. This process, called ray tracing, is usually accomplished on a computer. When this process is completed, it is typically found that not all the rays pass through the points or positions predicted by paraxial theory. These deviations from ideal imaging are called lens aberrations.

The direction of a light ray after refraction at the interface between two homogeneous, isotropic media of differing index of refraction is given by Snell's law:

$$n_1 \sin\theta_1 = n_2 \sin\theta_2 \qquad (1.20)$$

where θ_1 is the angle of incidence, θ_2 is the angle of refraction, and both angles are measured from the surface normal as shown in figure 1.14.
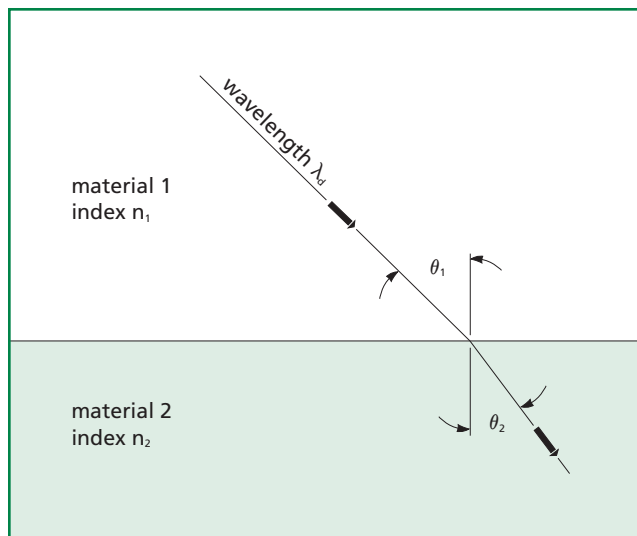

Figure 1.14 **Refraction of light at a dielectric boundary**

Even though tools for precise analysis of an optical system are becoming easier to use and are readily available, it is still quite useful to have a method for quickly estimating lens performance. This not only saves time in the initial stages of system specification, but can also help achieve a better starting point for any further computer optimization.

The first step in developing these rough guidelines is to realize that the sine functions in Snell's law can be expanded in an infinite Taylor series:

$$\sin \theta_1 = \theta_1 - \theta_1^3/3! + \theta_1^5/5! - \theta_1^7/7! + \theta_1^9/9! - \dots$$

The first approximation we can make is to replace all sine functions with their arguments (i.e., replace $\sin \theta_1$ with θ_1 itself and so on). This is called first-order or paraxial theory because only the first terms of the sine expansions are used. Design of any optical system generally starts with this approximation using the paraxial formulas.

The assumption that $\sin\theta = \theta$ is reasonably valid for θ close to zero (i.e., high f-number lenses). With more highly curved surfaces (and particularly marginal rays), paraxial theory yields increasingly large deviations from real performance because $\sin\theta \neq \theta$. These deviations are known as aberrations. Because a perfect optical system (one without any aberrations) would form its image at the point and to the size indicated by paraxial theory, aberrations are really a measure of how the image differs from the paraxial prediction.

As already stated, exact ray tracing is the only rigorous way to analyze real lens surfaces. Before the advent of computers, this was excessively tedious and time consuming. Seidel addressed this issue by developing a method of calculating aberrations resulting from the $\theta_1^3/3!$ term. The resultant third-order lens aberrations are therefore called Seidel aberrations.

To simplify these calculations, Seidel put the aberrations of an optical system into several different classifications. In monochromatic light they are spherical aberration, astigmatism, field curvature, coma, and distortion. In polychromatic light there are also chromatic aberration and lateral color. Seidel developed methods to approximate each of these aberrations without actually tracing large numbers of rays using all the terms in the sine expansions.

In actual practice, aberrations occur in combinations rather than alone. This system of classifying them, which makes analysis much simpler, gives a good description of optical system image quality. In fact, even in the era of powerful ray-tracing software, Seidel's formula for spherical aberration is still widely used.

SPHERICAL ABERRATION

Figure 1.15 illustrates how an aberration-free lens focuses incoming collimated light. All rays pass through the focal point F″. The lower figure shows the situation more typically encountered in single lenses. The farther from the optical axis the ray enters the lens, the nearer to the lens it focuses (crosses the optical axis). The distance along the optical axis between the intercept of the rays that are nearly on the optical axis (paraxial rays) and the rays that go through the edge of the lens (marginal rays) is called longitudinal spherical aberration (LSA). The height at which these rays intercept the paraxial focal plane is called transverse spherical aberration (TSA). These quantities are related by

$$\text{TSA} = \text{LSA} \times \tan u''. \tag{1.21}$$

Spherical aberration is dependent on lens shape, orientation, and conjugate ratio, as well as on the index of refraction of the materials present. Parameters for choosing the best lens shape and orientation for a given task are presented later in this chapter. However, the
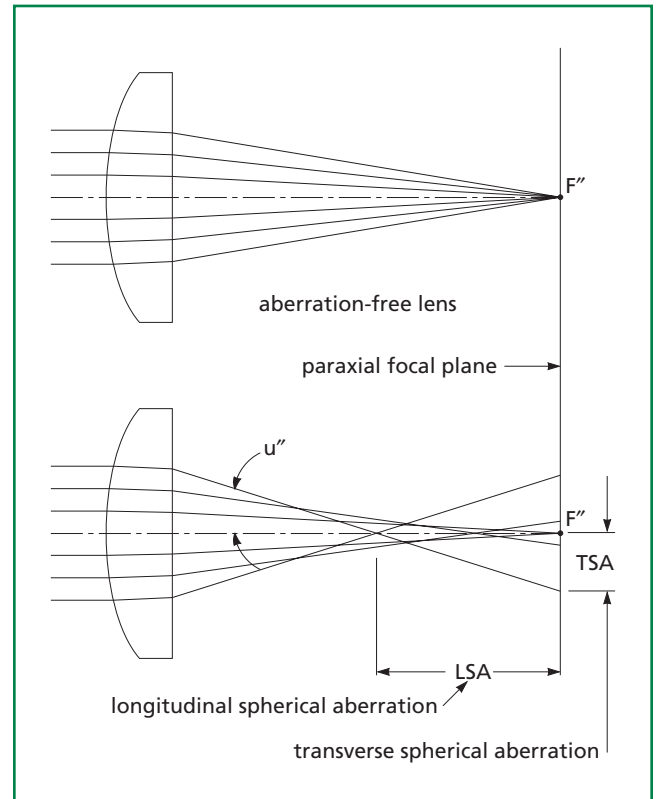

aberration-free lens

paraxial focal plane

u″

F″

TSA

LSA

longitudinal spherical aberration

transverse spherical aberration

Figure 1.15 **Spherical aberration of a plano-convex lens**

third-order, monochromatic, spherical aberration of a plano-convex lens used at infinite conjugate ratio can be estimated by

$$\text{spot size due to spherical aberration} = \frac{0.067\,f}{f/\#^3}.\qquad (1.22)$$

Theoretically, the simplest way to eliminate or reduce spherical aberration is to make the lens surface(s) with a varying radius of curvature (i.e., an aspheric surface) designed to exactly compensate for the fact that $\sin\theta \neq \theta$ at larger angles. In practice, however, most lenses with high surface quality are manufactured by grinding and polishing techniques that naturally produce spherical or cylindrical surfaces. The manufacture of aspheric surfaces is more complex, and it is difficult to produce a lens of sufficient surface accuracy to eliminate spherical aberration completely. Fortunately, these aberrations can be virtually eliminated, for a chosen set of conditions, by combining the effects of two or more spherical (or cylindrical) surfaces.

In general, simple positive lenses have undercorrected spherical aberration, and negative lenses usually have overcorrected spherical aberration. By combining a positive lens made from low-index glass with a negative lens made from high-index glass, it is possible to produce a combination in which the spherical aberrations cancel but the focusing powers do not. The simplest examples of this are cemented doublets, such as the 01 LAO series which produce minimal spherical aberration when properly used.

ASTIGMATISM

When an off-axis object is focused by a spherical lens, the natural asymmetry leads to astigmatism. The system appears to have two different focal lengths.

As shown in figure 1.16, the plane containing both optical axis and object point is called the tangential plane. Rays that lie in this plane are called tangential rays. Rays not in this plane are referred to as skew rays. The chief, or principal, ray goes from the object point through the center of the aperture of the lens system. The plane perpendicular to the tangential plane that contains the principal ray is called the sagittal or radial plane.

The figure illustrates that tangential rays from the object come to a focus closer to the lens than do rays in the sagittal plane. When the image is evaluated at the tangential conjugate, we see a line in the sagittal direction. A line in the tangential direction is formed at the sagittal conjugate. Between these conjugates, the image is either an elliptical or a circular blur. Astigmatism is defined as the separation of these conjugates.

The amount of astigmatism in a lens depends on lens shape only when there is an aperture in the system that is not in contact with the lens itself. (In all optical systems there is an aperture or stop, although in many cases it is simply the clear aperture of the lens element itself.) Astigmatism strongly depends on the conjugate ratio.
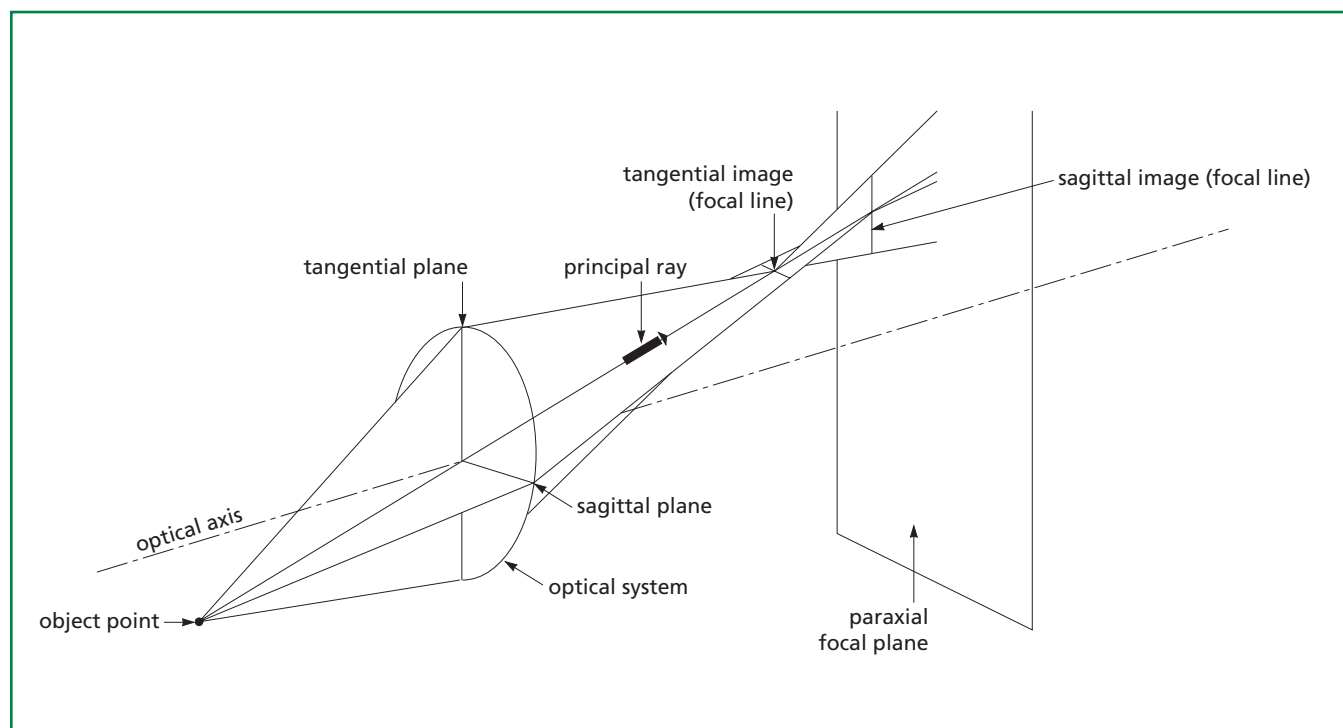

Figure 1.16 **Astigmatism represented by sectional views**

COMA

In spherical lenses, different parts of the lens surface exhibit different degrees of magnification. This gives rise to an aberration known as coma. As shown in figure 1.17, each concentric zone of a lens forms a ring-shaped image called a comatic circle. This causes blurring in the image plane (surface) of off-axis object points. An off-axis object point is not a sharp image point, but it appears as a characteristic comet-like flare. Even if spherical aberration is corrected and the lens brings all rays to a sharp focus on axis, a lens may still exhibit coma off axis. See figure 1.18.

As with spherical aberration, correction can be achieved by using multiple surfaces. Alternatively, a sharper image may be produced by judiciously placing an aperture, or stop, in an optical system to eliminate the more marginal rays.

FIELD CURVATURE

Even in the absence of astigmatism, there is a tendency of optical systems to image better on curved surfaces than on flat planes. This effect is called field curvature (see figure 1.19). In the presence of astigmatism, this problem is compounded because there are two separate astigmatic focal surfaces that correspond to the tangential and sagittal conjugates.

Field curvature varies with the square of field angle or the square of image height. Therefore, by reducing the field angle by one-half, it is possible to reduce the blur from field curvature to a value of 0.25 of its original size.
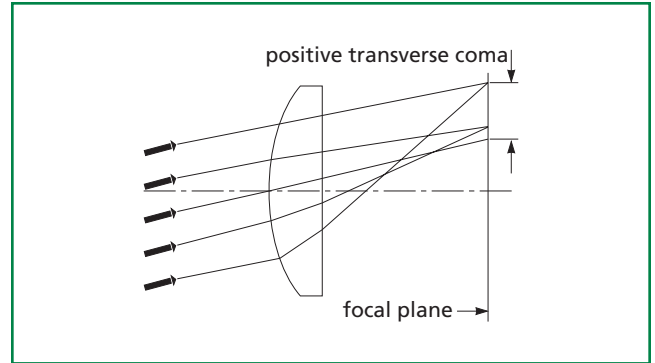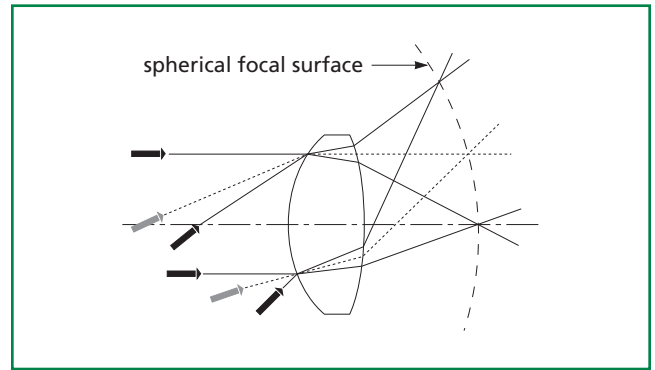

Figure 1.18   **Positive transverse coma**

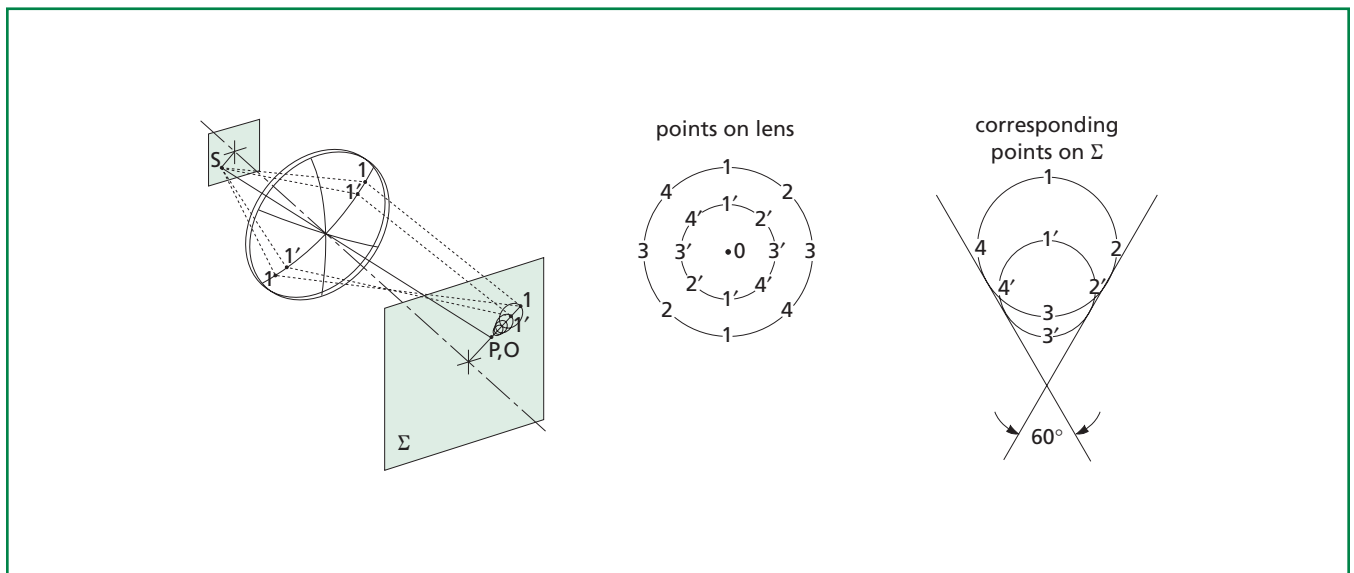Figure 1.19   **Field curvature**

Figure 1.17   **Imaging an off-axis point source by a lens with positive transverse coma**

Positive lens elements usually have inward curving fields, and negative lenses have outward curving fields. Field curvature can thus be corrected to some extent by combining positive and negative lens elements.

DISTORTION

The image field not only may have curvature but may also be distorted. The image of an off-axis point may be formed at a location on this surface other than that predicted by the simple paraxial equations. This distortion is different from coma (where rays from an off-axis point fail to meet perfectly in the image plane). Distortion means that even if a perfect off-axis point image is formed, its location on the image plane is not correct. Furthermore, the amount of distortion usually increases with increasing image height. The effect of this can be seen as two different kinds of distortion: pincushion and barrel (see figure 1.20). Distortion does not lower system resolution; it simply means that the image shape does not correspond exactly to the shape of the object. Distortion is a separation of the actual image point from the paraxially predicted location on the image plane and can be expressed either as an absolute value or as a percentage of the paraxial image height.

It should be apparent that a lens or lens system has opposite types of distortion depending on whether it is used forward or backward. This means that if a lens were used to make a photograph, and then used in reverse to project it, there would be no distortion in the final screen image. Also, perfectly symmetrical optical systems at 1:1 magnification have no distortion or coma.

CHROMATIC ABERRATION

The aberrations previously described are purely a function of the shape of the lens surfaces, and can be observed with monochromatic light. There are, however, other aberrations that arise when these optics are used to transform light containing multiple wavelengths.

The index of refraction of a material is a function of wavelength. Known as dispersion, this is discussed in Chapter 4, *Material Properties*. From Snell's law (see equation 1.20), it can be seen that light rays of different wavelengths or colors will be refracted at different angles since the index is not a constant. Figure 1.21 shows the result when polychromatic collimated light is incident on a positive lens element. Because the index of refraction is higher for shorter wavelengths, these are focused closer to the lens than the longer wavelengths. Longitudinal chromatic aberration is defined as the axial distance from the nearest to the farthest focal point.

As in the case of spherical aberration, positive and negative elements have opposite signs of chromatic aberration. Once again, by combining elements of nearly opposite aberration to form a doublet, chromatic aberration can be partially corrected. It is necessary to use two glasses with different dispersion characteristics, so that the weaker negative element can balance the aberration of the stronger, positive element.

Variations of Aberrations with Aperture, Field Angle, and Image Height

Aberration	Aperture (ϕ)	Field Angle (θ)	Image Height (y)
Lateral Spherical	ϕ^3	—	—
Longitudinal Spherical	ϕ^2	—	—
Coma	ϕ^2	θ	y
Astigmatism	ϕ	θ^2	y^2
Field Curvature	ϕ	θ^2	y^2
Distortion	—	θ^3	y^3
Chromatic	—	—	—

Figure 1.20 **Pincushion and barrel distortion**

Figure 1.21 **Longitudinal chromatic aberration**

LATERAL COLOR

Lateral color is the difference in image height between blue and red rays. Figure 1.22 shows the chief ray of an optical system consisting of a simple positive lens and a separate aperture. Because of the change in index with wavelength, blue light is refracted more strongly than red light, which is why rays intercept the image plane at different heights. Stated simply, magnification depends on color. Lateral color is very dependent on system stop location.

For many optical systems, the third-order term is all that may be needed to quantify aberrations. However, in highly corrected systems or in those having large apertures or a large angular field of view, third-order theory is inadequate. In these cases, exact ray tracing is absolutely essential.
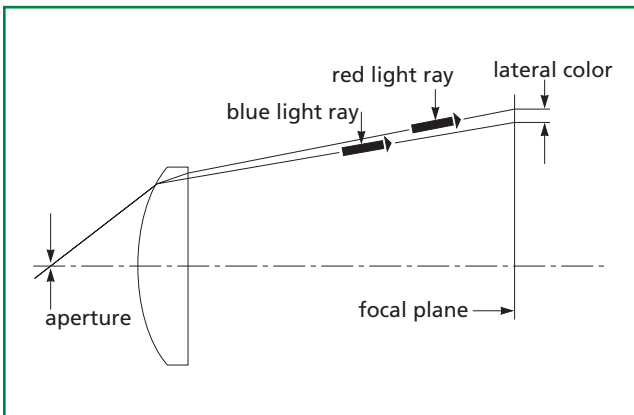
Figure 1.22   **Lateral color**
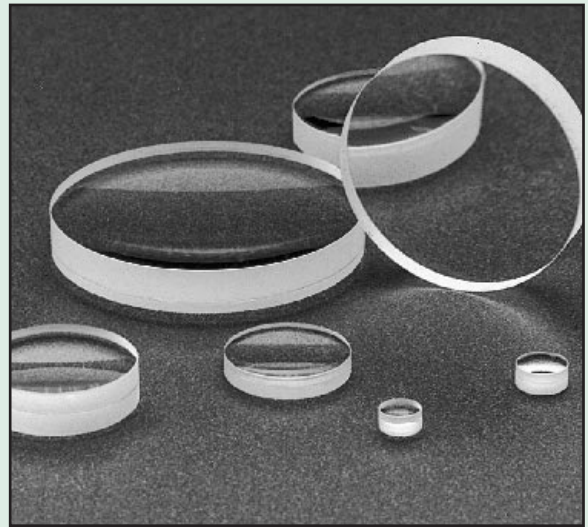
Lens Shape

Aberrations described in the preceding section are highly dependent on application, lens shape, and material of the lens (or, more exactly, its index of refraction). The singlet shape that minimizes spherical aberration at a given conjugate ratio is called best-form. The criterion for best-form at any conjugate ratio is that the marginal rays are equally refracted at each of the lens/air interfaces. This minimizes the effect of $\sin\theta \neq \theta$. It is also the criterion for minimum surface-reflectance loss. Another benefit is that absolute coma is nearly minimized for best-form shape, at both infinite and unit conjugate ratios.

To further explore the dependence of aberrations on lens shape, it is helpful to make use of the Coddington shape factor, q, defined as

$$q = \frac{(r_2 + r_1)}{(r_2 - r_1)}. \qquad (1.23)$$

Figure 1.23 shows the transverse and longitudinal spherical aberration of a singlet lens as a function of the shape factor, q. In this particular instance, the lens has a focal length of 100 mm, operates at f/5, has an index of refraction of 1.518722 (BK7 at the mercury green line, 546.1 nm), and is being operated at the infinite conjugate ratio. It is also assumed that the lens itself is the aperture stop. An asymmetric shape that corresponds to a q-value of about 0.7426 for this material and wavelength is the best singlet shape for on-axis imaging. Best-form shapes are used in Melles Griot laser-line-focusing singlet lenses. It is important to note that the best-form shape is dependent on refractive index. For example, with a high-index material, such as silicon, the best-form lens for the infinite conjugate ratio is a meniscus shape.

At infinite conjugate with a typical glass singlet, the plano-convex shape (q = 1), with convex side toward the infinite conjugate, performs nearly as well as the best-form lens. Because a plano-convex lens costs much less to manufacture than an asymmetric biconvex singlet, these lenses are quite popular. Furthermore, this lens shape exhibits near-minimum total transverse aberration and near-zero coma when used off axis, thus enhancing its utility.

For imaging at unit magnification (s = s″ = 2f), a similar analysis would show that a symmetric biconvex lens is the best shape. Not only is spherical aberration minimized, but coma, distortion, and lateral chromatic aberration exactly cancel each other out. These results are true regardless of material index or wavelength, which explains the utility of symmetric convex lenses, as well as symmetrical optical systems in general. However, if a remote stop is present, these aberrations may not cancel each other quite as well.

For wide-field applications, the best-form shape is definitely not the optimum singlet shape, especially at the infinite conjugate ratio, since it yields maximum field curvature. The ideal shape is determined by the situation and may require rigorous ray-tracing analysis.

It is possible to achieve much better correction in an optical system by using more than one element. The cases of an infinite conjugate ratio system and a unit conjugate ratio system are discussed in the following section.
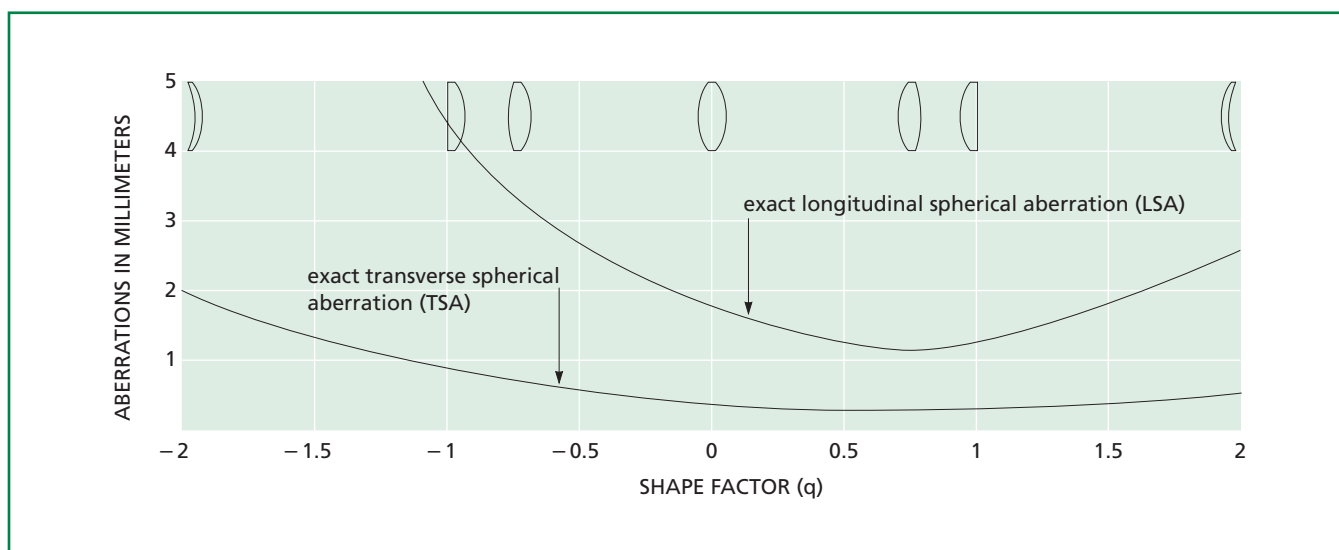
Figure 1.23 **Aberrations of positive singlets at infinite conjugate ratio as a function of shape**

Lens Combinations

INFINITE CONJUGATE RATIO

As shown in the previous discussion, the best-form singlet lens for use at infinite conjugate ratios is generally nearly plano-convex. Figure 1.24 shows a plano-convex lens (01 LPX 023) with incoming collimated light at a wavelength of 546.1 nm. This drawing, including the rays traced through it, is shown to exact scale. The marginal ray (ray f-number 1.5) strikes the paraxial focal plane significantly off the optical axis.

This situation can be improved by using a two-element system. The second part of the figure shows a precision achromat (01 LAO 014), which consists of a positive low-index (crown glass) element cemented to a negative meniscus high-index (flint glass) element. This is drawn to the same scale as the plano-convex lens. No spherical aberration can be discerned in the lens. Of course, not all of the rays pass exactly through the paraxial focal point; however, in this case, the departure is measured in micrometers, rather than in millimeters, as in the case of the plano-convex lens. Additionally, chromatic aberration (not shown) is much better corrected in the doublet. Even though these lenses are known as achromatic doublets, it is important to remember that even with monochromatic light the doublet's performance is superior.

Figure 1.24 also shows the f-number at which singlet performance becomes unacceptable. The ray with f-number 7.5 practically intercepts the paraxial focal point, and the f/3.8 ray is fairly close. This useful drawing, which can be scaled to fit a plano-convex lens of any focal length, can be used to estimate the magnitude of its spherical aberration, although lens thickness affects results slightly.

UNIT CONJUGATE RATIO

Figure 1.25 shows three possible systems for use at the unit conjugate ratio. All are shown to the same scale and using the same ray f-numbers with a light wavelength of 546.1 nm. The first system is a symmetric biconvex lens (01 LDX 027), the best-form singlet in this application. Clearly, significant spherical aberration is present in this lens at f/2.7. Not until f/13.3 does the ray closely approach the paraxial focus.

A dramatic improvement in performance is gained by using two identical plano-convex lenses with convex surfaces facing and nearly in contact. Those shown in figure 1.25 are both 01 LPX 081. The combination of these two lenses yields almost exactly the same focal length as the biconvex lens. To understand why this configuration improves performance so dramatically, consider that if the biconvex lens were split down the middle, we would have two identical plano-convex lenses, each working at an infinite conjugate ratio, but with the convex surface toward the focus. This orientation is opposite to that shown to be optimum for this shape lens. On the other hand, if these lenses are reversed, we have the system just described but with a better correction of the spherical aberration.
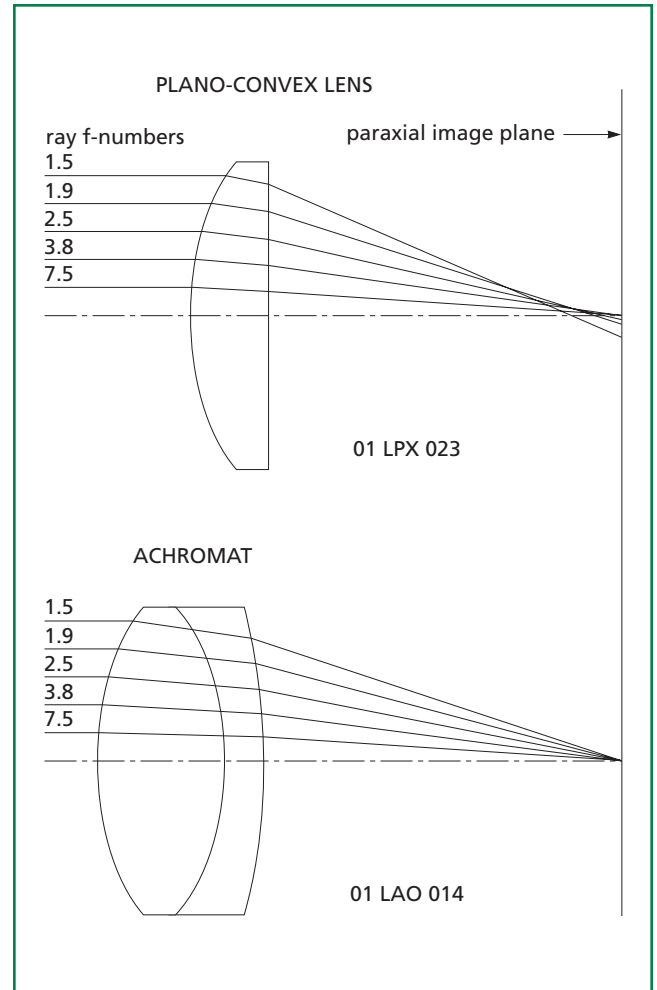
Figure 1.24 **Single-element plano-convex lens compared with a two-element achromat**

The previous examples indicate that an achromat is superior in performance to a singlet when used at the infinite conjugate ratio and at low f-numbers. Since the unit conjugate case can be thought of as two lenses, each working at the infinite conjugate ratio, the next step is to replace the plano-convex singlets with achromats, yielding a four-element system. The third part of figure 1.25 shows a system composed of two 01 LAO 037 lenses. Once again, spherical aberration is not evident, even in the f/2.7 ray.

SYMMETRIC BICONVEX LENS

ray f-numbers

2.7
3.3
4.4
6.7
13.3

paraxial image plane

01 LDX 027

IDENTICAL PLANO-CONVEX LENSES

2.7
3.3
4.4
6.7
13.3

01 LPX 081

IDENTICAL ACHROMATS

2.7
3.3
4.4
6.7
13.3

01 LAO 037

Figure 1.25 **Three possible systems for use at the unit conjugate ratio**

Diffraction Effects

In all light beams, some energy is spread outside the region predicted by rectilinear propagation. This effect, known as diffraction, is a fundamental and inescapable physical phenomenon.

Diffraction can be understood by considering the wave nature of light. Huygen's principle (figure 1.26) states that each point on a propagating wavefront is an emitter of secondary wavelets. The combined focus of these expanding wavelets forms the propagating wave. Interference between the secondary wavelets gives rise to a fringe pattern that rapidly decreases in intensity with increasing angle from the initial direction of propagation. Huygen's principle nicely describes diffraction, but rigorous explanation demands a detailed study of wave theory.

Diffraction effects are traditionally classified into either Fresnel or Fraunhofer types. Fresnel diffraction is primarily concerned with what happens to light in the immediate neighborhood of a diffracting object or aperture. It is thus only of concern when the illumination source is close to this aperture or object. Consequently, Fresnel diffraction is rarely important in most optical setups.

Fraunhofer diffraction, however, is often very important. This is the light-spreading effect of an aperture when the aperture (or object) is illuminated with an infinite source (plane-wave illumination) and the light is sensed at an infinite distance (far-field) from this aperture.

From these overly simple definitions, one might assume that Fraunhofer diffraction is important only in optical systems with infinite conjugate, whereas Fresnel diffraction equations should be considered at finite conjugate ratios. Not so. A lens or lens system of finit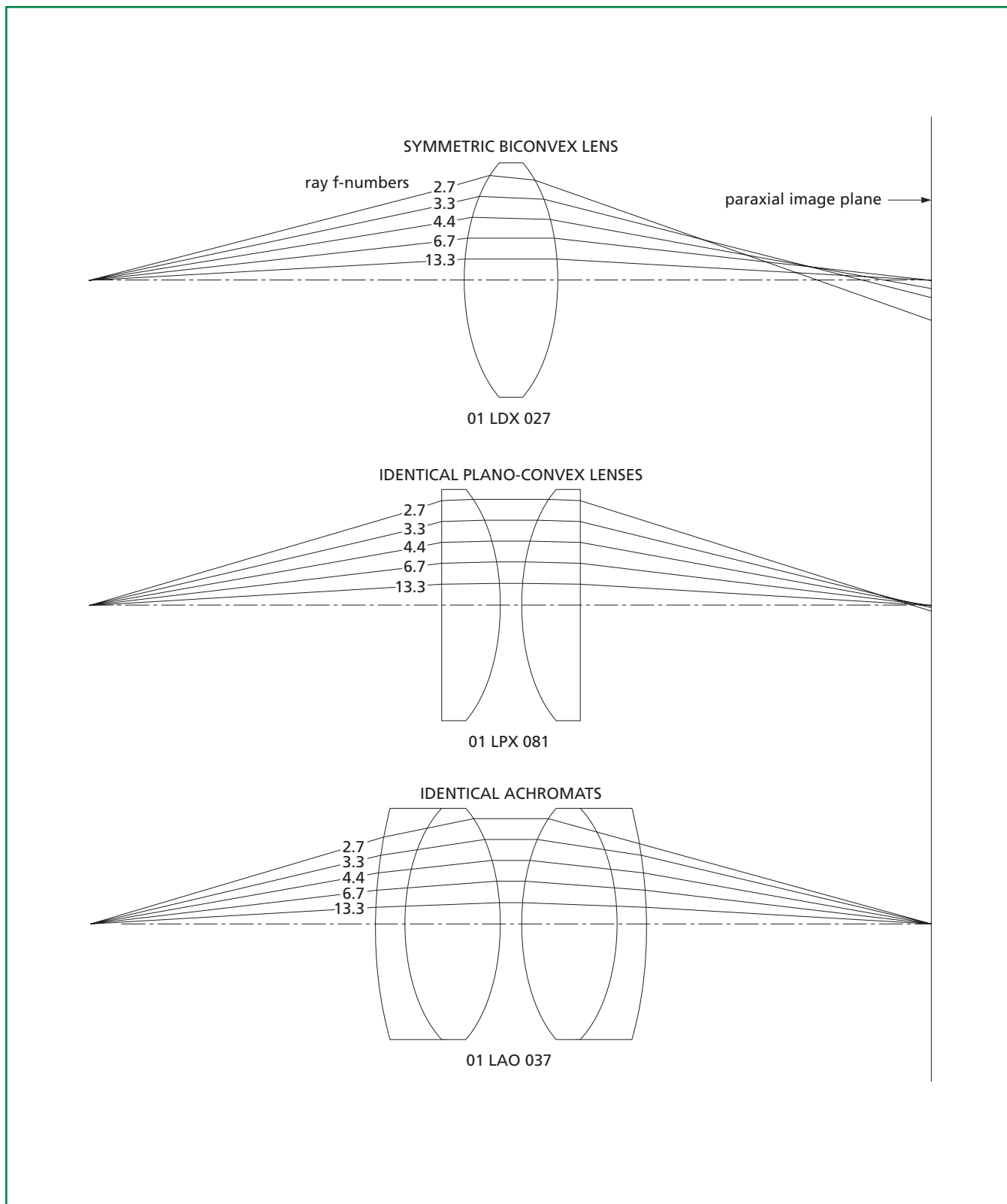e positive focal length with plane-wave input maps the far-field diffraction pattern of its aperture onto the focal plane; therefore, it is Fraunhofer diffraction that determines the limiting performance of optical systems. More generally, at any conjugate ratio, far-field angles are transformed into spatial displacements in the image plane.

CIRCULAR APERTURE

Fraunhofer diffraction at a circular aperture dictates the fundamental limits of performance for circular lenses. It is important to remember that the spot size, caused by diffraction, of a circular lens is

$$d = 2.44 \, \lambda \, f/\# \tag{1.24}$$

where d is the diameter of the focused spot produced from plane-wave illumination and λ is the wavelength of light being focused. Notice that it is the f-number of the lens, not its absolute diameter, that determines this limiting spot size.

The diffraction pattern resulting from a uniformly illuminated circular aperture actually consists of a central bright region, known as the Airy disc (see figure 1.27), which is surrounded by a number of much fainter rings. Each ring is separated by a circle of zero intensity. The irradiance distribution in this pattern can be described by

$$I_x = I_0 \left[\frac{2J_1(x)}{x} \right]^2 \tag{1.25}$$

where I_0 = peak irradiance in image

$$J_1(x) = x \sum_{n=1}^{\infty} (-1)^{n+1} \frac{x^{2n-2}}{(n-1)!n!2^{2n-1}}$$

$J_1(x)$ = Bessel function of the first kind of order unity

$$x = \frac{\pi D}{\lambda} \sin \theta$$

where λ = wavelength
 D = aperture diameter
 θ = angular radius from pattern maximum.

This useful formula shows the far-field irradiance distribution from a uniformly illuminated circular aperture of diameter, D.

APPLICATION NOTE

Rayleigh Criterion

In imaging applications, spatial resolution is ultimately limited by diffraction. Calculating the maximum possible spatial resolution of an optical system requires an arbitrary definition of what is meant by resolving two features. In the Rayleigh criterion, it is assumed that two separate point sources can be resolved when the center of the Airy disc from one overlaps the first dark ring in the diffraction pattern of the second. In this case, the smallest resolvable distance, d, is

$$d = \frac{0.61 \, \lambda}{N.A.} = 1.22 \, \lambda \, f/\#.$$

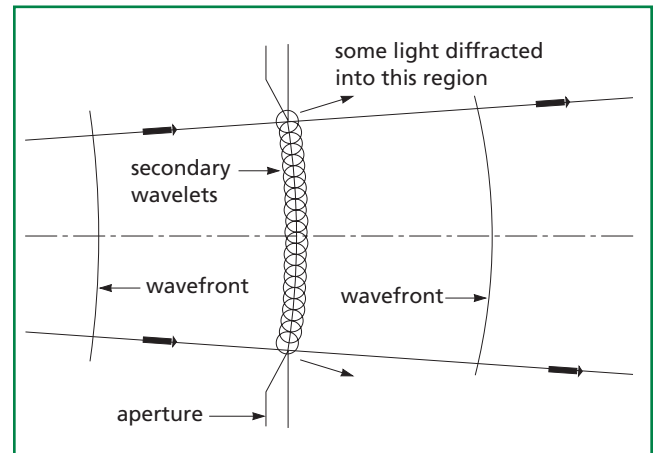

Figure 1.26 **Huygen's principle**
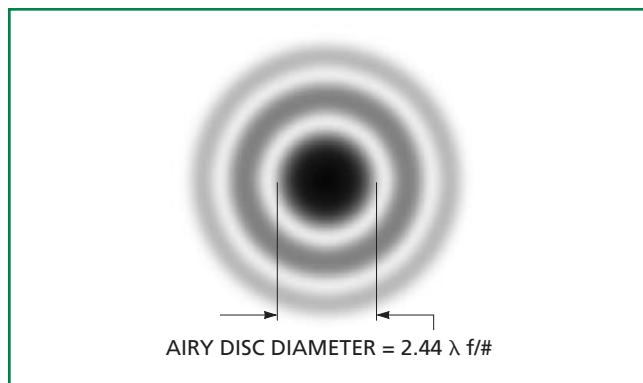
Figure 1.27 **Center of a typical diffraction pattern for a circular aperture**

AIRY DISC DIAMETER = 2.44 λ f/#

SLIT APERTURE

A slit aperture, which is mathematically simpler, is useful in relation to cylindrical optical elements. The irradiance distribution in the diffraction pattern of a uniformly illuminated slit aperture is described by

$$I_x = I_0 \left[\frac{\sin x}{x} \right]^2 \qquad (1.26)$$

where I_0 = peak irradiance in image

$$x = \frac{\pi w \sin \theta}{\lambda}$$

where λ = wavelength
$\quad$ w = slit width
$\quad$ θ = angular deviation from pattern maximum.

ENERGY DISTRIBUTION TABLE

The table below shows the major features of pure (unaberrated) Fraunhofer diffraction patterns of circular and slit apertures. The table shows the position, relative intensity, and percentage of total pattern energy corresponding to each ring or band. It is especially convenient to characterize positions in either pattern with the same variable x. This variable is related to field angle in the circular aperture case by

$$\sin \theta = \frac{\lambda x}{\pi D} \qquad (1.27)$$

where D is the aperture diameter. For a slit aperture, this relationship is given by

$$\sin \theta = \frac{\lambda x}{\pi w} \qquad (1.28)$$

where w is the slit width, π has its usual meaning, and D, w, and λ are all in the same units (preferably millimeters).

Linear instead of angular field positions are simply found from

$$r = s'' \tan (\theta) \qquad (1.29)$$

where s″ is the secondary conjugate distance. This last result is often seen in a different form, namely the diffraction-limited spot-size equation. For a circular lens that was stated at the outset of this section:

$$d = 2.44 \lambda \, f/\# \qquad (see\ 1.24)$$

This value represents the smallest spot size that can be achieved by an optical system with a circular aperture of a given f-number.

Energy Distribution in the Diffraction Pattern of a Circular or Slit Aperture

	Circular Aperture			Slit Aperture		
Ring or Band	Position (x)	Relative Intensity (I_x/I_0)	Energy in Ring (%)	Position (x)	Relative Intensity (I_x/I_0)	Energy in Band (%)
Central Maximum	0.0	1.0	83.8	0.0	1.0	90.3
First Dark	1.22π	0.0		1.00π	0.0	
First Bright	1.64π	0.0175	7.2	1.43π	0.0472	4.7
Second Dark	2.23π	0.0		2.00π	0.0	
Second Bright	2.68π	0.0042	2.8	2.46π	0.0165	1.7
Third Dark	3.24π	0.0		3.00π	0.0	
Third Bright	3.70π	0.0016	1.5	3.47π	0.0083	0.8
Fourth Dark	4.24π	0.0		4.00π	0.0	
Fourth Bright	4.71π	0.0008	1.0	4.48π	0.0050	0.5
Fifth Dark	5.24π	0.0		5.00π	0.0	

Note: Position variable (x) is defined in the text.

The graph in figure 1.28 shows the form of both circular and slit aperture diffraction patterns when plotted on the same normalized scale. Aperture diameter is equal to slit width so that patterns between x-values and angular deviations in the far-field are the same.

GAUSSIAN BEAMS

Apodization, or nonuniformity of aperture irradiance, alters diffraction patterns. If pupil irradiance is nonuniform, the formulas and results given previously do not apply. This is important to remember because most laser-based optical systems do not have uniform pupil irradiance. The output beam of a laser operating in the TEM_{00} mode has a smooth Gaussian irradiance profile. Formulas to determine the focused spot size from such a beam are discussed in Chapter 2, *Gaussian Beam Optics*. Furthermore,

when dealing with Gaussian beams, the location of the focused spot also departs from that predicted by the paraxial equations given in this chapter. This is also detailed in chapter 2.

CIRCULAR APERTURE

$$y_c = \left(\frac{2J_1(x)}{x} \right)^2$$

$$\text{where } J_1(x) = x \sum_{n=1}^{\infty} (-1)^{n+1} \frac{x^{2n-2}}{(n-1)!n!2^{2n-1}}$$

Note: $J_1(x)$ is the Bessel function of the first kind of order unity.

$x = \dfrac{\pi}{\lambda} D \sin\theta$

λ = wavelength

D = aperture diameter

θ = angular radius from pattern maximum

$$y_s = \left(\frac{\sin x}{x} \right)^2, \text{ where } x = \frac{\pi}{\lambda} w \sin \theta$$

λ = wavelength

w = slit width

θ = angular deviation direction of pattern maximum

91.0% within first bright ring

83.9% in Airy disc

90.3% in central maximum

95.0% within the two adjoining subsidiary maxima

SLIT APERTURE

NORMALIZED PATTERN IRRADIANCE (y): 1.0 .9 .8 .7 .6 .5 .4 .3 .2 .1 0.0

slit aperture — circular aperture

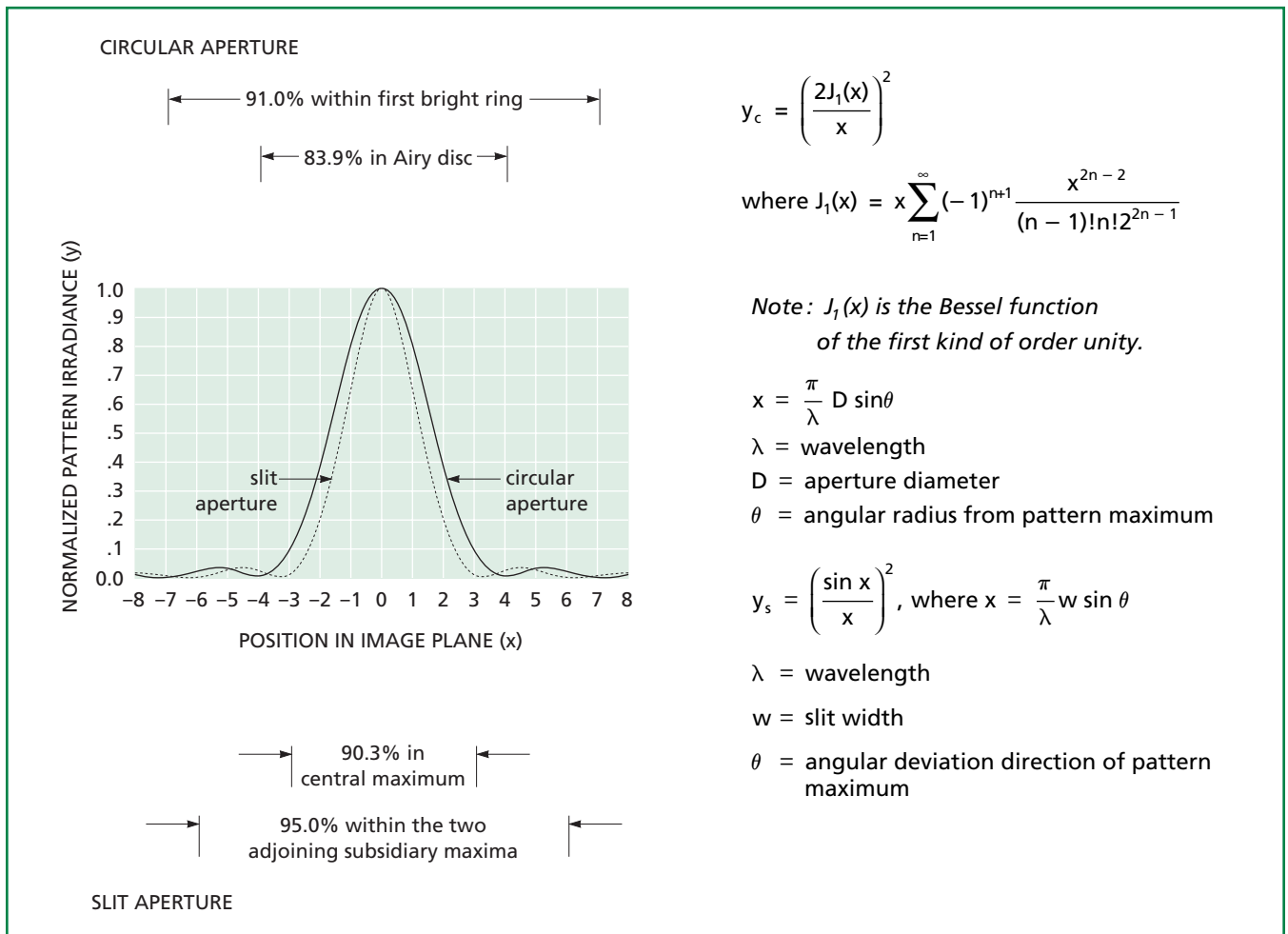POSITION IN IMAGE PLANE (x): −8 −7 −6 −5 −4 −3 −2 −1 0 1 2 3 4 5 6 7 8

Figure 1.28 **Fraunhofer diffraction pattern of a singlet slit superimposed on the Fraunhofer diffraction pattern of a circular aperture**

Lens Selection

Having discussed the most important factors that affect a lens or a lens system's performance, we will now address the practical matter of selecting the optimum catalog components for a particular task.

The following useful relationships are important to keep in mind throughout the selection process:

- Diffraction-limited spot size = 2.44 λ f/#

- Approximate on-axis spot size of a plano-convex lens at the infinite conjugate resulting from spherical aberration = $\dfrac{0.067\ f}{f/\#^3}$

- Optical invariant = $m = \dfrac{NA}{NA''}$.

Example 1: Collimating an Incandescent Source

Produce a collimated beam from a quartz halogen bulb having a 1-mm-square filament. Collect the maximum amount of light possible and produce a beam with the lowest possible divergence angle.

This problem, illustrated in figure 1.29, involves the typical trade-off between light-collection efficiency and resolution (where a beam is being collimated rather than focused, resolution is defined by beam divergence). To collect more light, it is necessary to work at a low f-number, but because of aberrations, higher resolution (lower divergence angle) will be achieved by working at a higher f-number.

In terms of resolution, the first thing to realize is that the minimum divergence angle (in radians) that can be achieved using any lens system is the source size divided by system focal length. An off-axis ray (from the edge of the source) entering the first principal point of the system exits the second principal point at the same angle. Therefore, increasing system focal length improves this limiting divergence because the source appears smaller.

An optic that can produce a spot size of 1 mm when focusing a perfectly collimated beam is therefore required. Since source size is inherently limited, it is pointless to strive for better resolution. This level of resolution can be achieved easily with a plano-convex lens.

While angular divergence decreases with increasing focal length, spherical aberration of a plano-convex lens increases with increasing focal length. To determine the appropriate focal length, set the spherical aberration formula for a plano-convex lens equal to the source (spot) size:

$$\frac{0.067\ f}{f/\#^3} = 1\ \text{mm}. \qquad \text{(see eq. 1.22)}$$

This ensures a lens that meets the minimum performance needed.

To select a focal length, make an arbitrary f-number choice. As can be seen from the relationship, as we lower the f-number (increase collection efficiency), we decrease the focal length, which will worsen the resultant divergence angle (minimum divergence = 1 mm/f).

In this example, we will accept f/2 collection efficiency, which gives us a focal length of about 120 mm. For f/2 operation we would need a minimum diameter of 60 mm. The 01 LPX 209 fits this specification exactly. Beam divergence would be about 8 mrad.

Finally, we need to verify that we are not operating below the theoretical diffraction limit. In this example, the numbers (1-mm spot size) indicate that we are not, since

diffraction-limited spot size = 2.44 $\times$ 0.5 μm $\times$ 2 = 2.44 μm.

Example 2: Coupling an Incandescent Source into a Fiber

On pages 1.6 and 1.7 we considered a system in which the output of an incandescent bulb with a filament of 1 mm in diameter was to be coupled into an optical fiber with a core diameter of 100 μm and a numerical aperture of 0.25. From the optical invariant and other constraints given in the problem, we determined that system focal length is 9.1 mm, diameter = 5 mm, s = 100 mm, s" = 10 mm, NA" = 0.25, and NA = 0.025 (or f/2 and f/20). The singlet lenses that match these specifications are the plano-convex 01 LPX 003 or biconvex lenses 01 LDX 003 and 01 LDX 005. The closest achromat would be the 01 LAO 001.
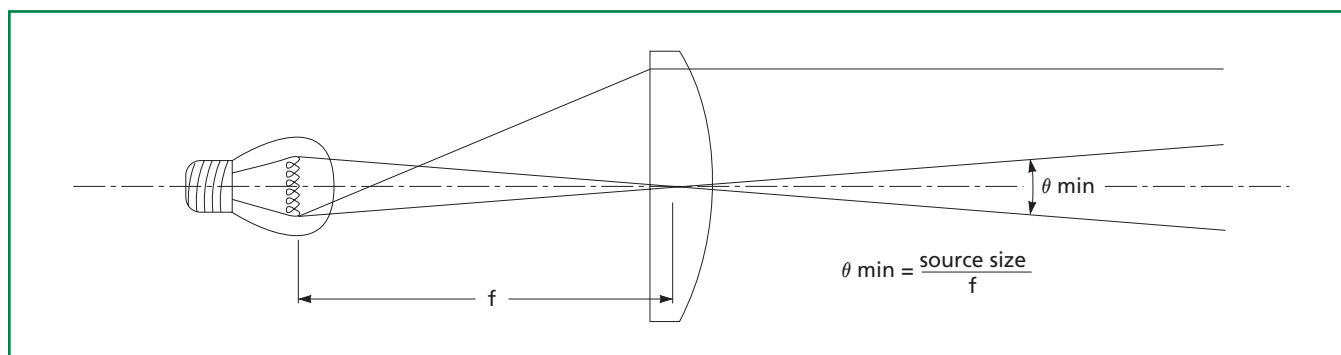

θ min

θ min = $\dfrac{\text{source size}}{f}$

f

Figure 1.29 **Collimating an incandescent source**

We can immediately reject the biconvex lenses because of spherical aberration. We can estimate the performance of the 01 LPX 003 on the focusing side by using our spherical aberration formula:

$$\text{spot size} = \frac{0.067\,(10)}{2^3} = 84\,\mu m.$$

We will ignore, for the moment, that we are not working at the infinite conjugate.

This is slightly smaller than the 100-μm spot size we're trying to achieve. However, since we are not working at infinite conjugate, the spot size will be larger than given by our simple calculation. This lens is therefore likely to be marginal in this situation, especially if we consider chromatic aberration. A better choice is the achromat. Although a computer ray trace would be required to determine its exact performance, it is virtually certain to provide adequate performance.

Example 3: Symmetric Fiber-to-Fiber Coupling

Couple an optical fiber with an 8-μm core and a 0.15 numerical aperture into another fiber with the same characteristics. Assume a wavelength of 0.5 μm.

This problem, illustrated in figure 1.30, is essentially a 1:1 imaging situation. We want to collect and focus at a numerical aperture of 0.15 or f/3.3, and we need a lens with an 8-μm spot size at this f-number. Based on the lens combination discussion on page 1.8, our most likely setup is either a pair of identical plano-convex lenses or achromats, faced front to front. To determine the necessary focal

length for a plano-convex lens, we again use the spherical aberration estimate formula:

$$\frac{0.067\,f}{3.3^3} = 0.008\text{ mm.}$$

This formula yields a focal length of 4.3 mm and a minimum diameter of 1.3 mm. The 01 LPX 423 meets these criteria. The biggest problem with utilizing these tiny, short focal length lenses is the practical considerations of handling, mounting, and positioning them. Since using a pair of longer focal length singlets would result in unacceptable performance, the next step might be to use a pair of the slightly longer focal length, larger achromats, such as the 01 LAO 001. The performance data, given on page 1.26, shows that this combination does provide the required 8-μm spot diameter.

Because fairly small spot sizes are being considered here, it is important to make sure that the system is not being asked to work below the diffraction limit:

$$2.44 \times 0.5\,\mu m \times 3.3 = 4\,\mu m.$$

Since this is half the spot size caused by aberrations, it can be safely assumed that diffraction will not play a significant role here.

An entirely different approach to a fiber-coupling task such as this would be a pair of spherical ball lenses (06 LMS series), listed on page 15.15, or one of the gradient-index lenses (06 LGT series), listed on page 15.19.
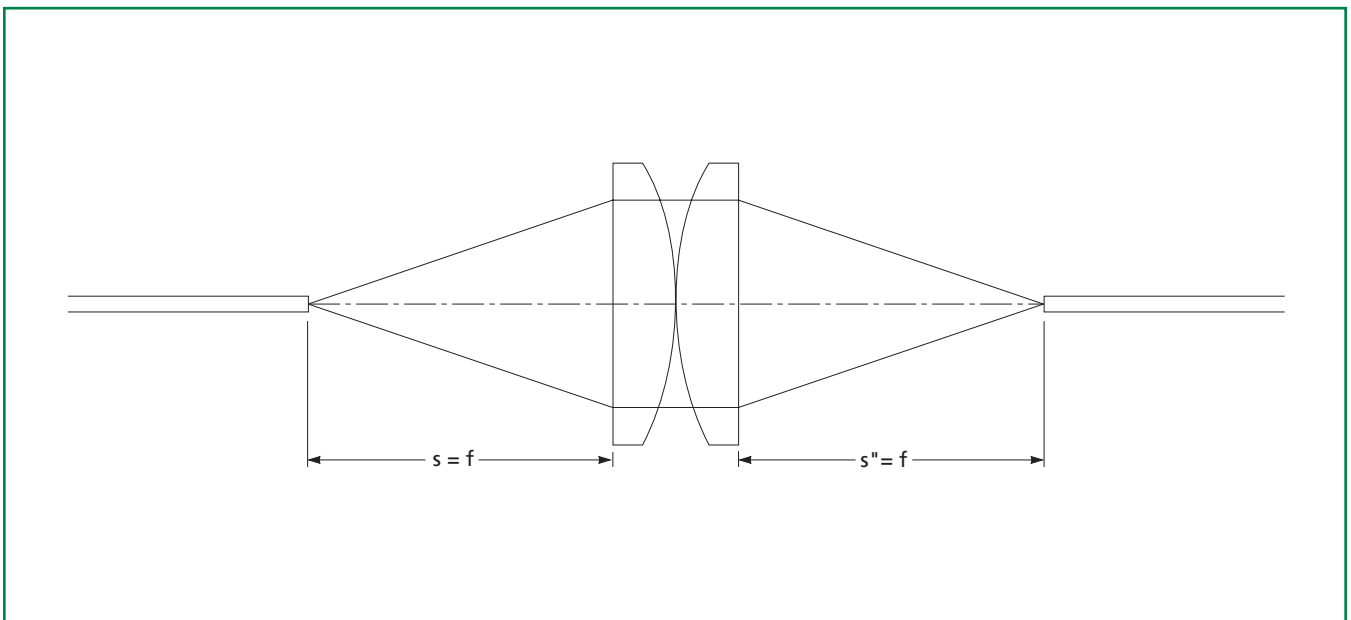

Figure 1.30   **Symmetric fiber-to-fiber coupling**

Example 4: Diffraction-Limited Performance

Determine at what f-number a plano-convex lens being used at an infinite conjugate ratio with 0.5-μm wavelength light becomes diffraction limited (i.e., the effects of diffraction exceed those caused by aberration).

To solve this problem, set the equations for diffraction-limited spot size and third-order spherical aberration equal to each other. The result depends upon focal length, since aberrations scale with focal length, while diffraction is solely dependent upon f-number. Substituting some common focal lengths into this formula, we get f/8.6 at f = 100 mm, f/7.2 at f = 50 mm, and f/4.8 at f = 10 mm.

$$2.44 \times 0.5\,\mu m \times f/\# = \frac{0.067 \times f}{f/\#^3}$$

or

$$f/\# = (54.9 \times f)^{1/4}.$$

When working with these focal lengths (and under the conditions previously stated), we can assume essentially diffraction-limited performance above these f-numbers. Keep in mind, however, that this treatment does not take into account manufacturing tolerances or chromatic aberration, which will be present in polychromatic applications.

MELLES GRIOT LENS DATABASE

A database containing prescription information for most of the optical components listed in this catalog is included in the Melles Griot catalog on CD-ROM. This database, in a Zemax format, facilitates the determination of

• Spot size

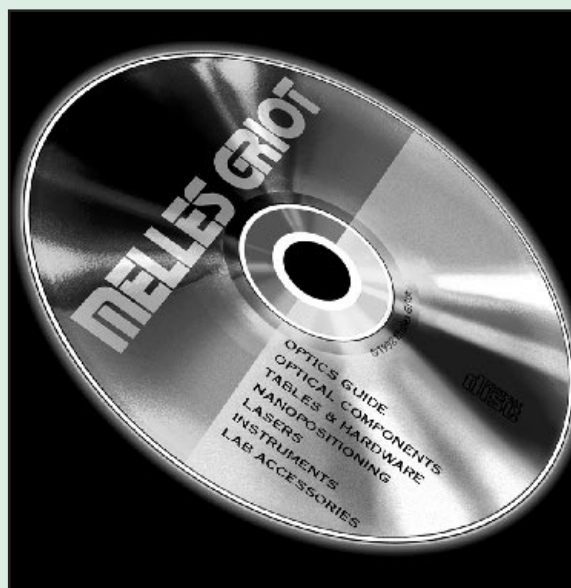• Prescription information

• Wavefront distortion.

Please contact our sales department for your free Melles Griot Catalog on CD-ROM:
Phone: 1-800-835-2626 / (949) 261-5600
FAX: (949) 261-7790
E-mail: mglit@irvine.mellesgriot.com

Non-US customers should contact the nearest Melles Griot office (see back cover).

Spot Size

In general, the performance of a lens or lens system in a specific circumstance should be determined by an exact trigonometric ray trace. Melles Griot applications engineers can supply ray-trace data for particular lenses and systems of catalog components on request. However, for certain situations, some simple guidelines can be used for lens selection. The optimum working conditions for some of the lenses in this catalog have already been presented. The following tables give some quantitative results for a variety of simple and compound lens systems that can be constructed from standard catalog optics.

In interpreting these tables, remember that these theoretical values obtained from computer ray tracing consider only the effects of ideal geometric optics. Effects of manufacturing tolerances have not been considered. Furthermore, remember that using more than one element provides a higher degree of correction but makes alignment more difficult. When actually choosing a lens or a lens system, it is important to note the tolerances and specifications clearly described for each Melles Griot lens in the product listings.

The tables give spot size for a variety of lenses used at several different f-numbers. All the tables are for on-axis, uniformly illuminated, collimated input light at 632.8 nm. They assume that the lens is facing in the direction that produces a minimum spot size. When the spot size caused by aberrations is smaller or equal to the diffraction-limited spot size, the notation "DL" appears next to the entry. The shorter focal length lenses produce smaller spot sizes because aberrations increase linearly as a lens is scaled up.

The effect on spot size caused by spherical aberration is strongly dependent on f-number. For a plano-convex singlet, spherical aberration is inversely dependent on the cube of the f-number. For doublets, this relationship can be even higher. On the other hand, the spot size caused by diffraction increases linearly with f-number. Thus, for some lens types, spot size at first decreases and then increases with f-number, meaning that there is some optimum performance point where both aberrations and diffraction combine to form a minimum.

Unfortunately, these results cannot be generalized to situations where the lenses are used off axis. This is particularly true of the achromat/aplanatic meniscus lens combinations because their performance degrades rapidly off axis.

Focal Length = 10 mm

	Spot Size (μm)*		
	01 LDX 005	01 LPX 005	01 LAO 001
f/2	550	95	4
f/3	120	25	5 (DL)
f/5	30	8 (DL)	8 (DL)
f/10	15 (DL)	15 (DL)	15 (DL)

Diffraction-limited performance is indicated by DL.

Focal Length = 30 mm

	Spot Size (μm)*		01 LAO 059 &
	01 LPX 049	01 LAO 024	01 LAM 059
f/2	350	80	4
f/3	90	11	5 (DL)
f/5	17	8 (DL)	8 (DL)
f/10	15 (DL)	15 (DL)	15 (DL)

Diffraction-limited performance is indicated by DL.

Focal Length = 60 mm

	Spot Size (μm)*			
	01 LDX 123	01 LPX 127	01 LAO 079	01 LAO 126 & 01 LAM 126
f/2	800	600	80	6
f/3	225	200	35	5 (DL)
f/5	42	30	9	8 (DL)
f/10	15 (DL)	15 (DL)	15 (DL)	15 (DL)

Diffraction-limited performance is indicated by DL.

Aberration Balancing

To improve system performance, optical designers make sure that the total aberration contribution from all surfaces taken together sums to nearly zero. Normally, such a process requires computerized analysis and optimization. However, there are some simple guidelines that can be used to achieve this with lenses available in this catalog. This approach can yield systems that operate at a much lower f-number than can usually be achieved with simple lenses.

Specifically, we will examine how to null the spherical aberration from two or more lenses in collimated, monochromatic light. Thus, this technique will be most useful for laser beam focusing and expanding.

Figure 1.31 shows the third-order longitudinal spherical aberration coefficients for four of the most common positive and negative lens shapes when used with parallel, monochromatic incident light. The plano-convex and plano-concave lenses both show minimum spherical aberration when oriented with their curved surface facing the incident parallel beam. All other configurations exhibit larger amounts of spherical aberration. With these lens types, it is now possible to show how various systems can be corrected for spherical aberration.

A two-element laser beam expander is a good starting example. In this case, two lenses are separated by a distance which is the sum of their focal lengths, so that the overall system focal length is infinite. This system will not focus incoming collimated light, but it will change the beam diameter. By definition, each of the lenses is operating at the same f-number.

The equation for longitudinal spherical aberration shows that for two lenses with the same f-number, aberration varies directly with the focal lengths of the lenses. The sign of the aberration is the same as focal length. Thus, it should be possible to correct the spherical

aberration of this Galilean-type beam expander, which consists of a positive focal length objective and a negative diverging lens.

If a plano-convex lens of focal length f_1 oriented in the normal direction is combined with a plano-concave lens of focal length f_2 oriented in its reverse direction, the total spherical aberration of the system is

$$\text{LSA} = \frac{0.272\,f_1}{f/\#^2} + \frac{1.069\,f_2}{f/\#^2}. \tag{1.30}$$

After setting this equal to zero, we obtain

$$\frac{f_1}{f_2} = -\frac{1.069}{0.272} = -3.93.$$

To make the magnitude of aberration contributions of the two elements equal so they will cancel out, and thus correct the system, select the focal length of the positive element to be 3.93 times that of the negative element.

Figure 1.32 shows a beam-expander system made up of catalog elements, in which the focal length ratio is 4:1. This simple system is corrected to about 1/6 wavelength at 632.8 nm, even though the objective is operating at f/4 with a 20-mm aperture diameter. This is remarkably good wavefront correction for such a simple system; one would normally assume that a doublet objective would be needed and a complex diverging lens as well. This analysis does not take into account manufacturing tolerances.

A beam expander of lower magnification can also be derived from this information. If a symmetric-convex objective is used together with a reversed plano-concave diverging lens, the aberration coefficients are in the ratio of 1.069/0.403 = 2.65. Figure 1.32 shows a system of catalog lenses that provides a magnification of
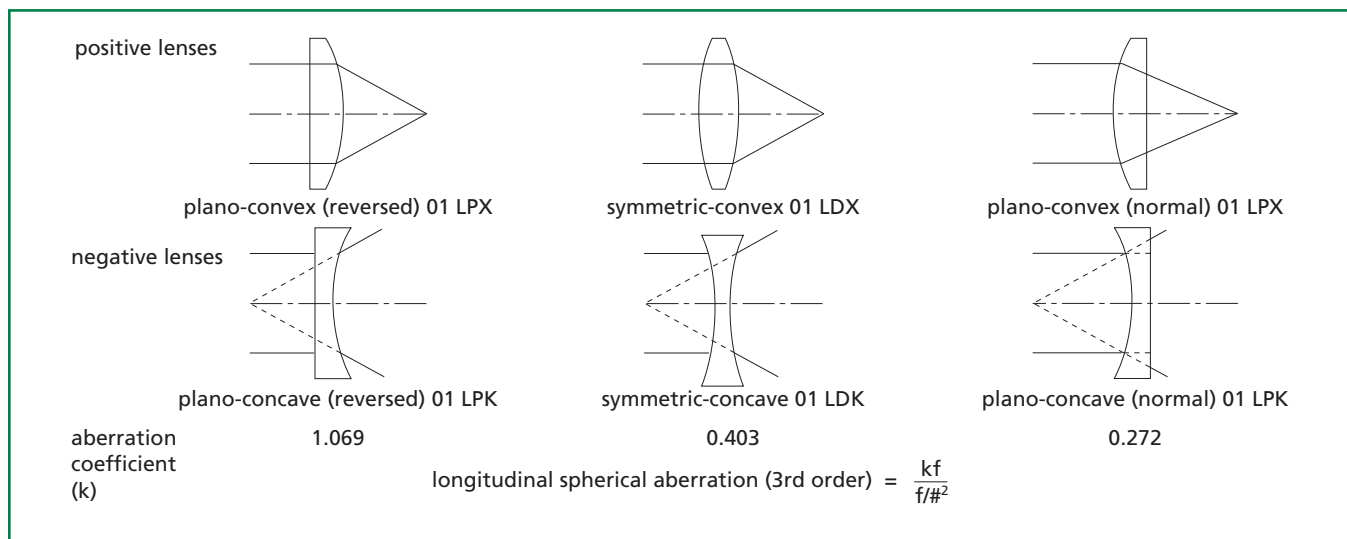

Figure 1.31 **Third-order longitudinal spherical aberration of typical lens shapes**
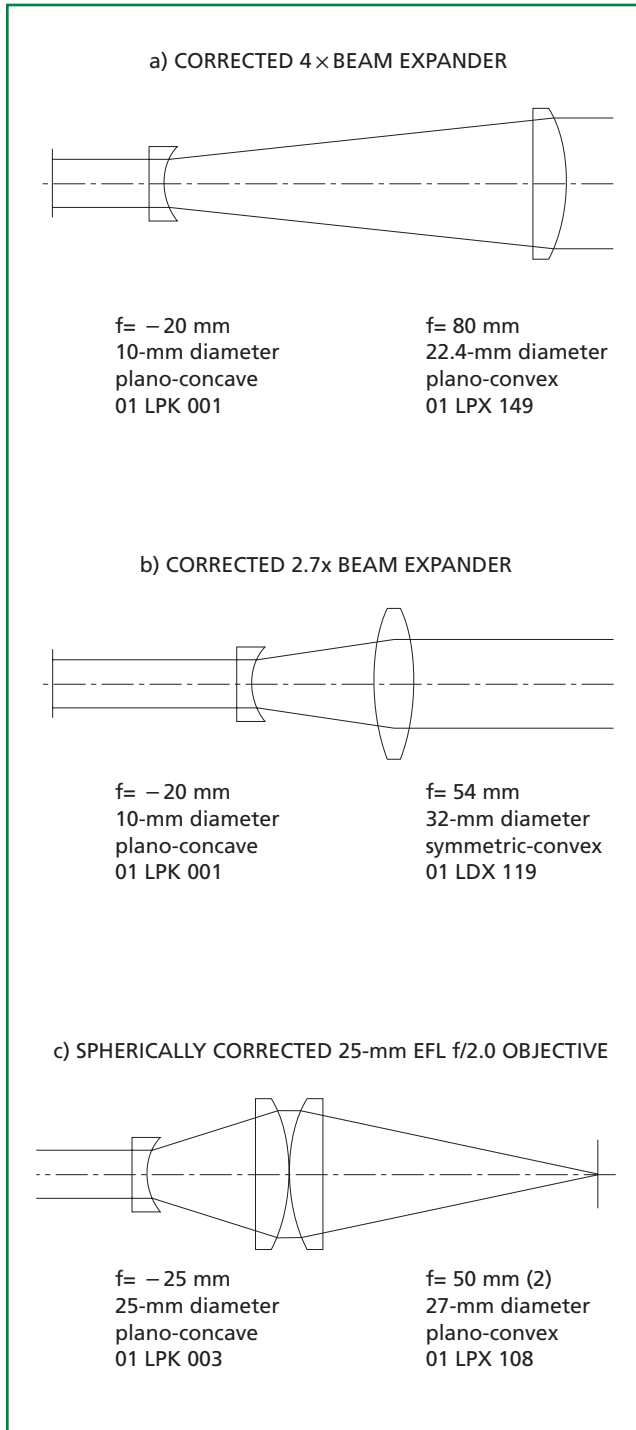
a) CORRECTED 4 × BEAM EXPANDER

f= − 20 mm
10-mm diameter
plano-concave
01 LPK 001

f= 80 mm
22.4-mm diameter
plano-convex
01 LPX 149

b) CORRECTED 2.7x BEAM EXPANDER

f= − 20 mm
10-mm diameter
plano-concave
01 LPK 001

f= 54 mm
32-mm diameter
symmetric-convex
01 LDX 119

c) SPHERICALLY CORRECTED 25-mm EFL f/2.0 OBJECTIVE

f= − 25 mm
25-mm diameter
plano-concave
01 LPK 003

f= 50 mm (2)
27-mm diameter
plano-convex
01 LPX 108

Figure 1.32 **Combining catalog lenses for aberration balancing**

The material presented in this section is based on the work of John F. Forkner.

2.7 (the closest possible given the available focal lengths). The maximum wavefront error in this case is only 1/4 wave, even though the objective is working at f/3.3.

The relatively fast speed of these objectives is a great advantage in minimizing the length of these beam expanders. They would be particularly useful with Nd:YAG and argon-ion lasers, which tend to have large output beam diameters.

These same principles can be utilized to create high numerical aperture objectives that might be used as laser focusing lenses. Figure 1.32 shows an objective consisting of an initial negative element, followed by two identical plano-convex positive elements. Again, all of the elements operate at the same f-number, so that their aberration contributions are proportional to their focal lengths. To obtain zero total spherical aberration from this configuration, we must satisfy

$$1.069\, f_1 + 0.272\, f_2 + 0.272\, f_2 = 0$$

or

$$\frac{f_1}{f_2} = -0.51.$$

Therefore, a corrected system should result if the focal length of the negative element is just about half that of each of the positive lenses. In this case, $f_1 = -25$ mm and $f_2 = 50$ mm yield a total system focal length of about 25 mm and an f-number of approximately f/2. This objective, corrected to 1/6 wave, has the additional advantage of a very long working distance.

UV OPTICS

Melles Griot now offers a selection of UV optics ranging from 193 to 355 nm. See Chapter 16, *UV Optics,* for details.

Definition of Terms

FOCAL LENGTH (f)

Two distinct terms describe the focal lengths associated with every lens or lens system. The effective focal length (EFL) or equivalent focal length (denoted f in figure 1.33) determines magnification and hence the image size. The term f appears frequently in the lens formulas and tables of standard lenses. Unfortunately, f is measured with reference to principal points which are usually inside the lens so the meaning of f is not immediately apparent when a lens is visually inspected.

The second type of focal length relates the focal plane positions directly to landmarks on the lens surfaces (namely the vertices) which are immediately recognizable. It is not simply related to image size but is especially convenient for use when one is concerned about correct lens positioning or mechanical clearances. Examples of this second type of focal length are the front focal length (FFL, denoted f_f in figure 1.33) and the back focal length (BFL, denoted f_b).

The convention in all of the figures (with the exception of a single deliberately reversed ray) is that light travels from left to right.

FOCAL POINT (F OR F″)

Rays that pass through or originate at either focal point must be, on the opposite side of the lens, parallel to the optical axis. This fact is the basis for locating both focal points.

PRIMARY PRINCIPAL SURFACE

Let us imagine that rays originating at the front focal point F (and therefore parallel to the optical axis after emergence from the opposite side of the lens) are singly refracted at some imaginary surface, instead of twice refracted (once at each lens surface) as actually happens. There is a unique imaginary surface, called the principal surface, at which this can happen.

To locate this unique surface, consider a single ray traced from the air on one side of the lens, through the lens and into the air on the other side. The ray is broken into three segments by the lens. Two of these are external (in the air), and the third is internal (in the glass). The external segments can be extended to a common point of intersection (certainly near, and usually within, the lens). The

Figure 1.33 **Focal length and focal points**

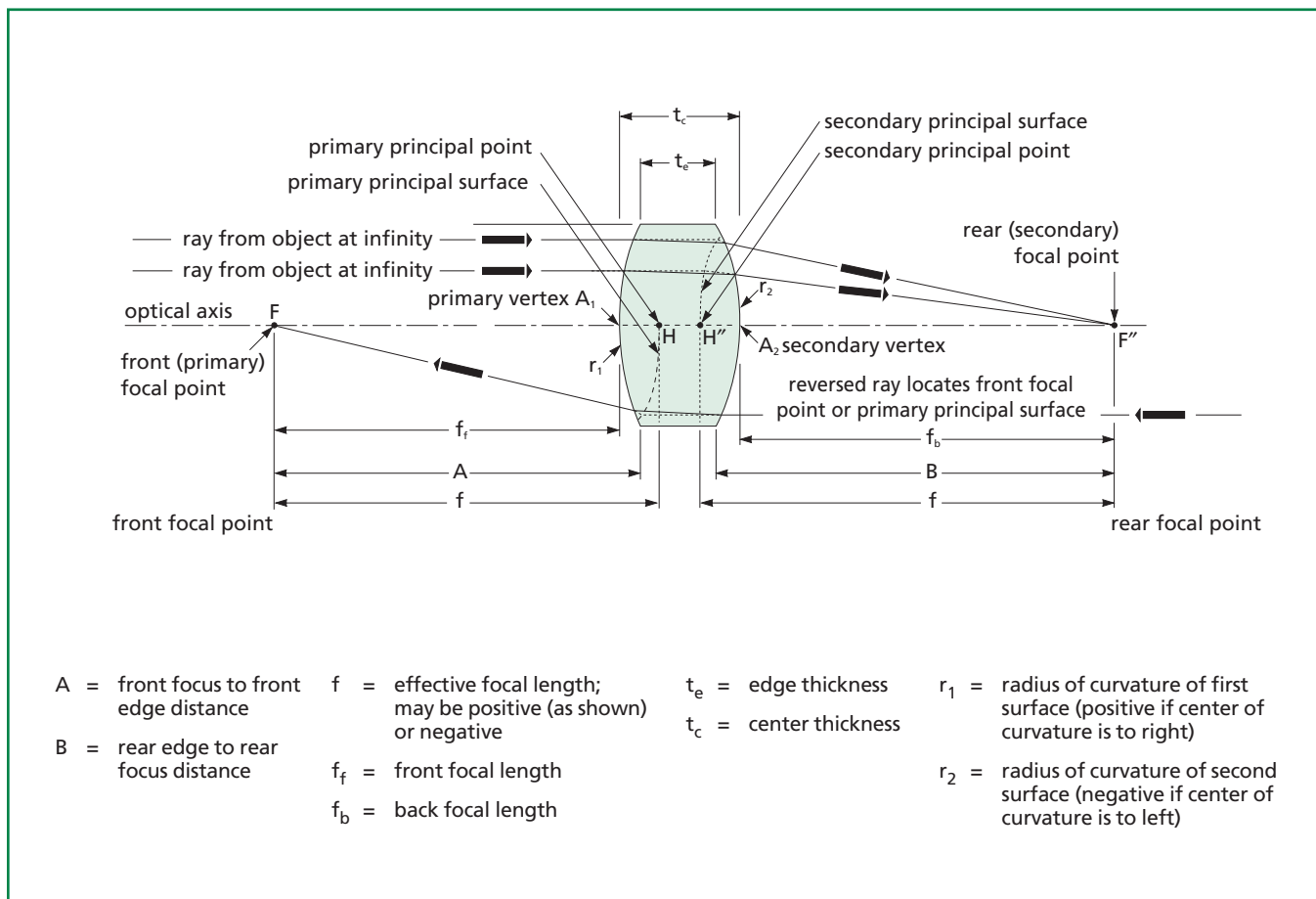|			
A = front focus to front edge distance	f = effective focal length; may be positive (as shown) or negative	t_e = edge thickness	r_1 = radius of curvature of first surface (positive if center of curvature is to right)
B = rear edge to rear focus distance	f_f = front focal length	t_c = center thickness	r_2 = radius of curvature of second surface (negative if center of curvature is to left)
	f_b = back focal length		

principal surface is the locus of all such points of intersection of extended external ray segments. The principal surface of a perfectly corrected optical system is a sphere centered on the focal point.

Near the optical axis, the principal surface is nearly flat, and for this reason, it is sometimes referred to as the *principal plane*.

SECONDARY PRINCIPAL SURFACE

This term is defined analogously to the primary principal surface, but it is used for a collimated beam incident from the left and focused to the rear focal point F≤ on the right. Rays in that part of the beam nearest the axis can be thought of as once refracted at the secondary principal surface, instead of being refracted by both lens surfaces.

PRIMARY PRINCIPAL POINT (H)
OR FIRST NODAL POINT

This point is the intersection of the primary principal surface with the optical axis.

SECONDARY PRINCIPAL POINT (H≤)
OR SECONDARY NODAL POINT

This point is the intersection of the secondary principal surface with the optical axis.

CONJUGATE DISTANCES (S AND S″)

The conjugate distances are the object distance, s, and image distance, s″. Specifically, s is the distance from the object to H, and s″ is the distance from H″ to the image location. The term infinite conjugate ratio refers to the situation in which a lens is either focusing incoming collimated light, or being used to collimate a source (therefore either s or s″ is infinity).

PRIMARY VERTEX (A$_1$)

The primary vertex is the intersection of the primary lens surface with the optical axis.

SECONDARY VERTEX (A$_2$)

The secondary vertex is the intersection of the secondary lens surface with the optical axis.

EFFECTIVE FOCAL LENGTH (EFL, f)

Assuming that the lens is surrounded by air or vacuum (refractive index 1.0), this is both the distance from the front focal point (F) to the primary principal point (H) and the distance from the secondary principal point (H″) to the rear focal point (F″). Later we use f to designate the paraxial effective focal length for the design wavelength (λ_0).

FRONT FOCAL LENGTH (f$_f$)

This length is the distance from the front focal point (F) to the primary vertex (A$_1$).

BACK FOCAL LENGTH (f$_b$)

This length is the distance from the secondary vertex (A$_2$) to the rear focal point (F″).

EDGE-TO-FOCUS DISTANCES (A AND B)

A is the distance from the front focal point to the front edge of the lens. B is the distance from the rear edge of the lens to the rear focal point. Both distances are presumed always to be positive.

REAL IMAGE

A real image is one in which the light rays actually converge; if a screen were placed at the point of focus, an image would be formed on it.

VIRTUAL IMAGE

A virtual image does not represent an actual convergence of light rays. A virtual image can be viewed only by looking back through the optical system, such as in the case of a magnifying glass.

F-NUMBER (F/#)

The f-number (also known as the focal ratio, relative aperture, or speed) of a lens system is defined to be the effective focal length divided by system clear aperture. Ray f-number is the conjugate distance for that ray divided by the height at which it intercepts the principal surface.

$$f/\# \ = \ \frac{f}{\phi}.$$

NUMERICAL APERTURE (NA)

The numerical aperture of a lens system is defined to be the sine of the angle, θ_1, that the marginal ray (the ray that exits the lens system at its outer edge) makes with the optical axis multiplied by the index of refraction (n) of the medium. The numerical aperture can be defined for any ray as the sine of the angle made by that ray with the optical axis multiplied by the index of refraction:

$$NA = n \sin \theta. \tag{1.31}$$

MAGNIFICATION POWER

Often, positive lenses intended for use as simple magnifiers are rated with a single magnification, such as 4× . To create a virtual image for viewing with the human eye, in principle, any positive lens can be used at an infinite number of possible magnifications. However, there is usually a narrow range of magnifications that will be comfortable for the viewer. Typically, when the viewer adjusts the object distance so that the image appears to be essentially at

infinity (which is a comfortable viewing distance for most individuals), magnification is given by the relationship

$$\text{magnification} = \frac{250 \text{ mm}}{f} \quad \text{(f in mm)}. \tag{1.32}$$

Thus, a 25-mm focal length positive lens would be a 10× magnifier.

DIOPTERS

Diopter is a term used to define the reciprocal of the focal length, which is commonly used for ophthalmic lenses. The inverse focal length of a lens expressed in diopters is

$$\text{diopters} = \frac{1000}{f} \quad \text{(f in mm)}. \tag{1.33}$$

Thus, the smaller the focal length, the larger the power in diopters.

DEPTH OF FIELD AND DEPTH OF FOCUS

In an imaging system, depth of field refers to the distance in object space over which the system delivers an acceptably sharp image. The criteria for what is acceptably sharp is arbitrarily chosen by the user; depth of field increases with increasing f-number.

For an imaging system, depth of focus is the range in image space over which the system delivers an acceptably sharp image. In other words, this is the amount that the image surface (such as a screen or piece of photographic film) could be moved while maintaining acceptable focus. Again, criteria for acceptability are defined arbitrarily.

In nonimaging applications, such as laser focusing, depth of focus refers to the range in image space over which the focused spot diameter remains below an arbitrary limit.

APPLICATION NOTE

Technical Reference

For further reading about the definitions and formulas presented here, refer to the following publications:

Rudolph Kingslake, *Lens Design Fundamentals* (Academic Press)

Rudolph Kingslake, *Optical System Design* (Academic Press)

Warren Smith, *Modern Optical Engineering* (McGraw Hill).

If you need help with the use of definitions and formulas presented in this catalog, our applications engineers will be pleased to assist you.

Paraxial Lens Formulas

PARAXIAL FORMULAS FOR LENSES IN AIR

The following formulas are based on the behavior of paraxial rays, which are always very close and nearly parallel to the optical axis. In this region, lens surfaces are always very nearly normal to the optical axis, and hence all angles of incidence and refraction are small. As a result, the sines of the angles of incidence and refraction are small (as used in Snell's law) and can be approximated by the angles themselves (measured in radians).

The paraxial formulas do not include effects of spherical aberration experienced by a marginal ray — a ray passing through the lens near its edge or margin. All effective focal length values (f) tabulated in this catalog are paraxial values which correspond to the paraxial formulas.

The following paraxial formulas are valid for both thick and thin lenses unless otherwise noted. The refractive index of the lens glass, n, is the ratio of the speed of light in vacuum to the speed of light in the lens glass. All other variables are defined in figure 1.33.

Focal Length

$$\frac{1}{f} = (n-1)\left(\frac{1}{r_1} - \frac{1}{r_2}\right) + \frac{(n-1)^2}{n}\frac{t_c}{r_1 r_2} \qquad (1.34)$$

where n is the refractive index, t_c is the center thickness, and the sign convention previously given for the radii r_1 and r_2 applies. For thin lenses, $t_c \cong 0$, and for plano lenses either r_1 or r_2 is infinite. In either case the second term of the above equation vanishes, and we are left with the familiar Lens Maker's formula

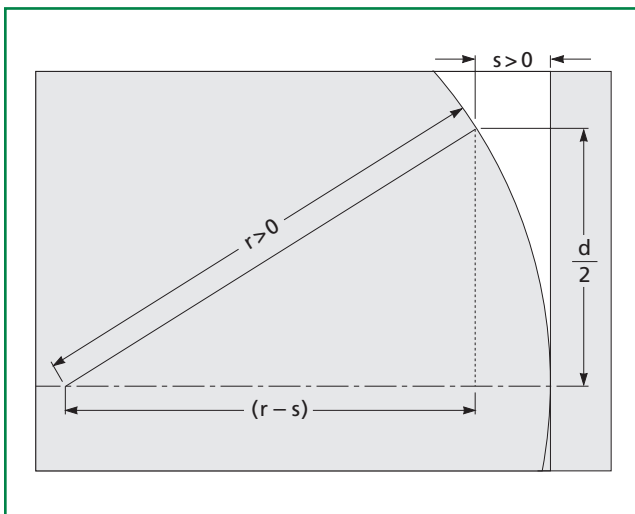$$\frac{1}{f} = (n-1)\left(\frac{1}{r_1} - \frac{1}{r_2}\right). \qquad (1.35)$$

Figure 1.34 **Surface sagitta and radius of curvature**

Surface Sagitta and Radius of Curvature
(refer to figure 1.34)

$$r^2 = (r-s)^2 + \left(\frac{d}{2}\right)^2 \qquad (1.36)$$

$$s = r - \sqrt{r^2 - \left(\frac{d}{2}\right)^2} > 0 \qquad (1.37)$$

$$r = \frac{s}{2} + \frac{d^2}{8s}. \qquad (1.38)$$

An often useful approximation is to neglect s/2.

Symmetric Lens Radii ($r_2 = -r_1$)

With center thickness constrained,

$$r_1 = (n-1)\left[f \pm \sqrt{f^2 - \left(\frac{ft_c}{n}\right)}\right] \qquad (1.39)$$

$$= (n-1)\ f\left[1 + \sqrt{1 - \left(\frac{t_c}{nf}\right)}\right]$$

where, in the first form, the + sign is chosen for the square root if f is positive, but the − sign must be used if f is negative. In the second form, the + sign must be used regardless of the sign of f. With edge thickness constrained, the equation for r_1 becomes transcendental:

$$\frac{1}{f} = \frac{2(n-1)}{r_1} - \frac{(n-1)^2}{nr_1^2}\left[t_c + 2r_1\left\{1 - \cos\left(\arcsin\frac{\phi}{2r_1}\right)\right\}\right] \qquad (1.40)$$

where ϕ is the lens diameter. This equation can be solved by numerical methods.

Plano Lens Radius

Since r_2 is infinite,

$$r_1 = (n-1)\ f. \qquad (1.41)$$

Principal-Point Locations (signed distances from vertices)

$$A_2 H'' = \frac{-r_2 t_c}{n(r_2 - r_1) + t_c(n-1)} \qquad (1.42)$$

$$A_1 H = \frac{-r_1 t_c}{n(r_2 - r_1) + t_c(n-1)} \qquad (1.43)$$

where the above sign convention applies.

For symmetric lenses ($r_2 = -r_1$),

$$A_1H = -A_2H''$$
$$= \frac{r_1 t_c}{2nr_1 - t_c(n-1)}. \qquad (1.44)$$

If either r_1 or r_2 is infinite, l'Hôpital's rule from calculus must be used. Thus, referring to page 1.27, for plano-convex lenses in the correct orientation,

$$A_1H = 0$$

and

$$A_2H'' = -\frac{t_c}{n}. \qquad (1.45)$$

For flat plates, by letting $r_1 \rightarrow \infty$ in a symmetric lens, we obtain $A_1H = A_2H'' = t_c/2n$. These results are useful in connection with the following paraxial lens combination formulas.

Hiatus or Interstitium (principal-point separation)

$$HH'' = t_c \left\{ 1 - \frac{f}{n}\left[\frac{1}{f} - \frac{(n-1)^2}{n}\frac{t_c}{r_1 r_2} \right] \right\} \qquad (1.46)$$

which, in the thin-lens approximation (exact for plano lenses), becomes

$$HH'' = t_c \left(1 - \frac{1}{n} \right). \qquad (1.47)$$

Solid Angle

The solid angle subtended by a lens, for an observer situated at an on-axis image point, is

$$\Omega = 2\pi (1 - \cos\theta)$$
$$= 4\pi \sin^2\left(\frac{\theta}{2}\right) \qquad (1.48)$$

where this result is in steradians, and where

$$\theta = \arctan\left(\frac{\phi}{2s''} \right)$$

is the apparent angular radius of the lens clear aperture. For an observer at an on-axis object point, use s instead of s''. To convert from steradians to the more intuitive *sphere* units, simply divide Ω by 4π. If the Abbé sine condition is known to apply, θ may be calculated using the arc sine function instead of the arc tangent.

Back Focal Length

$$f_b = f'' + A_2H''$$
$$= f'' - \frac{r_2 t_c}{n(r_2 - r_1) + t_c(n-1)} \qquad (1.49)$$

where the sign convention presented above applies to A_2H'' and to the radii. If r_2 is infinite, l'Hôpital's rule from calculus must be used, whereby

$$f_b = f'' - \frac{t_c}{n}. \qquad (1.50)$$

Front Focal Length

$$f_f = f - A_1H \qquad (1.51)$$
$$= f + \frac{r_1 t_c}{n(r_2 - r_1) + t_c(n-1)}$$

where the sign convention presented above applies to A_1H and to the radii. If r_1 is infinite, l'Hôpital's rule from calculus must be used, whereby

$$f_f = f - \frac{t_c}{n}. \qquad (1.52)$$

Edge-to-Focus Distances

For positive lenses,

$$A = f_f + s_1 \qquad (1.53)$$

and

$$B = f_b + s_2 \qquad (1.54)$$

where s_1 and s_2 are the sagittas of the first and second surfaces. Bevel is neglected.

Magnification or Conjugate Ratio

$$m = \frac{s''}{s}$$
$$= \frac{f}{s - f}$$
$$= \frac{s'' - f}{f}. \qquad (1.55)$$

PARAXIAL FORMULAS FOR LENSES IN ARBITRARY MEDIA

These formulas allow for the possibility of distinct and completely arbitrary refractive indices for the object space medium (refractive index n'), lens (refractive index n''), and image space medium (refractive index n). In this situation, the effective focal length assumes two distinct values, namely f in object space and f'' in image space. It is also necessary to distinguish the principal points from the nodal points. The lens serves both as a lens and as a window separating the object space and image space media.

The situation of a lens immersed in a homogenous fluid (figure 1.35) is included as a special case ($n = n''$). This case is of considerable practical importance. The two values f and f'' are again equal, so that the lens-combination formulas are applicable to systems immersed in a common fluid. The general case (two different fluids) is more difficult, and it must be approached by ray tracing on a surface-by-surface basis.

LENS CONSTANT (k)

This number appears frequently in the following formulas. It is an explicit function of the complete lens prescription (both radii, t_c and n') and both media indices (n and n''). This dependence is implicit anywhere that k appears.

$$k = \frac{n'-n}{r_1} + \frac{n''-n'}{r_2} - \frac{t_c(n'-n)(n''-n')}{n'r_1r_2}. \tag{1.56}$$

Effective Focal Lengths

$$f = \frac{n}{k} \qquad f'' = \frac{n''}{k}. \tag{1.57}$$

Lens Formula (Gaussian form)

$$\frac{n}{s} + \frac{n''}{s''} = k. \tag{1.58}$$

Lens Formula (Newtonian form)

$$xx'' = ff'' = \frac{nn''}{k^2} \tag{1.59}$$

where $x = s - f$ and $x'' = s'' - f''$.

Principal-Point Locations

$$A_1H = \frac{nt_c}{k}\left(\frac{n''-n'}{n'r_2}\right) \tag{1.60}$$

$$A_2H'' = \frac{-n''t_c}{k}\left(\frac{n'-n}{n'r_1}\right). \tag{1.61}$$

Object-to-First-Principal-Point Distance

$$s = \frac{ns''}{ks'' - n''}. \tag{1.62}$$

Second Principal-Point-to-Image Distance

$$s'' = \frac{n''s}{ks - n}. \tag{1.63}$$

Magnification

$$m = \frac{ns''}{n''s}. \tag{1.64}$$

Lens Maker's Formula

$$\frac{n}{f} = \frac{n''}{f''} = k. \tag{1.65}$$

Nodal-Point Locations

$$A_1N = A_1H + HN \tag{1.66}$$
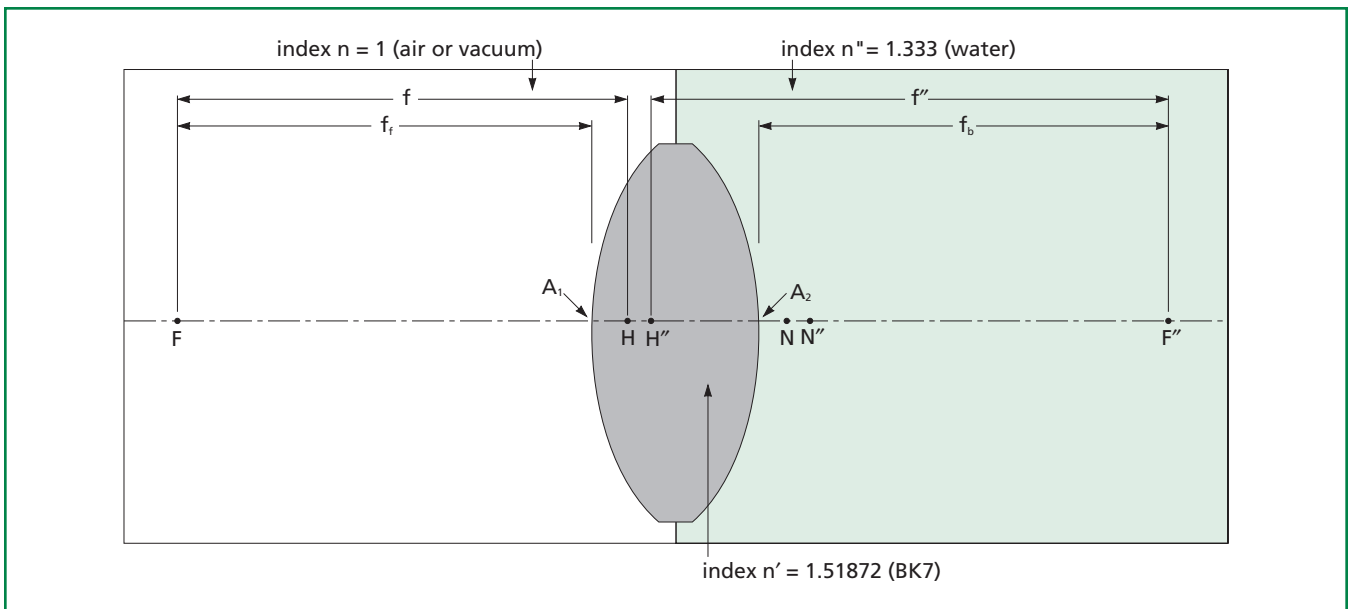
$$A_2N'' = A_2H'' + H''N''. \tag{1.67}$$

Figure 1.35 **Symmetric lens with disparate object and image space indices**

Separation of Nodal Point
from Corresponding Principal Point

$HN = H''N'' = (n'' - n)/k$, positive for N to right of H and N'' to right of H''.

Back Focal Length

$$f_b = f'' + A_2 H''. \qquad \text{(see eq. 1.49)}$$

Front Focal Length

$$f_f = f - A_1 H. \qquad \text{(see eq. 1.51)}$$

Focal Ratios

The focal ratios are f/ϕ and f''/ϕ, where ϕ is the diameter of the clear aperture of the lens.

Numerical Apertures

$$n \sin \theta$$

$$\text{where } \theta = \arcsin\left(\frac{\phi}{2s}\right)$$

and

$$n'' \sin \theta''$$

$$\text{where } \theta'' = \arcsin\left(\frac{\phi}{2s''}\right).$$

Solid Angles (in steradians)

$$\Omega = 2\pi (1 - \cos \theta) \qquad \text{(see eq.1.48)}$$

$$= 4\pi \sin^2\left(\frac{\theta}{2}\right)$$

$$\text{where } \theta = \arctan\left(\frac{\phi}{2s}\right)$$

$$\Omega'' = 2\pi (1 - \cos \theta'')$$

$$= 4\pi \sin^2\left(\frac{\theta}{2}\right) \qquad (1.68)$$

$$\text{where } \theta'' = \arctan\left(\frac{\phi}{2s''}\right).$$

To convert from steradians to spheres, simply divide by 4π.

APPLICATION NOTE

For Quick Approximations

Much time and effort can be saved by ignoring the differences among f, f_b, and f_f in these formulas (assume $f = f_b = f_f$) by thinking of s as the lens-to-object distance, by thinking of s'' as the lens-to-image distance, and by thinking of the sum of conjugate distances s + s'' as being the object-to-image distance. This is known as the thin-lens approximation.

APPLICATION NOTE

Physical Significance of the Nodal Points

A ray directed at the primary nodal point N of a lens appears to emerge from the secondary nodal point N'' without change of direction. Conversely, a ray directed at N'' appears to emerge from N without change of direction. At the infinite conjugate ratio, if a lens is rotated about a rotational axis orthogonal to the optical axis at the secondary nodal point (i.e., if N'' is the center of rotation), the image remains stationary during the rotation. This fact is the basis for the nodal slide method for measuring nodal-point location. The nodal points coincide with their corresponding principal points when the image space and object space refractive indices are equal (n = n''). This makes the nodal slide method the most precise method of principal-point location.

Principal-Point Locations

Figure 1.36 indicates approximately where the principal points fall in relation to the lens surfaces for various standard lens shapes. The exact positions depend on the index of refraction of the lens material, and on the lens radii, and can be found by formula. In extreme meniscus lens shapes (short radii or steep curves), it is possible that both principal points will fall outside the lens boundaries. For symmetric lenses, the principal points divide that part of the optical axis between the vertices into three approximately equal segments. For plano lenses, one principal point is at the curved vertex, and the other is approximately one-third of the way to the plane vertex.
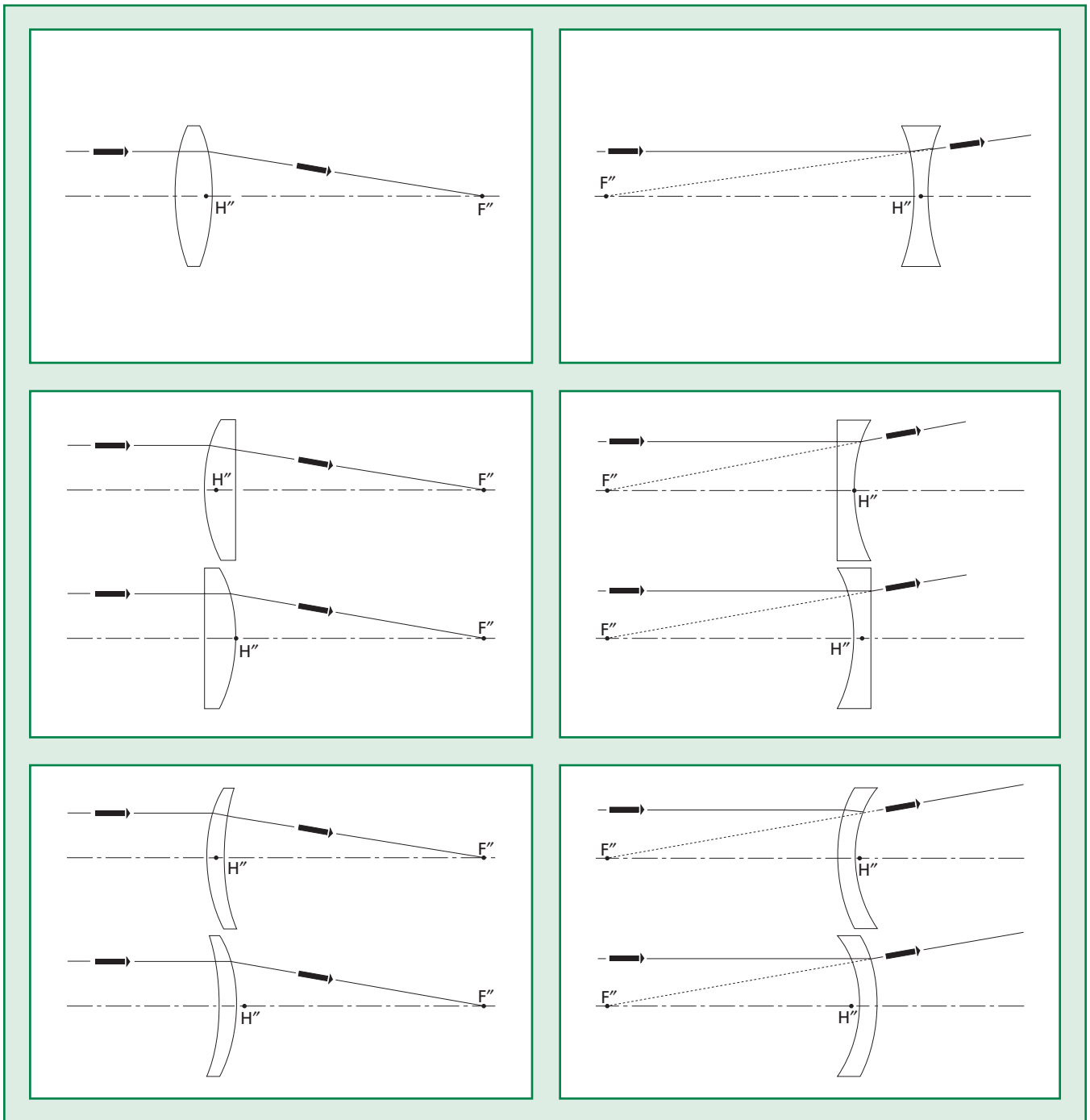

Figure 1.36   **Principal points of common lenses**