Part I: Thin Converging Lens

This experiment is a classic exercise in geometric optics. The goal is to measure the radius of curvature and focal length of a single *converging* lens from which you can calculate the index of refraction *n*. We shall explicitly consider the errors that accompany any measurement and how errors are analyzed to yield a quantitative estimate of uncertainty. This includes quantities derived from measurements, in this case the index of refraction.

In the procedures for this lab, you are asked to estimate the uncertainties. Please see Appendix I and II for reference material and some relevant equations. The questions, labeled **Q1**, **Q2**, … should be explicitly addressed in your write-up in the Analysis & Discussion section.

Procedures

A. By definition, the focal length *f* of a lens is the image distance from the lens center for an infinitely distance object. To obtain a rough estimate of *f*, project an image of the trees outside the lab onto the white paper near the door. Use a 3" diameter lens labeled A,B,C, or D.
   Q1. Why do the trees appear upside down?
   Describe a method to measure the height of a tree based on this imaging method.
   Apply this method to "estimate" the height of a tree around the BPS building.
   Estimate the image size of the moon and Sun using this given lens?

B. Use a spherometer to measure the radius of curvature of both surfaces of your lens. See Appendix I. You begin by finding the "zero" position, x_0, using a scratch-free spot on your bench (a good approximation to a flat surface). Then perform the measurement with your lens in place, x_1; the distance *h* is then $|x_0 - x_1|$. Repeat the measurements a few times to obtain an estimate of the spherometer's precision. Rulers, calipers, graph paper, and markers are available.
   Q2. Having estimated measurement uncertainties δ_{x0} and δ_{x1}, write an expression for σ_h and evaluate it using your data.

C. Arrange an object (the T on the lamp window) and screen with a separation greater than $4f$ on the optical rail. Locate the lens position which gives a sharp image on the screen. Record the object and image distances measured from the center of the lens including uncertainties. Use the thin lens equation to calculate *f* and σ_f. Repeat this for 4 positions of the screen increasing the object-screen separation in units of about 2 cm. Find the best value for the focal length.
   Q3. How does the best value for *f* compare to your original rough estimate?

D. Insert a variable iris before/after the lens. Observe the image as the aperture size is changed. Specifically note whether it affects your ability to focus the image.
   Q4. What is the meaning of the term "depth of field" (DOF)?
   Estimate the depth of field for your setup or a camera.
   Refs: Depth of Field Calculator - http://www.dofmaster.com/dofjs.html
   Depth of field - Wikipedia - http://en.wikipedia.org/wiki/Depth_of_field
   The f-number (f-stop, f/#) is the focal length divided by the "effective" aperture diameter.

E. Place the light source a distance less than f from the lens. Try to position the screen to bring the object into focus.

Q5. How do you explain your observations?

Calculate the index of refraction (including uncertainty) for the glass of your lens using the lensmaker's Equation. Compare with known values for n.

Part II: Thin Divergent Lens

For a divergent lens, all principles and conventions used for a convergent lens will apply. The key difference is that a divergent lens cannot by itself form a real image of a real object. Hence, in this experiment we will measure *f* using a virtual object. The virtual object and real image are on the *same* side of the lens. You will measure the radius of curvature and focal length, then calculate the index of refraction of the glass. Be sure to consider error propagation in your final results.

Procedure:

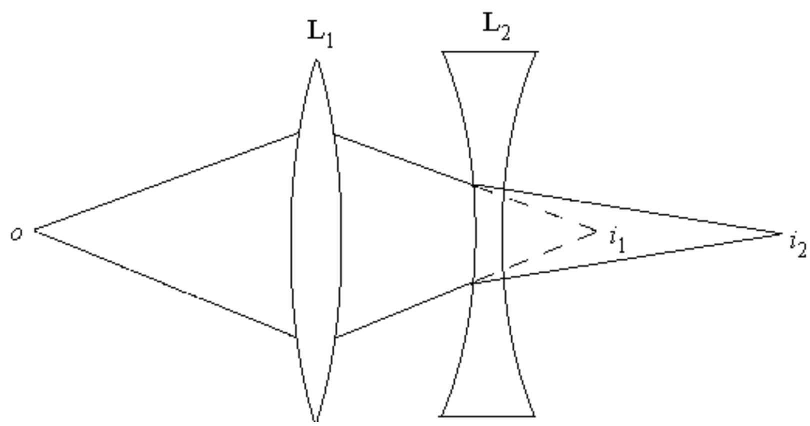
A. Use a spherometer to measure the radius of curvature of a divergent lens.

Determine the focal length using the lensmaker's equation.

B. Use a convergent lens L_1 to form a sharp image i_1 of your object on a screen using the lamp as a source. Next, place a divergent lens L_2 between L_1 and i_1 as shown below. Measure the distances to i_1 (=S_{i1}) and i_2 (=S_{i2}), the distance between two lenses (d), and the object distance (S_0) to calculate *f* for the divergent lens.
[You may use the formula given in Appendix II-1.]

Repeat this for 3 positions of i_2 by changing the lens-screen separation in units of about 1 cm.
Find the best value for the focal length using the thin-lens equation.

Q6. Make a quantitative comparison of the L_2 focal length obtained by the two different methods. Do they agree within experimental error? How could you improve either measurement?



C. Calculate the index of refraction (including uncertainty) for the glass of your lens using the lensmaker's equation. Compare this with the value found in Part I.

Before leaving the laboratory, make rough estimates of all quantities that need to be calculated or included in your report.

Appendix I-1: Lens Equations

Thin Lens Equation

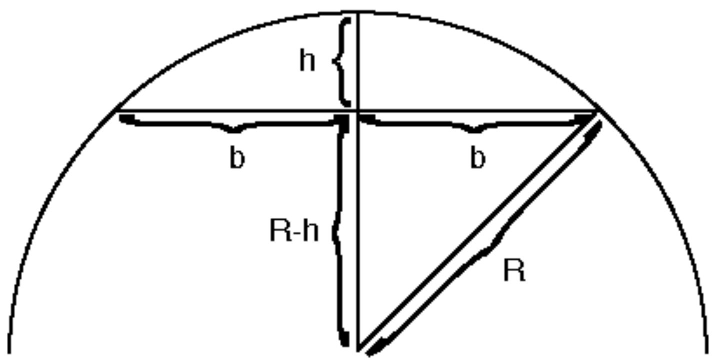
$$\frac{1}{s}+\frac{1}{s^{'}}=\frac{1}{f}$$

Lensmaker's Equation

$\frac{1}{f}=(n-1)\left(\frac{1}{R_1}-\frac{1}{R_2}\right)$, assuming that $n_{\mathrm{air}} =$ 1.000.

Spherometer Equation

$$R=\frac{b^2}{2h}+\frac{h}{2}$$



From Pythagoras:

$$R^2 = (R-h)^2 + b^2$$

$$R^2 = R^2 - 2Rh + h^2 + b^2$$

$$R=\frac{b^2}{2h}+\frac{h}{2}$$

Appendix I-2: Error Analysis

Random fluctuations in the measurement process lead to a Gaussian distribution about the true value. This distribution gives us a parameter, σ, called the "standard deviation". (Systematic errors lead to a non-Gaussian distribution.) Essentially, if many measurements x_i are taken, 68% of the data points lie within $\bar{x} \pm \sigma_x$, where $\bar{x}$ is the mean value of x.

Now, suppose an arbitrary function f(x,y) depends on the variables x and y, assumed to be independent of each other. How do we compute the uncertainty in f, σ_f, given σ_y and σ_x? Under the assumption that the uncertainties are small compared to the absolute value of the quantities in question

$$\sigma_f = \sqrt{\left(\frac{\partial f}{\partial x}\right)^2 \sigma_x{}^2 + \left(\frac{\partial f}{\partial y}\right)^2 \sigma_y{}^2}$$

For errors that are much smaller than the measured values, specific functions yield:

$$f = ax + by$$
$$\sigma_f = \sqrt{a^2\sigma_x^2 + b^2\sigma_y^2}$$

$$f = cxy$$
$$\frac{\sigma_f}{f} = \sqrt{\left(\frac{\sigma_x}{x}\right)^2 + \left(\frac{\sigma_y}{y}\right)^2}$$

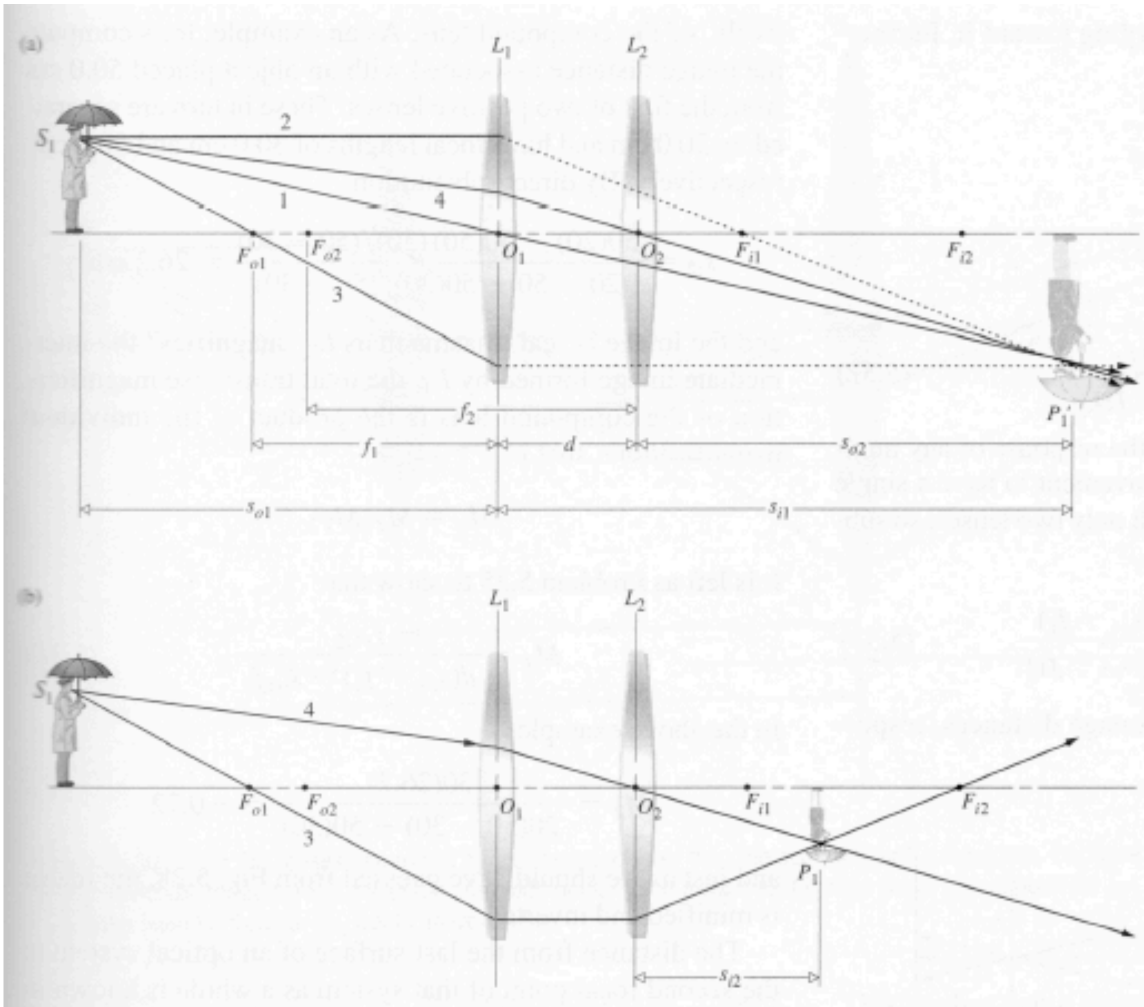
$$f = cx^a y^b$$
$$\frac{\sigma_f}{f} = \sqrt{\left(\frac{a\sigma_x}{x}\right)^2 + \left(\frac{b\sigma_y}{y}\right)^2}$$

$$f = ce^{bx}$$
$$\frac{\sigma_f}{f} = b\sigma_x$$

$$f = ca^{bx}$$
$$\frac{\sigma_f}{f} = (b \ln a)\sigma_x$$

When we make N measurements of the same quantity x, each with an uncertainty δ_x, we expect that after averaging the measurement will have uncertainty smaller than δ_x. In fact, the value of σ_x varies as $1/\sqrt{N}$ when N is large.

Appendix II-1: Thin Lens Combination



The image position for two thin lenses is

$$s_{i2} = \frac{f_2 d - \dfrac{f_2 s_{o1} f_1}{(s_{o1} - f_1)}}{d - f_2 - \dfrac{s_{o1} f_1}{(s_{o1} - f_1)}}$$

where s_o is the position of the object (before either lens), and d is the distance between the lenses. If $s_o = \infty$, and d=f_1+f_2, then $s_i = \infty$. This is a Galilean telescope.

For L_1,

$$\frac{1}{s_{i1}}=\frac{1}{f_1}-\frac{1}{s_{o1}}$$

Let $s_{o1}>f_1$ and $f_1>0$.

For L_2

$$s_{o2}=d-s_{i1}$$

$$\frac{1}{s_{i2}}=\frac{1}{f_2}-\frac{1}{s_{o2}}$$

Thus,

$$s_{i2}=\frac{f_2 d-f_2 s_{o1}f_1/(s_{o1}-f_1)}{d-f_2-s_{o1}f_1/(s_{o1}-f_1)}$$

and

$$M_T=M_{T1}M_{T2}=\frac{f_1 s_{i2}}{d(s_{o1}-f_1)-s_{o1}f_1}$$

Front focal length:

$$\left.\frac{1}{s_{o1}}\right|_{s_{i2}=\infty}=\frac{1}{f_1}-\frac{1}{d-f_2}=\frac{d-(f_1+f_2)}{f_1(d-f_2)}\ \Rightarrow\ bfl=\frac{f_1(d-f_2)}{d-(f_1+f_2)}$$

Back focal length:

$$\left.\frac{1}{s_{i2}}\right|_{s_{o1}=\infty}=\frac{1}{f_2}-\frac{1}{d-f_1}=\frac{d-(f_1+f_2)}{f_2(d-f_1)}\ \Rightarrow\ bfl=\frac{f_2(d-f_1)}{d-(f_1+f_2)}$$

If $d\to 0$, $f=bfl=ffl=\frac{f_2 f_1}{f_2+f_1}\ \Rightarrow\ \frac{1}{f}=\frac{1}{f_1}+\frac{1}{f_2}$

(Source: Ch. 5.2 "Optics", by Hecht)