Spontaneous Parametric Down Conversion: A closer look

In the previous labs you were able to, using elementary conservation laws, calculate the angle at which the idler and signal beam leave the crystal and, furthermore, observe this angle on a camera. As the crystal was rotated you also noticed that the angle of the cone was changing. It is clear then that the down converted light is dependant on the orientation of the crystal, but how? In addition to energy conservation, parametric down-conversion requires that the photon momentum $p = \hbar k$ is conserved inside the crystal. The wave number inside the crystal is $|\mathbf{k}| = nk = 2\pi n / \lambda$, where n is the index of refraction of the crystal. This momentum conservation condition can be expressed as

$$k_p = k_s + k_i \tag{1}$$

For degenerate down conversion i.e. $k_s = k_i = k_p/2$. Thus, (1) reduces to

$$n_p = n_s \cos\theta_c \tag{2}$$

where θ_c is the angle that the signal photons form with the direction of propagation of the pump beam inside the crystal.

It is not possible to satisfy Eq. (2) in an isotropic medium, because for normal dispersion the index of refraction decreases with increasing wavelength, that is, $n_p > n_s$. This problem can be solved using a birefringent crystal, which has two indices of refraction commonly known as the extra-ordinary and ordinary index of refraction of the crystal. We use nonlinear crystals cut for type-I parametric down-conversion to produce a pair of down-conversion photons with linear polarizations parallel to each other but orthogonal to the polarization of the pump beam. The directions taken by the down-conversion photons of specific but complementary wavelengths are determined by the angle formed by the optic axis of the crystal (OA) and the propagation direction of the pump beam, the phase-matching angle θ_m. The crystals are mounted on a rotation stage so that OA was in a horizontal plane. In this way we could easily fine tune the phase-matching angle of the crystal.
The ordinary index of refraction is used for light with the polarization perpendicular to the OA of the crystal. If the polarization is in the same plane as OA, the index of refraction, known as the extraordinary index of refraction, depends on the angle θ_m. This relation is

$$\begin{aligned} \frac{1}{\tilde{n}_e^2(\theta_m)} &= \frac{\cos^2\theta_m}{n_o^2} + \frac{\sin^2\theta_m}{n_e^2} \\ \tilde{n}_e(\theta_m) &= \left(\frac{\cos^2\theta_m}{n_o^2} + \frac{\sin^2\theta_m}{n_e^2} \right)^{-1/2} \end{aligned} \tag{3}$$

By selecting the correct phase-matching angle between the optic axis and the propagation vector we can tune the index of refraction between n_o and n_e to satisfy Eq. 2.

The indices of refraction correspond to those of the negative uniaxial beta-barium-borate crystal, with the index of refraction given by

$$n_{e,o} = \left[A_{e,o} + \frac{B_{e,o}}{\lambda^2 + C_{e,o}} + D_{e,o}\lambda^2 \right]^{1/2}$$

where the constants for n_o and n_e are A_o=2.7359, B_o=0.01878 μm^2, C_o= -0.01822 μm^2, D_o= -0.01354 μm^{-2} and A_e=2.3753, B_e=0.01224 μm^2, C_e= -0.01667 μm^2, and D_e= -0.01516 μm^{-2}.

Under the type-I phase matching, the pump photons are subject to the extraordinary index of refraction $\tilde{n}_e(\theta_m)$ and the down-conversion photons are subject to the ordinary index of refraction.

Q1. Determine n_o and n_e at 405 nm and 810 nm.
Q2. Determine θ_L (angle in the lab frame) and θ_c (angle in the crystal) based on the CCD images of the down-conversion photon cones. (Use Snell's law $\sin\theta_L = n_s \sin\theta_c$).
Q3. Determine θ_m for the corresponding θ_c in Q2 by equating $n_p = \tilde{n}_e \; at \; \lambda = 405\, nm$. Explain the underlying mechanism allowing for tuning of the angle formed by the signal and idler beams.
Q4. If we want the signal and idler beams to from a laboratory angle of $\theta_L = 2.5^\circ$ with the pump beam outside the crystal, what is the phase matching angle θ_m? (Use Snell's law $\sin\theta_L = n_s \sin\theta_c$).

LAB: Monitor the parametric down-conversion cone imaged by the CCD, tune the crystal such that the above condition $\theta_L = 2.5^\circ$ is met.

Reference:

E. J. Galvez, C. H. Holbrow, M. J. Pysher, J. W. Martin, N. Courtemanche, L. Heilig, and J. Spencer, "Interference with correlated photons: Five quantum mechanics experiments for undergraduates," Am. J. Phys. 73, 127 (2005)

In[1]:= **Phase matching conditions for BBO crystal**

In[28]:= $\lambda_p = 0.405;\ \lambda_s = 0.810;$

Wavelength of pump and signal/idler in microometers

Indices of refraction of the BBO crystal calculated using the Sellmeier equations

In[29]:= $n_o[\lambda\_] := \sqrt{2.7359 + \frac{0.01878}{\lambda^2 - 0.01822} - 0.01354 * (\lambda^2)};$

In[30]:= $n_e[\lambda\_] := \sqrt{2.3753 + \frac{0.01224}{\lambda^2 - 0.01667} - 0.01516 * (\lambda^2)};$

In[31]:=
$\{n_o[\lambda_s],\ n_e[\lambda_p],\ n_o[2 * \lambda_p]\}$

Out[31]= {1.66026, 1.56712, 1.66026}

Extraordinary index of refraction as a function of the phase matching angle between the propagation direction and the optic axis of the crystal

In[32]:= $n_{pe}[\theta\_,\ \lambda\_] := \left(\frac{\text{Cos}[\theta]^2}{n_o[\lambda]^2} + \frac{\text{Sin}[\theta]^2}{n_e[\lambda]^2}\right)^{-1/2};$

Now we wanted a lab angle of $\theta_L = 3^o$ so by Snell' s Law we found the angle within the crystal by

In[33]:= $\theta_c = \text{ArcSin}\left[\frac{\text{Sin}[\pi / 60]}{n_e[\lambda_s]}\right]$

Out[33]= 0.0338989

By energy conservation we have $n_p = n_s \cos\theta_c$ so using our value for θ_c we found that n_p must be

In[34]:= $n_p = n_o[\lambda_s] * \text{Cos}[\theta_c]$

Out[34]= 1.6593

Thus we wanted to solve for θ by setting $n_{pe}[\theta,\ \lambda_p]$ = 1.65658

In[36]:= $\theta_p = \text{Solve}[n_{pe}[\theta, \lambda_p] == n_p,\ \theta]$

Out[36]= $\{\{\theta \to -2.63017\},\ \{\theta \to -0.511426\},\ \{\theta \to 0.511426\},\ \{\theta \to 2.63017\}\}$

We ignored the other values and converted to radians to find

In[46]:= $\theta_{pm} = N[0.5114264328849383 * 180 / \pi]$

Out[46]= 29.3026