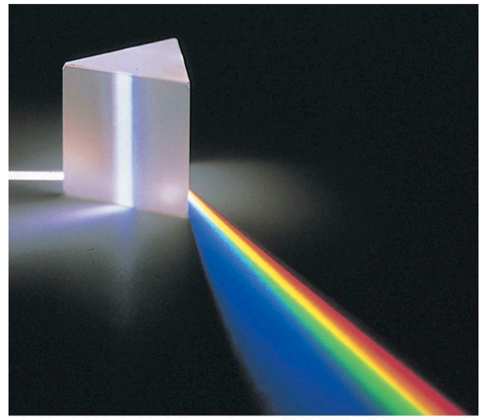
Prisms [Reading Pedrotti^3 Ch. 3-3]

Dispersion

This variation of the *refractive index* with the wavelength or frequency of the light is called **dispersion**. Dispersion is a property of *all* transparent materials.

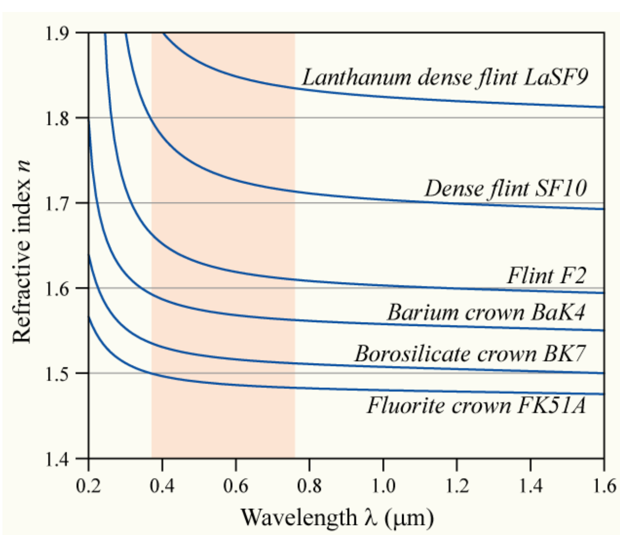
White light passing through a prism is broken down into its constituent colors.

Color	Frequency	Wavelength
violet	668–789 THz	380–450 nm
blue	631–668 THz	450–475 nm
cyan	606–630 THz	476–495 nm
green	526–606 THz	495–570 nm
yellow	508–526 THz	570–590 nm
orange	484–508 THz	590–620 nm
red	400–484 THz	620–750 nm



The above visible spectrum does not show all the colors seen in nature. Many of the colors we see are a mixture of wavelengths. For practical purposes, most natural colors can be reproduced using three primary colors. They are red, green, and blue (R-G-B) for direct source viewing such as TV and computer monitors. For inks used in printing, the primary colors are cyan, yellow, and magenta (C-Y-M) (& black CYMK).

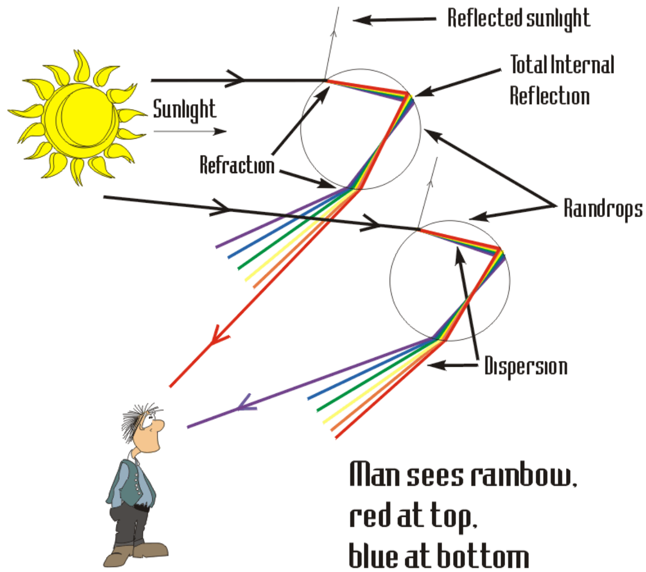
Index of refraction as a function of wavelength for various transparent solids.

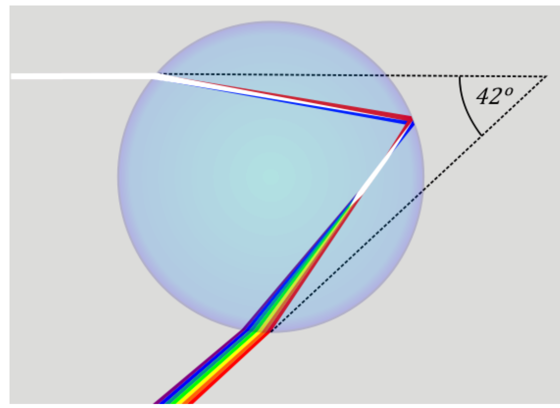


(image source: Wikipedia.org)

Rainbows

Rainbows are a spectacular example of dispersion – by drops of water. You can see rainbows when you look at falling water droplets with the Sun behind you. Red and violet rays are bent by spherical water droplets and are reflected off the back surfaces (total internal reflection). Red is bent the least and so reaches the observer's eyes from droplets higher in the sky, as shown in the diagram. Thus the top of the (primary) rainbow is red.





(image source: http://rebeccapaton.net/rainbows/formatn.htm & wikipedia.org)

Diamond

Diamonds achieve their brilliance from a combination of dispersion and total internal reflection. Because diamonds have a very high index of refraction of about 2.4, the critical angle for total internal reflection is only 25^o. The light dispersed into a spectrum inside the diamond therefore strikes many of the internal surfaces before it strikes one at less than 25^o and emerges. After many such reflections, the light has traveled far enough that the colors have become sufficiently separated to be seen individually and brilliantly by the eye after leaving the crystal.

Angular deviation of a prism [Pedrotti^3 Ch. 3-3]

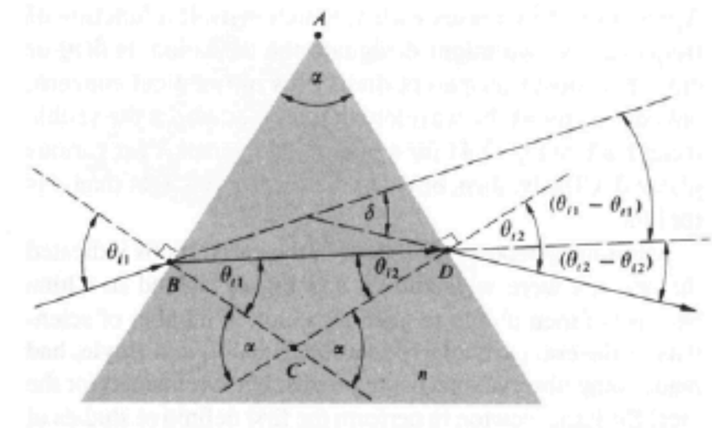


Figure 5.56 Geometry of a dispersing prism.

The total deviation:

$$\delta=(\theta_{i1}-\theta_{t1})+(\theta_{t2}-\theta_{i2})$$

Since $\alpha=\theta_{t1}+\theta_{i2}$,

$$\delta=\theta_{i1}+\theta_{t2}-\alpha$$

From Snell's law

$$\theta_{t2}=\sin^{-1}(n\sin\theta_{i2})=\sin^{-1}[(\sin\alpha)(n^2-\sin^2\theta_{i1})^{1/2}-\sin\theta_{i1}\cos\alpha]$$

The deviation is then

$$\delta=\theta_{i1}+\sin^{-1}[(\sin\alpha)(n^2-\sin^2\theta_{i1})^{1/2}-\sin\theta_{i1}\cos\alpha]-\alpha$$

δ Increases with n. For visible light, n increases as frequency increases. Therefore, blue light deviates more than red light.

At minimum deviation angle

$$\frac{d\delta}{d\theta_{i1}}=1+\frac{d\theta_{t2}}{d\theta_{i1}}=0 \rightarrow \frac{d\theta_{t2}}{d\theta_{i1}}=-1$$

Also

$$\alpha=\theta_{t1}+\theta_{i2} \rightarrow d\theta_{t1}=-d\theta_{i2}$$

Taking derivatives of Snell's law at the two interface, we get,

$$\cos\theta_{i1}d\theta_{i1}=n\cos\theta_{t1}d\theta_{t1}$$
$$\cos\theta_{t2}d\theta_{t2}=n\cos\theta_{i2}d\theta_{i2}$$

therefore

$$\frac{\cos\theta_{i1}}{\cos\theta_{t2}}=\frac{\cos\theta_{t1}}{\cos\theta_{i2}} \text{ or } \frac{1-\sin^2\theta_{i1}}{1-\sin\theta_{t2}}=\frac{n^2-\sin^2\theta_{i1}}{n^2-\sin^2\theta_{t2}}$$

Since $n\neq1$, we have $\theta_{i1}=\theta_{t2}$, and $\theta_{i2}=\theta_{t1}$

At the minimum deviation angle δ_m,

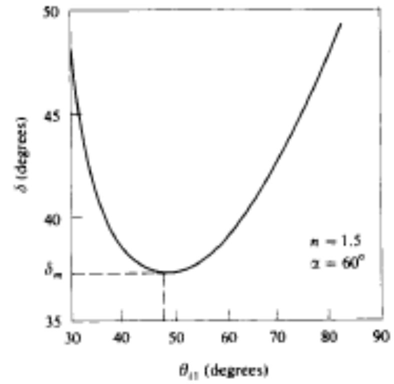
$$\theta_{i1}=(\delta_m+\alpha)/2,$$

$$\theta_{t1}=\alpha/2,$$

and

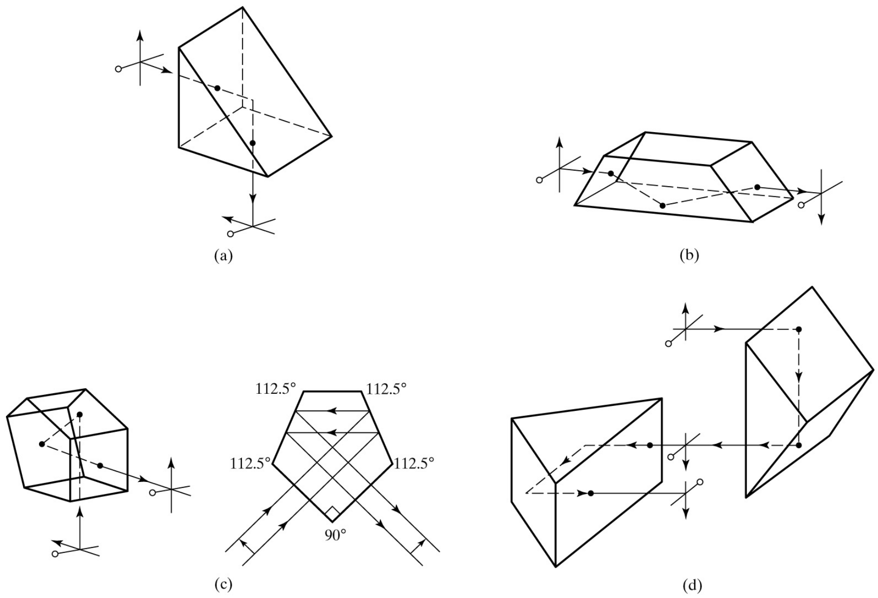
$$n=\frac{\sin[(\delta_m+\alpha)/2]}{\sin\alpha/2}$$

One of the most accurate technique for determining the refractive index.



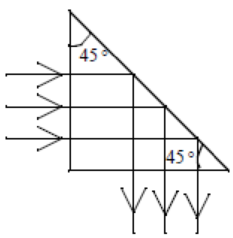
The deviation increases with decreasing index n. For most materials n increases with decreasing λ. This is the basis for the splitting of white light into colors by the prism.

Selected Applications of Prisms



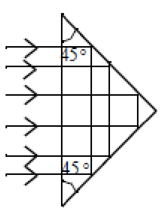
Pedrotti^3 Figure 3-18. Image manipulation by reflecting prisms. (a) Righ-angle prism. (b) Dove prism. (d) Penta prism. (d) Porro prism.

Right Angle Prism



common building block in non-dispersive prism devices

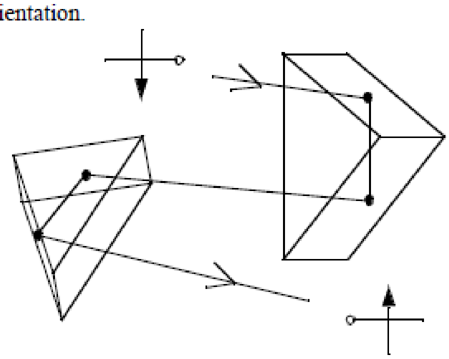
Porro prism



retro reflector (only folds back on itself in one meridian)

Erecting Prisms

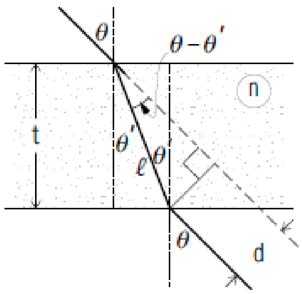
Most telescopes produce an inverted image (both U-D, L-R) to the eye. Erecting prisms re-invert the image to the proper orientation.



2 porro prisms used together.

Generally contacted

Lateral displacement through a plane parallel glass plate



$$d = \frac{t\sin\left(\theta-\theta'\right)}{\cos\left(\theta'\right)}$$

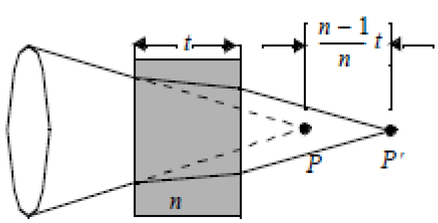
Using the trigonometric identity
$\sin(\theta-\theta') = \sin\theta\cos\theta' - \cos\theta\sin\theta'$;
we get:
$d/t = \sin\theta[1-\cos\theta/(n\cos\theta')]$
where:
$\cos\theta' = \sqrt{1-(\sin^2\theta)/n^2}$

This can be used to laterally displace an image. One application of this very simple device is in a specialized high speed camera. The film has to move so fast that it is driven continuously (rather than actually stopping briefly for each frame as in a conventional camera). A rotating plate is used to make the image track the moving film during exposure of a given frame to prevent blur.

But the plate introduces aberrations.

- Chromatic effect: longitudinal and lateral displacements depend on n which is λ dependent.
- For a plate used in convergant or divergent light, the amount of displacement is greater for larger angles which gives spherical aberration.

Plane parallel plate placed in between a lens and its focus:



A simple calculation based on the paraxial approximation shows that the focus is displaced by amount $\boxed{\frac{n-1}{n}t}$. However, at steeper incidence angles, the focal shift becomes a function of the incidence angle, which leads to spherical aberration.

Thick Lens [Optional, won't appear in exams]

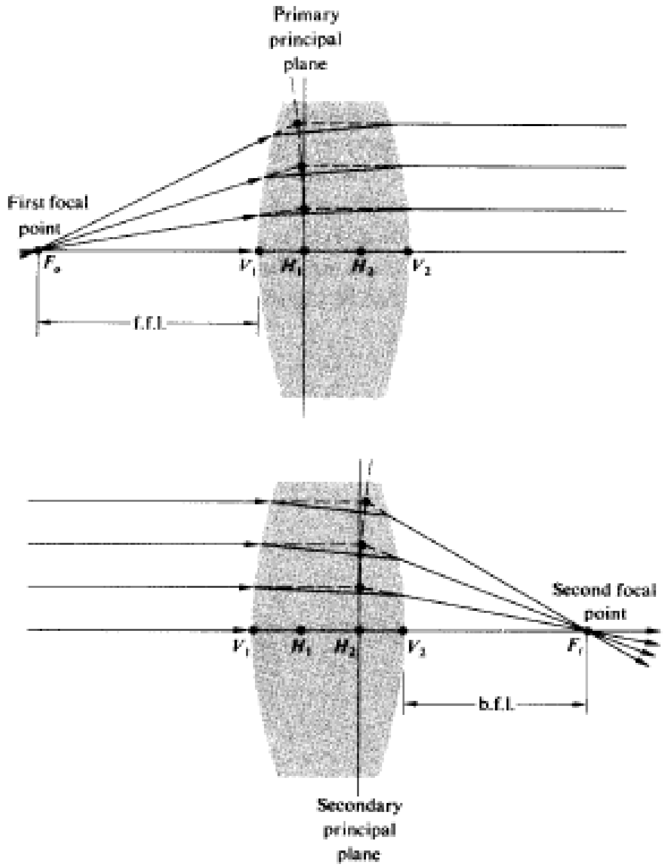


Figure 6.1 A thick lens.

Principle plane: the plane on which the extension lines of the ray incident from the first focus and the ray emerged from the lens intercept.

Secondary Plane: the same as the principle plane except that the ray is from the second focus.

First principal point H_1 : the intersection of the **Principle plane** and the optical axis.

Second principal point H_2 : the intersection of the **secondary plane** and the optical axis.

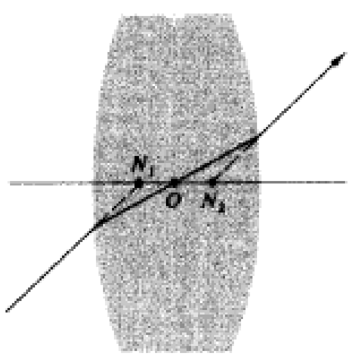


Figure 6.2 Nodal points.

Nodal points N_1 and N_2 : the interception of the incident and emerged rays which pass the optical center with optical axis.

Cardinal Points: the two focal, two principal and two nodal points.

Thick Lens Formula

Single Lens

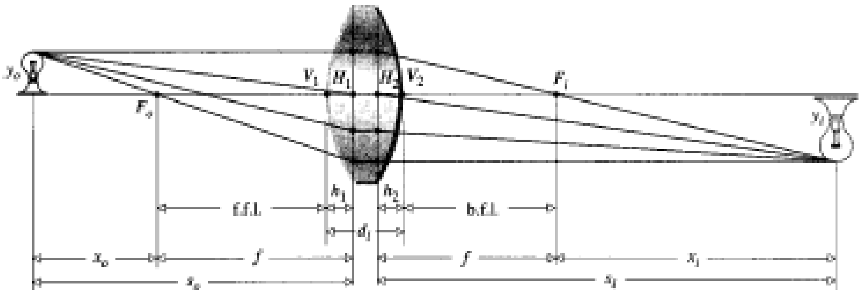


Figure 6.4 Thick-lens geometry.

If consider the thick lens as the combination of two spherical refracting surface separated by a distance d_l, the result is

$$\frac{1}{s_o}+\frac{1}{s_i}=\frac{1}{f}$$

$$\frac{1}{f}=(n_l-1)\left[\frac{1}{R_1}-\frac{1}{R_2}+\frac{(n_l-1)d_l}{n_l R_1 R_2}\right]$$

Note that s_o, s_i and f are measured from the first and second principal planes. Also the distance of the principal points and the vertices $\overline{V_1H_1}=h_1$ and $\overline{V_2H_2}=h_2$ are

$$h_1=-\frac{f(n_l-1)d_l}{R_2 n_l}$$

$$h_2=-\frac{f(n_l-1)d_l}{R_1 n_l}$$

which are positive when the principal points lie to the right of their respective vertices.

Chromatic Aberration [Reading Pedrotti^3 Ch. 3-5 & Ch. 20-7]

1. Due to the dependency of the refraction index on frequency. Lights with different wavelengths have different focuses.
2. A·CA: axial chromatic aberration.
3. L·CA: lateral chromatic aberration.

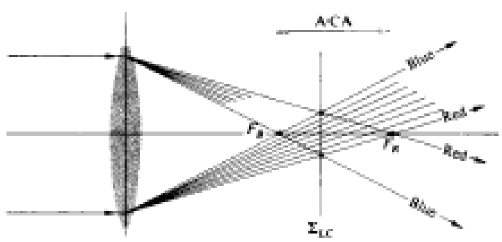


Figure 6.36 Axial chromatic aberration.

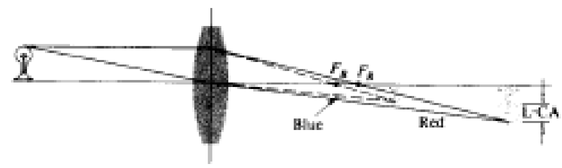


Figure 6.37 Lateral chromatic aberration.

Thin Achromatic Doublets

Purpose: to bring the focus of the red and blue lights together by a combination of two thin lens separated by a distance $\boldsymbol{d}$.

$$\frac{1}{f}=\frac{1}{f_1}+\frac{1}{f_2}-\frac{d}{f_1 f_2}$$

Let

$$\frac{1}{f_1}=(n_1-1)\left(\frac{1}{R_{11}}-\frac{1}{R_{12}}\right)=(n_1-1)\rho_1,\quad \frac{1}{f_2}=(n_2-1)\left(\frac{1}{R_{21}}-\frac{1}{R_{22}}\right)=(n_2-1)\rho_2.$$

then

$$\frac{1}{f}=\frac{1}{f_1}+\frac{1}{f_2}-\frac{d}{f_1 f_2}=(n_1-1)\rho_1+(n_2-1)\rho_2-d(n_1-1)\rho_1(n_2-1)\rho_2.$$

Let the focus of red light be f_R and blue light f_B. What we want is $\frac{1}{f_R}=\frac{1}{f_B}$.

This leads to

$$(n_{1R}-1)\rho_1+(n_{2R}-1)\rho_2-d(n_{1R}-1)\rho_1(n_{2R}-1)\rho_2=(n_{1B}-1)\rho_1+(n_{2B}-1)\rho_2-d(n_{1B}-1)\rho_1(n_{2B}-1)\rho_2$$

Case 1: Select $d=0$, we have

$$\frac{\rho_1}{\rho_2}=-\frac{n_{2B}-n_{2R}}{n_{1B}-n_{1R}}.$$

Let the focus of yellow light be f_Y, then

Consider yellow as the center of the spectrum from blue to red.

$$\frac{1}{f_{1Y}}=(n_{1Y}-1)\rho_1,\ \frac{1}{f_{2Y}}=(n_{2Y}-1)\rho_2 \rightarrow \frac{\rho_1}{\rho_2}=\frac{(n_{2Y}-1)f_{2Y}}{(n_{1Y}-1)f_{1Y}}.$$

Therefore,

$$\frac{f_{2Y}}{f_{1Y}}=-\frac{(n_{2B}-n_{2R})/(n_{2Y}-1)}{(n_{1B}-n_{1R})/(n_{1Y}-1)}.$$

Definition:

1. Dispersive power: $\frac{n_B-n_R}{n_Y-1}$.
2. Dispersive index, or V-number, or Abbe number: $V=\frac{n_Y-1}{n_B-n_R}$.

Thus,

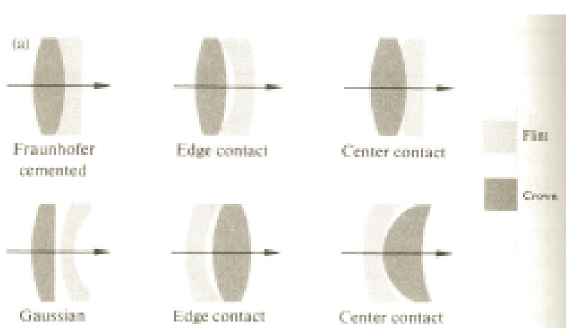
$$\frac{f_{2Y}}{f_{1Y}}=-\frac{V_1}{V_2} \rightarrow f_{1Y}V_1+f_{2Y}V_2=0$$

Case 2: Select $n_1=n_2$, then

$$d=\frac{1}{n_B+n_R-2}\left(\frac{1}{\rho_1}+\frac{1}{\rho_2}\right)=\frac{(f_{1Y}+f_{2Y})(n_Y-1)}{n_B+n_R-2}.$$

If $n_Y=\frac{n_B+n_R}{2}$, we have

$$d=\frac{f_{1Y}+f_{2Y}}{2}$$



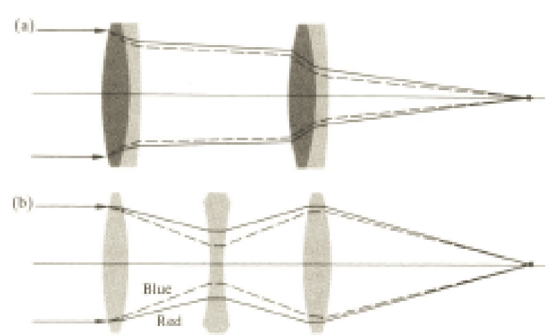


Figure 6.41 Achromatized lenses.

Aberrations (Additional online video resources)

Spherical Aberration: Youtube Clip http://www.youtube.com/watch?v=E85FZ7WLvao&NR=1

Chromatic Aberration: Youtube Clip http://www.youtube.com/watch?v=yOR4WHgRfvI&NR=1

Coma Aberration: Youtube Clip http://www.youtube.com/watch?v=EXmaY2txEBo&NR=1

MiniTutorial of Geometrical Aberrations (Seidel Aberrations):
Youtube clips 1. http://www.youtube.com/watch?v=wzEQX1tMLdY
2. http://www.youtube.com/watch?v=TWGXKj_RIs0