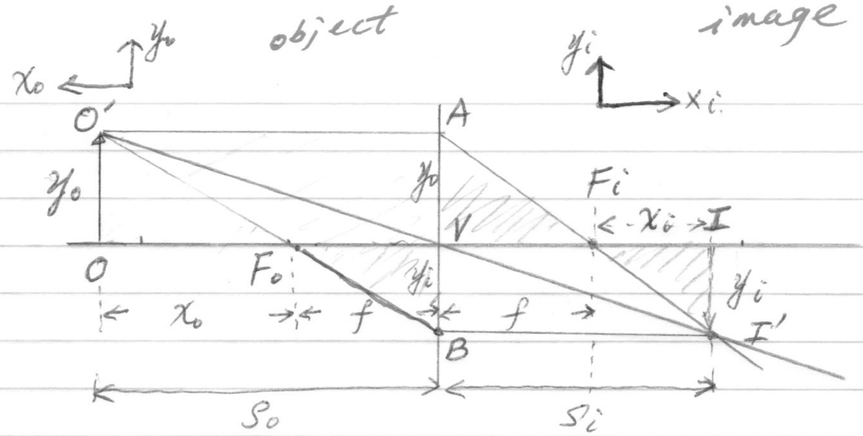
# Derivation of the lens formula




$x_o \leftrightarrow x$

$x_i \leftrightarrow x'$

$S_o \leftrightarrow s$

$S_i \leftrightarrow s'$

$$\frac{y_o - y_i}{S_o} = -\frac{y_i}{f} \quad (1) \quad \Delta BVF_o \sim \Delta BAO'$$

$$\frac{y_o - y_i}{S_i} = \frac{y_o}{f} \quad (2) \quad \Delta AVF_i \sim \Delta ABI'$$

sum of (1) & (2) $\quad \dfrac{y_o - y_i}{S_o} + \dfrac{y_o - y_i}{S_i} = \dfrac{y_o - y_i}{f}$

$$\frac{1}{S_o} + \frac{1}{S_i} = \frac{1}{f}$$

This is the lens formula in the gaussian form.

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$$\frac{y_o}{x_o} = -\frac{y_i}{f} \quad \text{--- (a)} \quad \Delta O'OF_o \sim \Delta BVF_o$$

$$-\frac{y_i}{x_i} = \frac{y_o}{f} \quad \text{--- (b)} \quad \Delta AVF_i \sim \Delta I'IF_i$$

multiplication of (a) & (b) $\quad -\dfrac{y_o y_i}{x_o x_i} = -\dfrac{y_o y_i}{f^2}$

$$x_o x_i = f^2$$

The lens formula is ~~the~~ $x_o x_i = f_o f_i$

This is the lens formula in the newtonian form. In the more general case where the medium on the two sides of the lens is different, it ~~will~~ can be shown that the front and ~~back~~ focal length are different, $f_o$ & $f_i$.

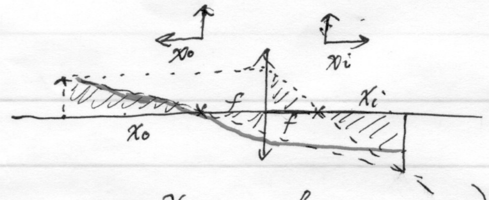
object left ← → image right

2)

| + | + |
|---|---|
| $S_o$ | $S_i$ |

$S_o$ + for object to left of the lens

$S_i$ + for image to right of the lens




$$\frac{x_o}{f} = \frac{f}{x_i}$$

use $\frac{1}{S_o} + \frac{1}{S_i} = \frac{1}{f}$

$$\frac{1}{x_o+f} + \frac{1}{x_i+f} = \frac{1}{f}$$

$$f(x_i+f+x_o+f) = x_o x_i + x_i f + x_o f + f^2$$

$$x_o x_i = f^2$$

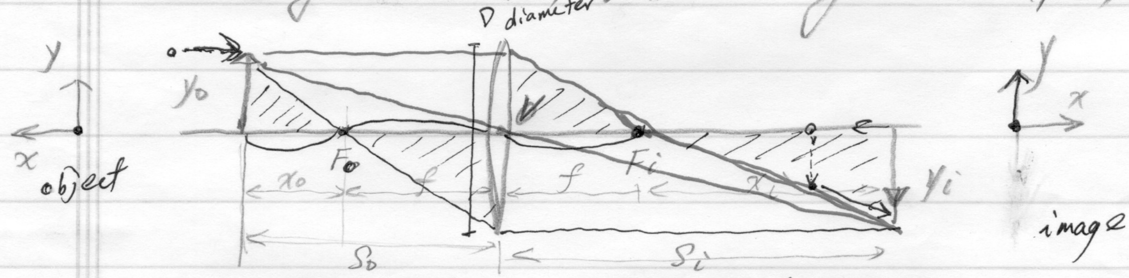
$f$-number

$f/\#$

$f/\phi$

$\phi$: clear aperture effective diameter

See Fig 2-22 (page 37) & Fig. 2-26 (p.42)




front focal point back focal point

| Quantity | Sign + | − |
|---|---|---|
| $S_o$ | real object | virtual object |
| $S_i$ | real image | virtual image |
| $f$ | Converging lens (convex) | Diverging lens (concave) |
| $y_o$ | Upright object | inverted object |
| $y_i$ | Upright image | inverted image |
| $M_T \equiv \frac{y_i}{y_o} = -\frac{S_i}{S_o}$ | Upright image | inverted image |

$$\boxed{\begin{array}{l} x \leftrightarrow x_o \\ x' \leftrightarrow x_i \end{array}}$$

$M_L \simeq -M_T^2$ longitudinal magnification

$$x_o x_i = f^2$$

$$x_i = \frac{f^2}{x_o}$$

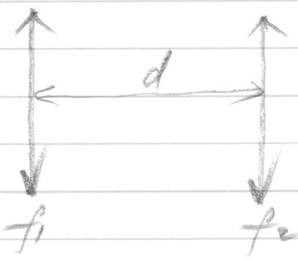
$$\frac{dx_i}{dx_o} = -\frac{f^2}{x_o^2}$$

$$\Rightarrow \quad M_L \equiv \frac{dx_i}{dx_o} = -\frac{f^2}{x_o^2} = -M_T^2$$

$M_L < 0$, which implies that a positive $dx_o$ corresponds to a negative $dx_i$ and vice versa.

~~(Fig. 2-27)~~ Fig. 2-26

lens combination formula




positive/convergent $f > 0$

negative/divergent $f < 0$

Effective focal length of two thin lens

$$f' = \frac{f_1 f_2}{f_1 + f_2 - d}$$

or $$\frac{1}{f'} = \frac{1}{f_1} + \frac{1}{f_2} - \frac{d}{f_1 f_2}$$

Note that the formula is symmetric with respect to interchange of the lenses / end-for-end rotation of the combination at constant d.

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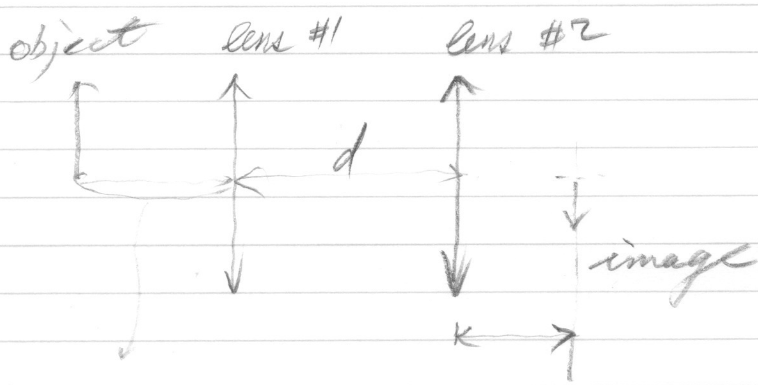


$S_{o1}$ (object to lens #1)

$S_{i2}$ (image to lens #2)