Review: Basic Concepts

Simulations

1. Radio Waves http://phet.colorado.edu/en/simulation/radio-waves
2. Propagation of EM Waves http://www.phys.hawaii.edu/~teb/java/ntnujava/emWave/emWave.html
3. 2D EM Waves http://www.falstad.com/emwave1/

Maxwell's Equations

$\oint \vec{E} \cdot d\vec{A} = \dfrac{Q_{\text{in}}}{\epsilon_0}$	Gauss's law
$\oint \vec{B} \cdot d\vec{A} = 0$	Gauss's law for magnetism
$\oint \vec{E} \cdot d\vec{s} = -\dfrac{d\Phi_{\text{m}}}{dt}$	Faraday's law
$\oint \vec{B} \cdot d\vec{s} = \mu_0 I_{\text{through}} + \epsilon_0 \mu_0 \dfrac{d\Phi_{\text{e}}}{dt}$	Ampère-Maxwell law

The Fundamental Ideas of Electromagnetism

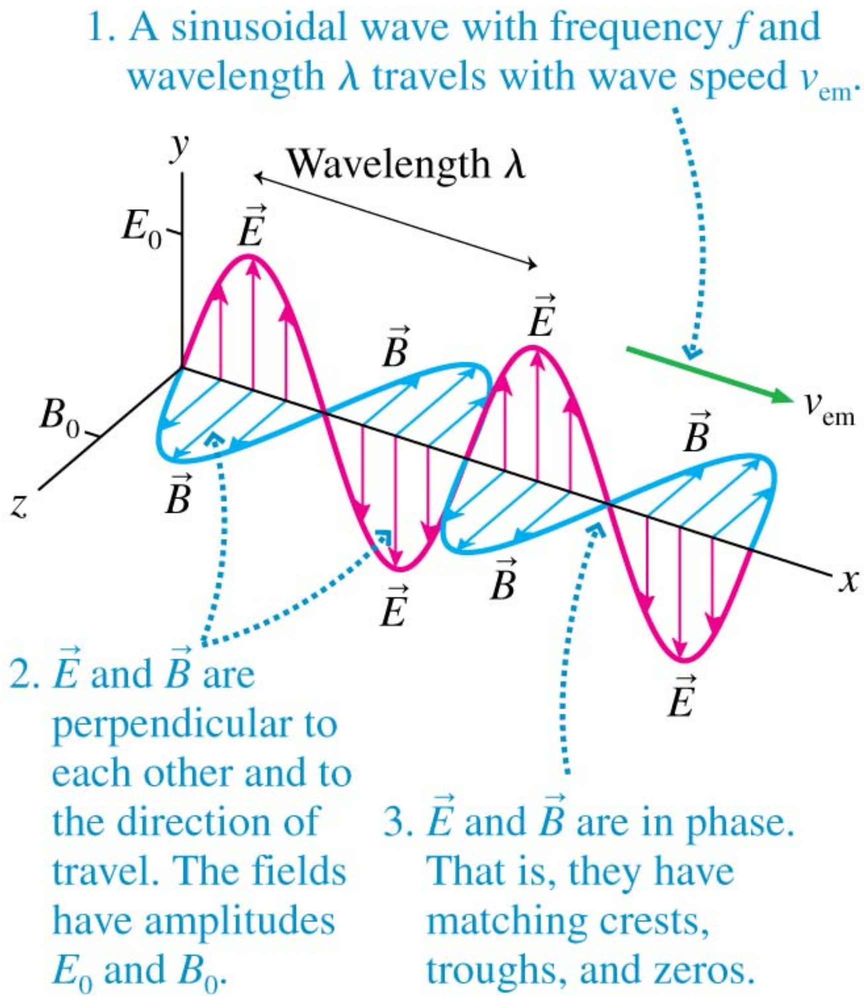
- **Gauss's law:** Charged particles create an electric field.
- **Faraday's law:** An electric field can also be created by a changing magnetic field.
- **Gauss's law for magnetism:** There are no magnetic monopoles.
- **Ampère-Maxwell law, first half:** Currents create a magnetic field.
- **Ampère-Maxwell law, second half:** A magnetic field can also be created by a changing electric field.
- **Lorentz force law, first half:** An electric force is exerted on a charged particle in an electric field.
- **Lorentz force law, second half:** A magnetic force is exerted on a charge moving in a magnetic field.

Electromagnetic Waves

Maxwell, using his equations of the electromagnetic field, was the first to understand that light is an oscillation of the electromagnetic field. Maxwell was able to predict that

- Electromagnetic waves can exist at any frequency, not just at the frequencies of visible light. This prediction was the harbinger of radio waves.
- All electromagnetic waves travel in a vacuum with the same speed, a speed that we now call the *speed of light*

$$v_{em} = \frac{1}{\sqrt{\epsilon_0 \mu_0}} = 3.00 \times 10^8 \text{ m/s} = c$$


1. A sinusoidal wave with frequency f and wavelength λ travels with wave speed v_{em}.
Wavelength λ
y
E_0
$\vec{E}$
$\vec{B}$
v_{em}
B_0
z
x
2. $\vec{E}$ and $\vec{B}$ are perpendicular to each other and to the direction of travel. The fields have amplitudes E_0 and B_0.
3. $\vec{E}$ and $\vec{B}$ are in phase. That is, they have matching crests, troughs, and zeros.

Properties of Electromagnetic Waves

Any electromagnetic wave must satisfy four basic conditions:

1. The fields $\boldsymbol{E}$ and $\boldsymbol{B}$ and are perpendicular to the direction of propagation $\boldsymbol{v}_{\text{em}}$.Thus an electromagnetic wave is a transverse wave.
2. $\boldsymbol{E}$ and $\boldsymbol{B}$ are perpendicular to each other in a manner such that $\boldsymbol{E} \times \boldsymbol{B}$ is in the direction of $\boldsymbol{v}_{\text{em}}$.
3. The wave travels in vacuum at speed $v_{\text{em}} = c$
4. $E = cB$ at any point on the wave.

Properties of Electromagnetic Waves

The energy flow of an electromagnetic wave is described by the **Poynting vector** defined as

$$\vec{S} \equiv \frac{1}{\mu_0}\vec{E} \times \vec{B}$$

The magnitude of the Poynting vector is

$$S = \frac{EB}{\mu_0} = \frac{E^2}{c\mu_0}$$

The intensity of an electromagnetic wave whose electric field amplitude is E_0 is

$$I = \frac{P}{A} = S_{\text{avg}} = \frac{1}{2c\mu_0}E_0^2 = \frac{c\epsilon_0}{2}E_0^2$$

EXAMPLE: The electric field of a laser beam

A helium-neon laser, the laser commonly used for classroom demonstrations, emits a 1.0-mm-diameter laser beam with a power of 1.0 mW. What is the amplitude of the oscillating electric field in the laser beam?

MODEL The laser beam is an electromagnetic plane wave.

SOLVE 1.0 mW, or 1.0×10^{-3} J/s, is the energy transported per second by the light wave. This energy is carried within a 1.0-mm-diameter beam, so the light intensity is

$$I = \frac{P}{A} = \frac{P}{\pi r^2} = \frac{1.0 \times 10^{-3}\ \mathrm{W}}{\pi (0.00050\ \mathrm{m})^2} = 1270\ \mathrm{W/m^2}$$

We can use Equation 35.37 to relate this intensity to the electric field amplitude:

$$E_0 = \sqrt{\frac{2I}{c\epsilon_0}} = \sqrt{\frac{2(1270\ \mathrm{W/m^2})}{(3.00 \times 10^8\ \mathrm{m/s})(8.85 \times 10^{-12}\ \mathrm{C^2/N\,m^2})}}$$

$$= 980\ \mathrm{V/m}$$

Radiation Pressure

It's interesting to consider the force of an electromagnetic wave exerted on an object per unit area, which is called the **radiation pressure** p_{rad}. The radiation pressure on an object that absorbs all the light is

$$p_{\text{rad}} = \frac{F}{A} = \frac{P/A}{c} = \frac{I}{c}$$

$$\Delta p = \frac{\text{energy absorbed}}{c} \quad (E = pc)$$

$$F = \frac{\Delta p}{\Delta t} = \frac{(\text{energy absorbed})/\Delta t}{c} = \frac{P}{c}$$

where P is the power (joules per second) of the light.

where *I* is the intensity of the light wave. The subscript on p_{rad} is important in this context to distinguish the radiation pressure from the momentum *p.*

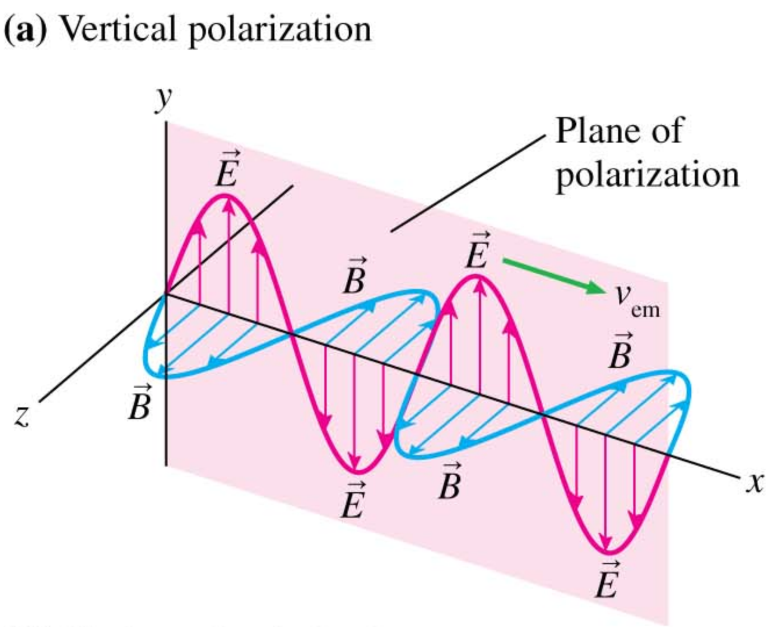
Example Solar sailing

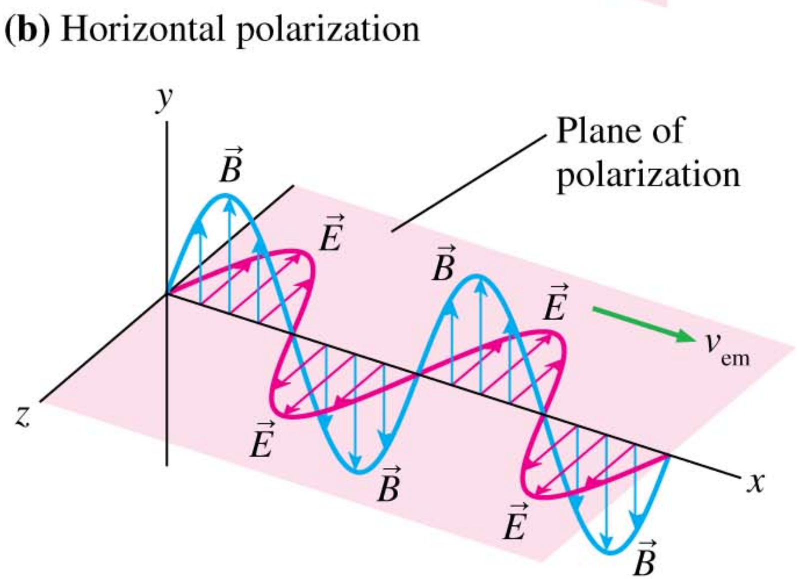
A low-cost way of sending spacecraft to other planets would be to use the radiation pressure on a solar sail. The intensity of the sun's electromagnetic radiation at distances near the earth's orbit is about 1300 W/m^2. What size sail would be needed to accelerate a 10,000 kg spacecraft toward Mars at 0.010 m/s^2?

SOLVE The force that will create a 0.010 m/s^2 acceleration is $F = ma = 100$ N. We can use Equation 35.39 to find the sail area that, by absorbing light, will receive a 100 N force from the sun:

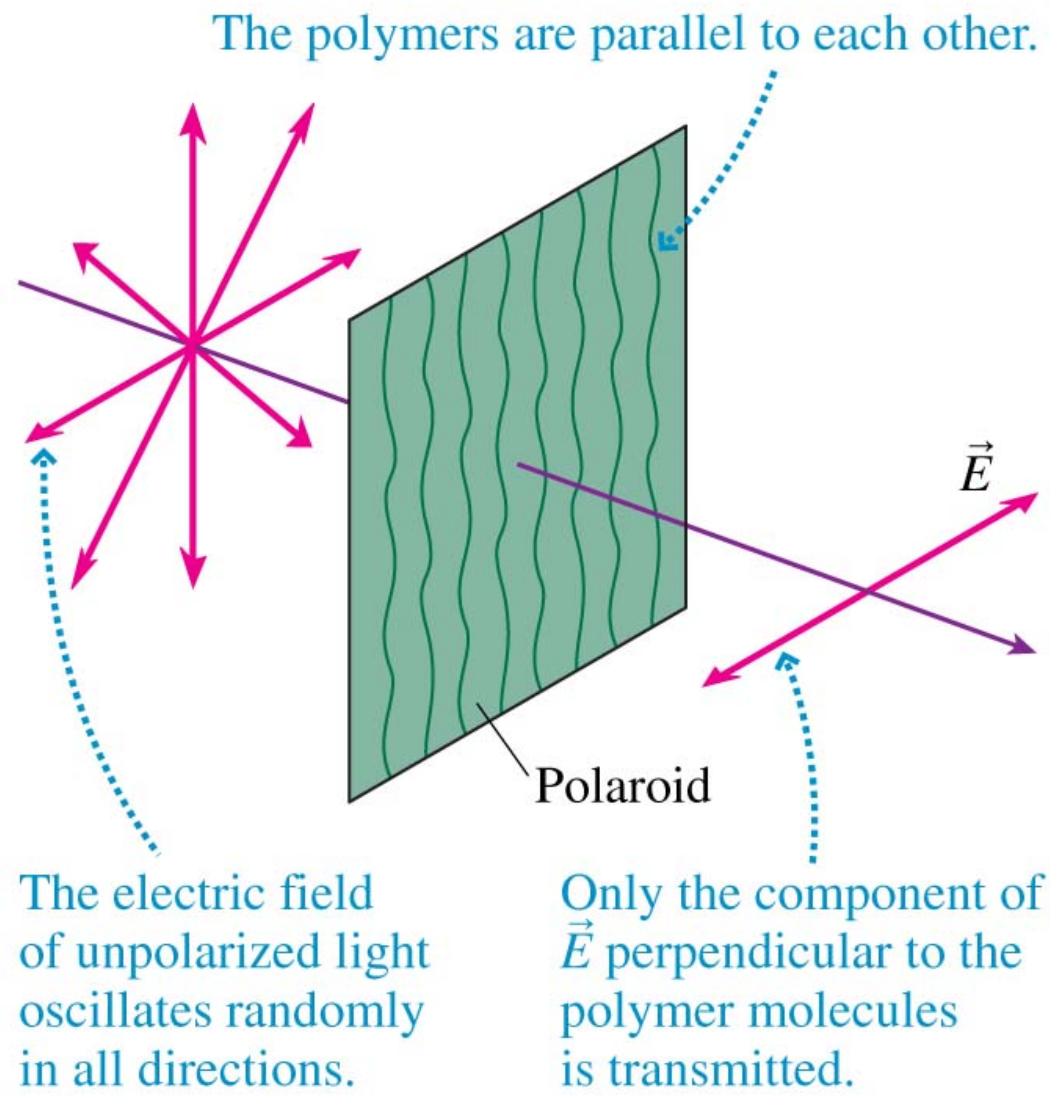
$$A = \frac{cF}{I} = \frac{(3.00 \times 10^8 \text{ m/s})(100 \text{ N})}{1300 \text{ W/m}^2} = 2.3 \times 10^7 \text{ m}^2$$

Polarization & Plane of Polarization





A Polarizing Filter



Malus' s Law

Suppose a *polarized* light wave of intensity I_0 approaches a polarizing filter. ϑ is the angle between the incident plane of polarization and the polarizer axis. The transmitted intensity is given by Malus' s Law:

$$I_{\text{transmitted}} = I_0 \cos^2\theta \qquad \text{(incident light polarized)}$$

If the light incident on a polarizing filter is *unpolarized*, the transmitted intensity is

$$I_{\text{transmitted}} = \frac{1}{2} I_0 \qquad \text{(incident light unpolarized)}$$

In other words, a polarizing filter passes 50% of unpolarized light and blocks 50%.

Intermediate/Advanced Concepts

Wave equations in a medium

The induced polarization in Maxwell's Equations yields another term in the wave equation:

$$\frac{\partial^2 E}{\partial z^2} - \mu\varepsilon \frac{\partial^2 E}{\partial t^2} = 0 \qquad \frac{\partial^2 E}{\partial z^2} - \frac{1}{v^2}\frac{\partial^2 E}{\partial t^2} = 0$$

This is the **Inhomogeneous Wave Equation**.

The polarization is the driving term for a new solution to this equation.

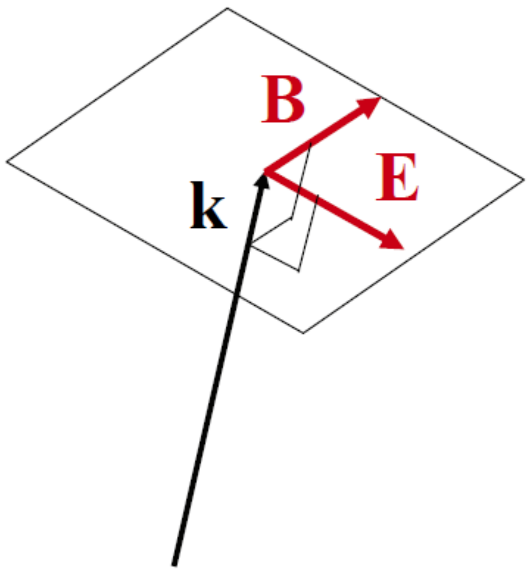
$$\frac{\partial^2 E}{\partial z^2} - \mu_0\varepsilon_0 \frac{\partial^2 E}{\partial t^2} = 0 \qquad \frac{\partial^2 E}{\partial z^2} - \frac{1}{c^2}\frac{\partial^2 E}{\partial t^2} = 0$$

Homogeneous (Vacuum) Wave Equation

$$\begin{aligned}\mathbf{E}(z,t) &= \mathrm{Re}\{\mathbf{E}_0 e^{i(kz-\omega t)}\} \\ &= \tfrac{1}{2}\{\mathbf{E}_0 e^{i(kz-\omega t)} + \mathbf{E}_0^* e^{-i(kz-\omega t)}\} \\ &= |\mathbf{E}_0| \cos(kz-\omega t)\end{aligned}$$

$$n^2 = \frac{c^2}{v^2} = \frac{\mu\varepsilon}{\mu_0\varepsilon_0} \qquad \frac{c}{v} = n$$

Propagation of EM Waves



$$\nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t} \quad \text{where} \quad \mathbf{E} = \hat{\mathbf{x}} E_0 \, e^{i(\mathbf{k}\cdot\mathbf{r} - \omega t)}$$

$$\Rightarrow \nabla \times \equiv i\mathbf{k} \times \quad \text{and} \quad \frac{\partial}{\partial t} \equiv -i\omega$$

$$\Rightarrow \mathbf{B} = \frac{1}{\omega} \mathbf{k} \times \mathbf{E}$$

Vectors **k**, **E**, **B** form a right-handed triad.

<u>Note:</u> free space or isotropic media only

Polarization and Propagation

In isotropic media (e.g. free space, amorphous glass, etc.)

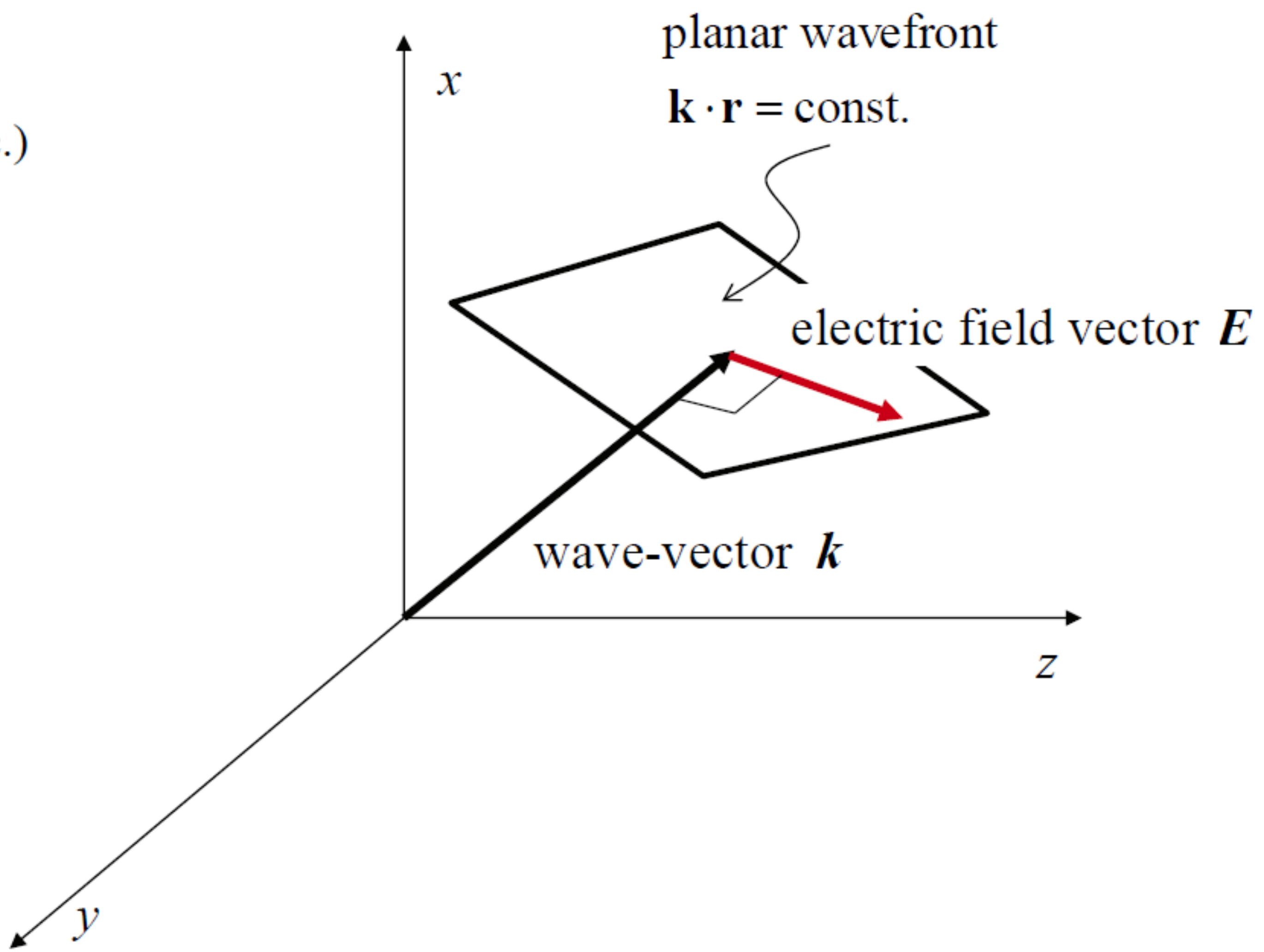
$\mathbf{k} \cdot \mathbf{E} = 0$

i.e. $\mathbf{k} \perp \mathbf{E}$

More generally,

$\mathbf{k} \cdot \mathbf{D} = 0$

(reminder: in anisotropic media, e.g. crystals, one could have $\mathbf{E}$ not parallel to $\mathbf{D}$)



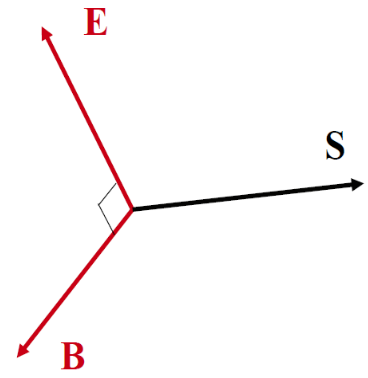
Energy and Intensity

$$\mathbf{S}=\mathbf{E}\times\mathbf{H}$$

Summary (free space or isotropic media)

$$\mathbf{S}=\frac{1}{\mu_0}\mathbf{E}\times\mathbf{B};\quad \|\mathbf{S}\|=c\varepsilon_0\|\mathbf{E}\|^2$$ Poynting vector

$$\langle\|\mathbf{S}\|\rangle=\frac{1}{T}\int_t^{t+T}\|\mathbf{S}\|\mathrm{d}t$$ Irradiance (or intensity)



$$\mathbf{S}=\frac{1}{\mu_0}\mathbf{E}\times\mathbf{B}=c^2\varepsilon_0\mathbf{E}\times\mathbf{B}$$

so in free space

$$\mathbf{S}\parallel\mathbf{k}$$

S has units of W/m^2 so it represents energy flux (energy per unit time & unit area)

- **Poynting vector** describes flows of E-M power
- Power flow is directed along this vector (usually parallel to **k)**
- Intensity is average energy transfer (i.e. the time averaged Poyning vector: I=<**S**>=P/A, where P is the power (energy transferred per second) of a wave that impinges on area A.

$$\langle\sin^2(kx-\omega t)\rangle=\langle\cos^2(kx-\omega t)\rangle=\frac{1}{2}$$

$$|\langle\mathbf{S}\rangle|=I\equiv|\langle\mathbf{E}(t)\times\mathbf{H}(t)\rangle|=\frac{c\varepsilon_0}{2}E^2=\frac{c\varepsilon_0}{2}\left(E_x^{\ 2}+E_y^{\ 2}\right)$$

$$c\varepsilon_0\approx 2.654\times10^{-3}\,A/V$$

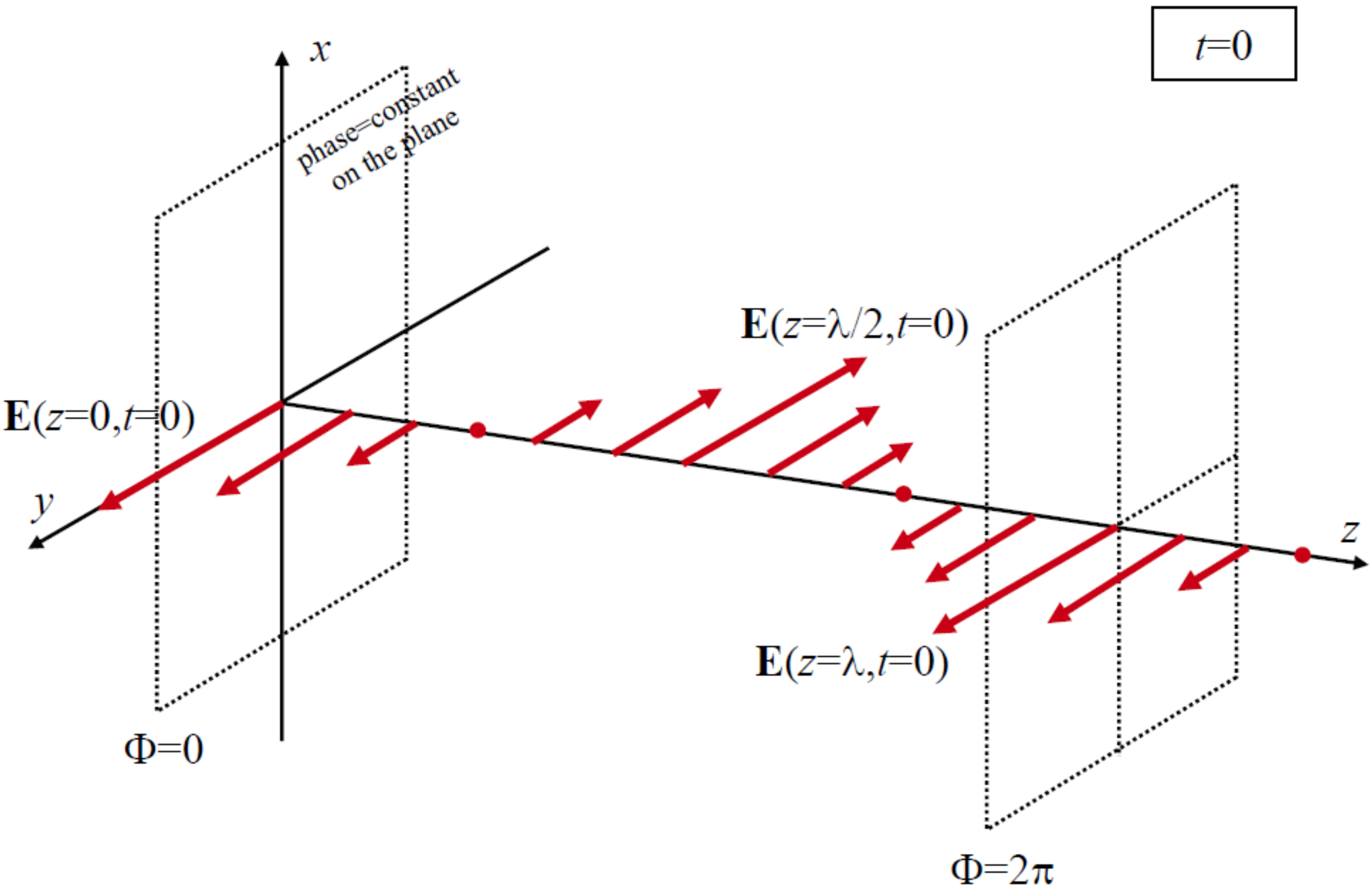
example $E=1V/m$

$I=?\,W/m^2$

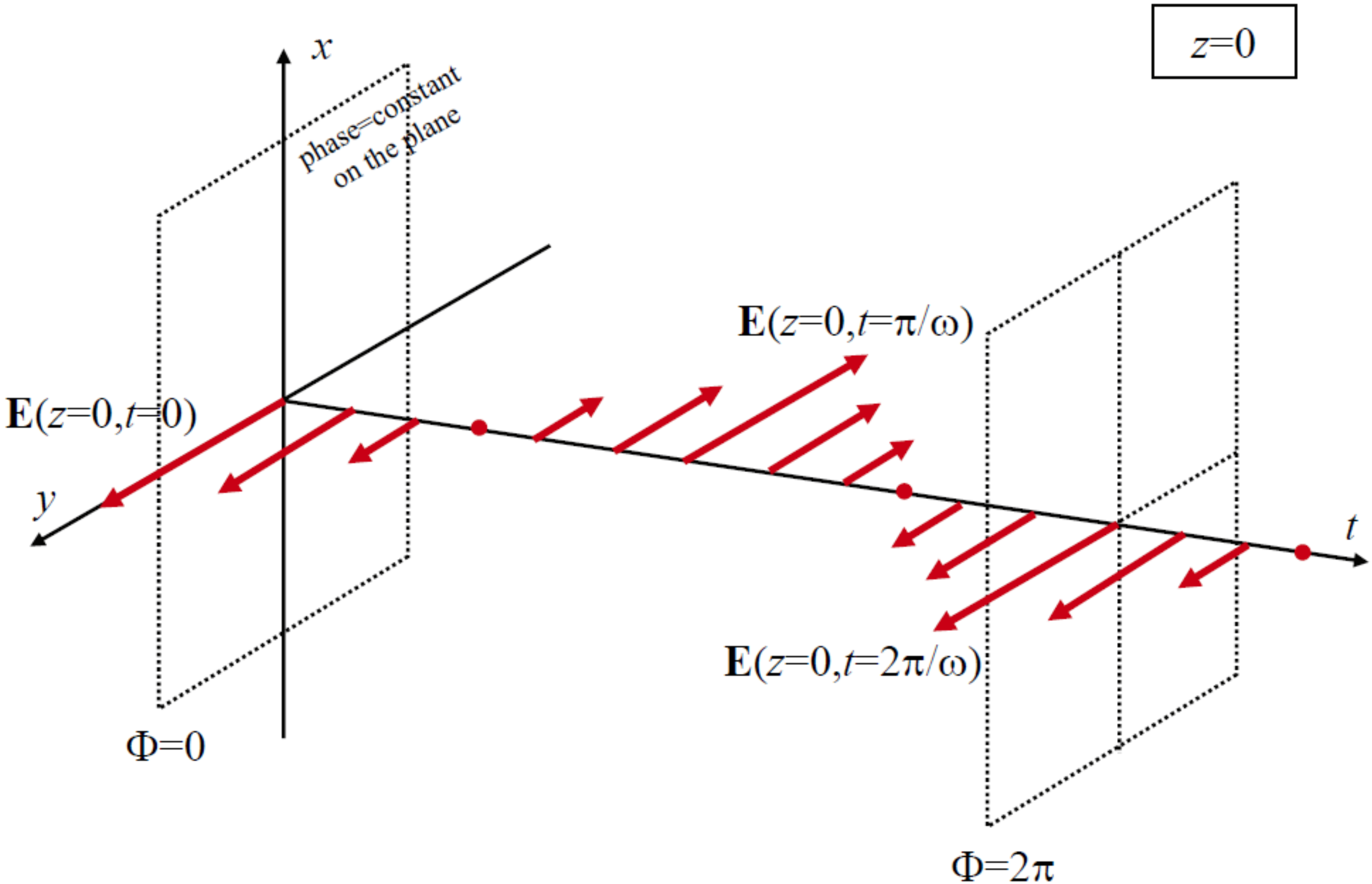
$$\hbar\omega[eV]=\frac{1239.85}{\lambda[nm]}$$

$$\hbar=1.05457266\times10^{-34}\,Js$$

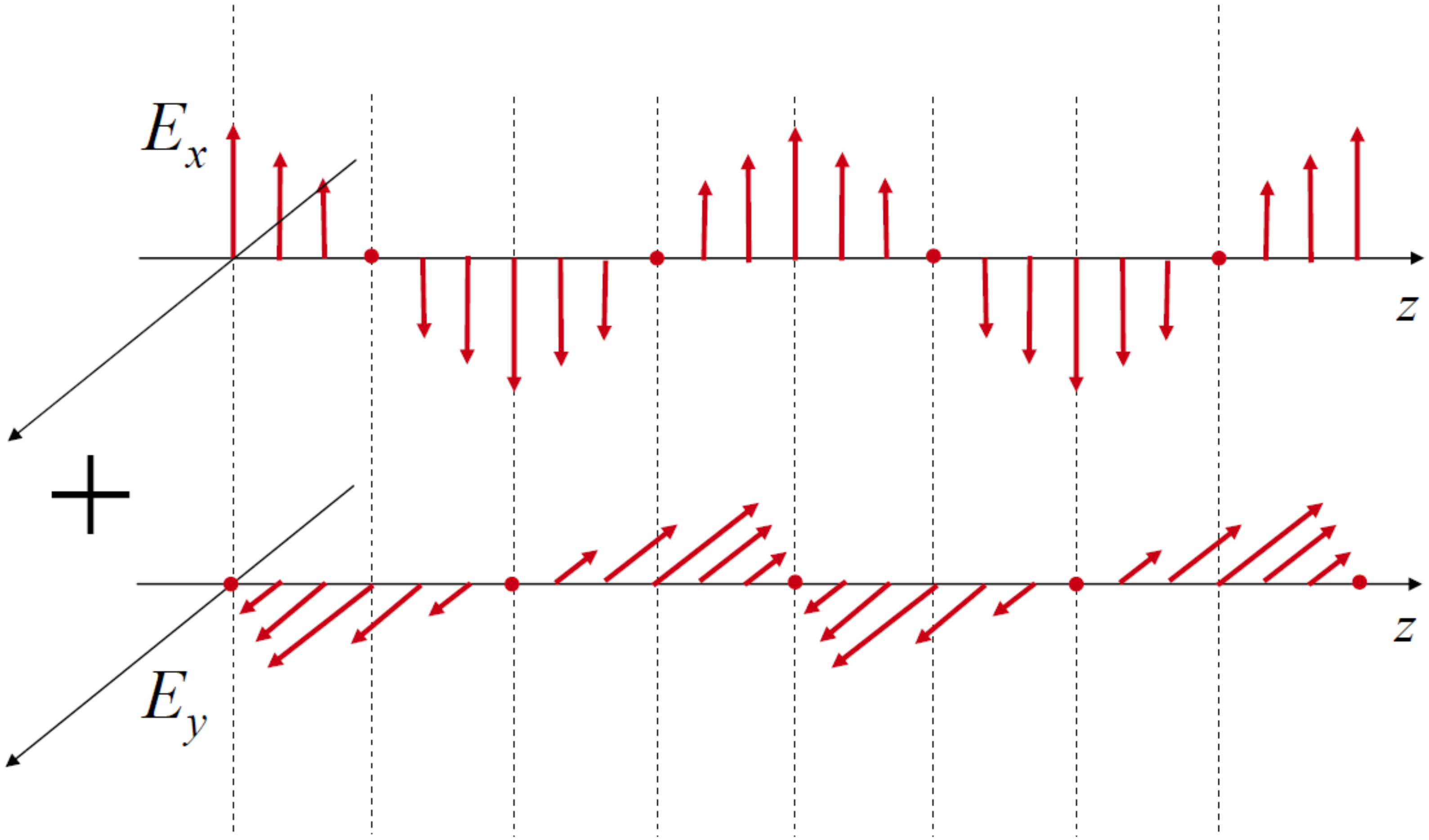
Linear polarization (frozen time)



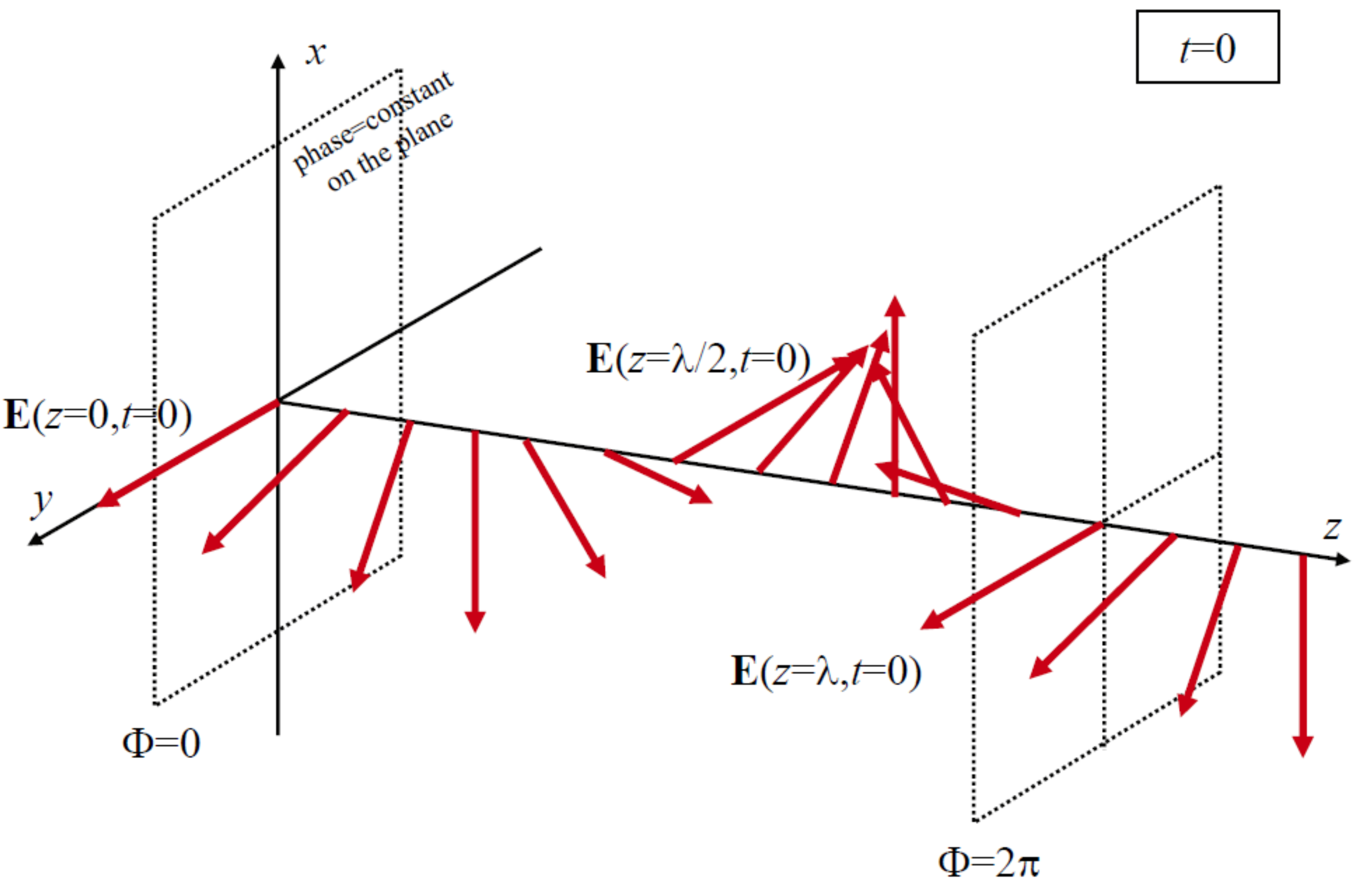
Linear polarization (fixed space)



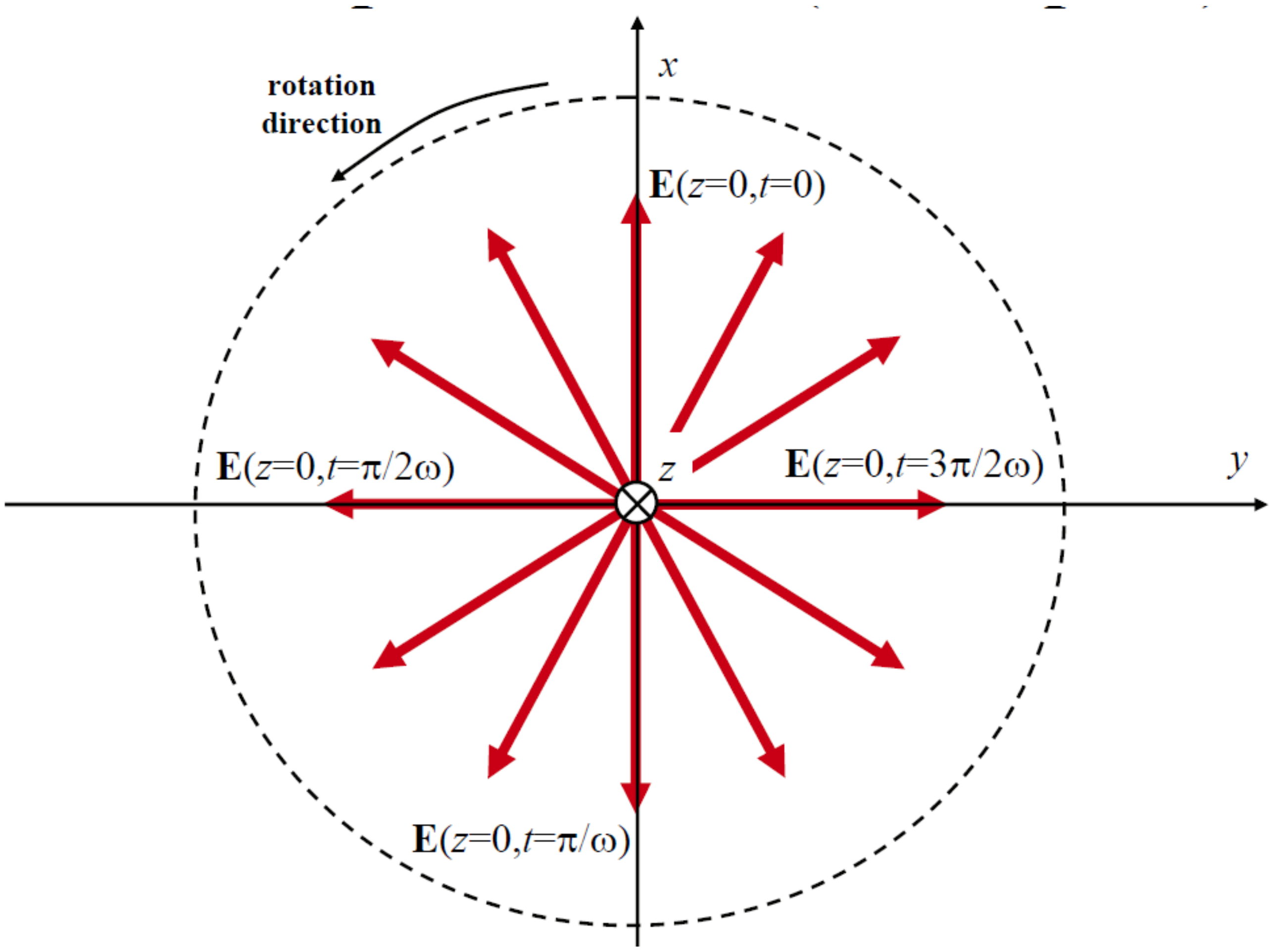
Circular polarization (linear components)



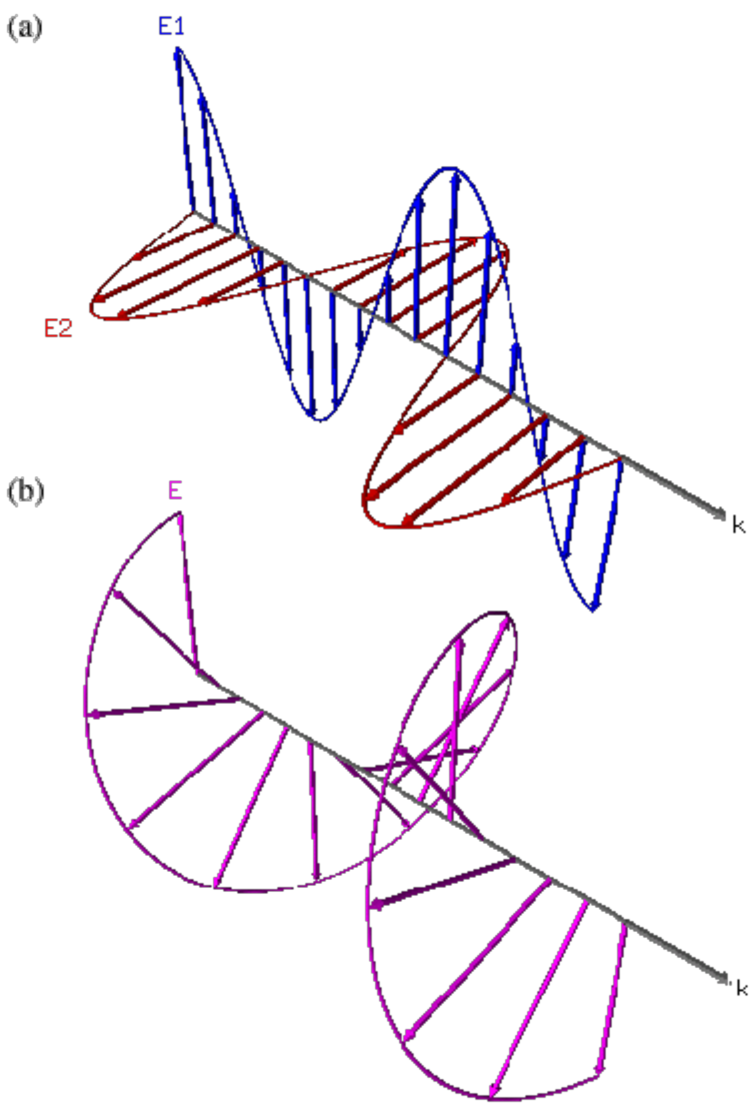
Circular polarization (frozen time)

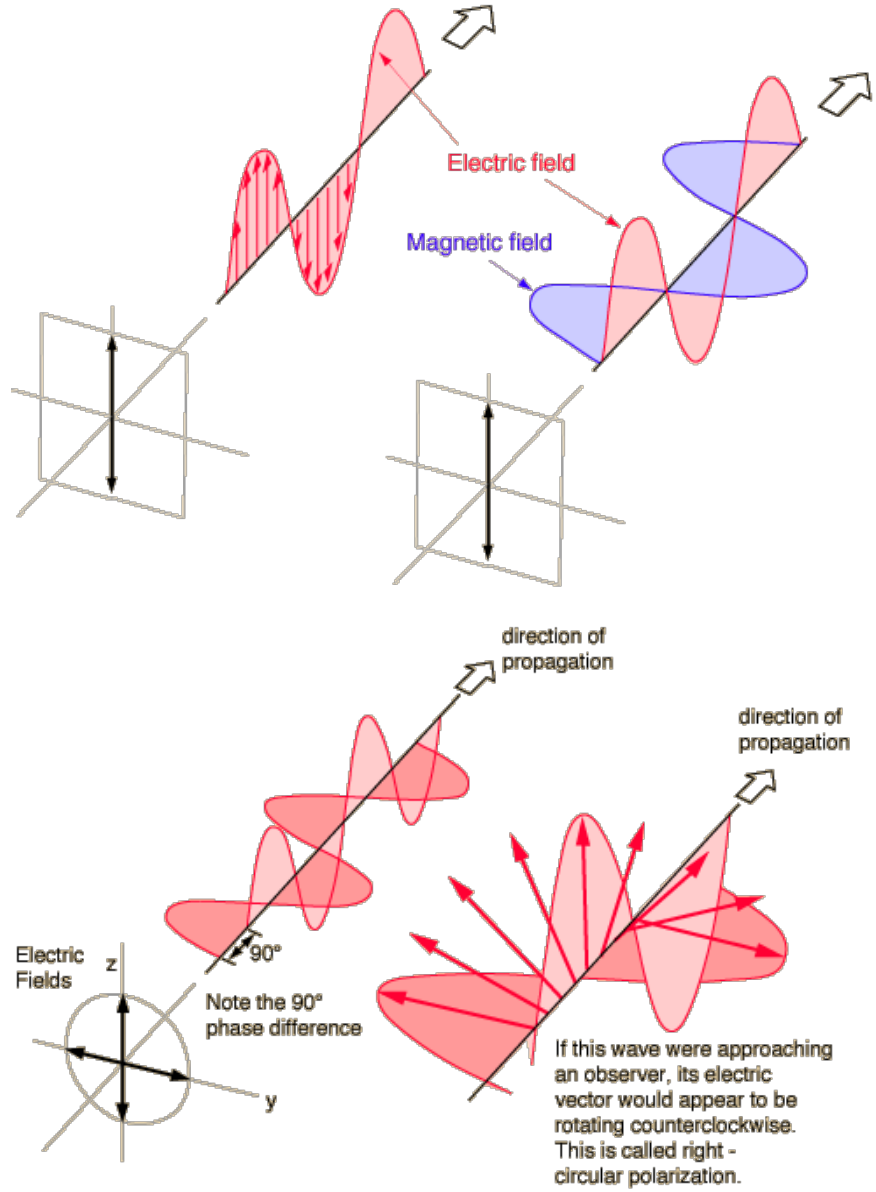


Circular polarization (fixed space)

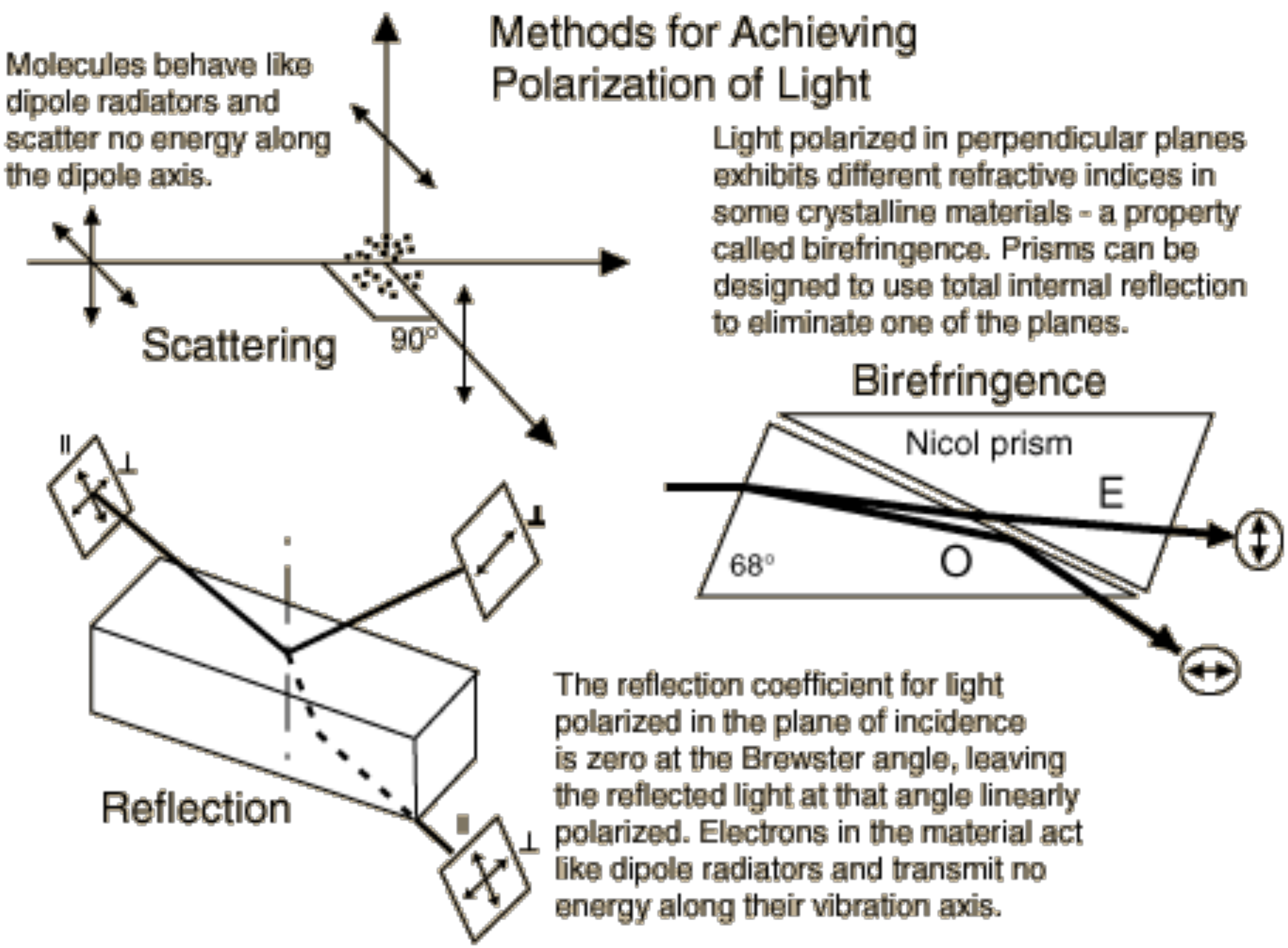


Linear versus Circular Polarization





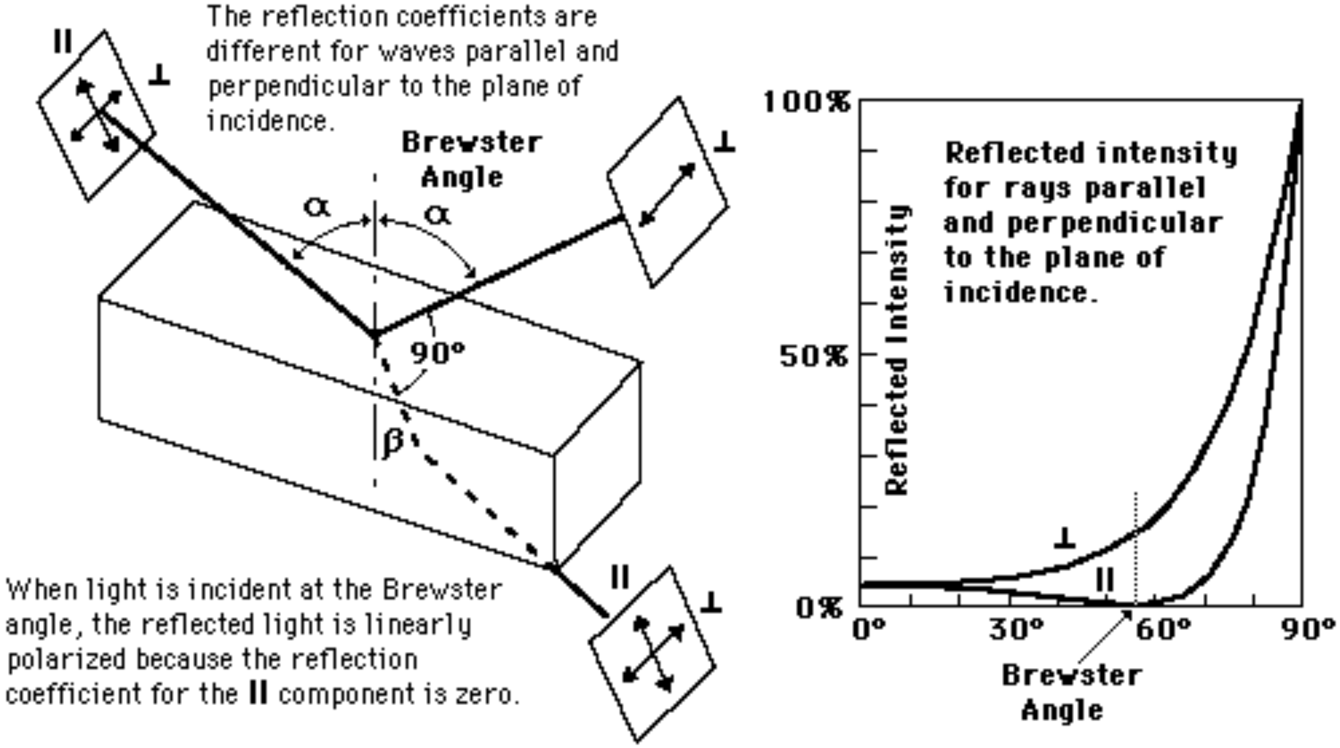
Methods for generating polarized light



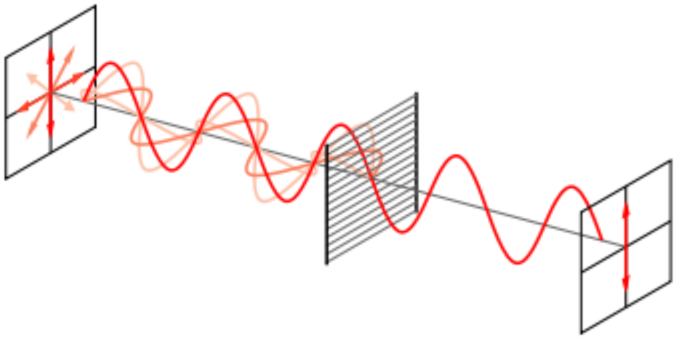
http://hyperphysics.phy-astr.gsu.edu/hbase/phyopt/polar.html

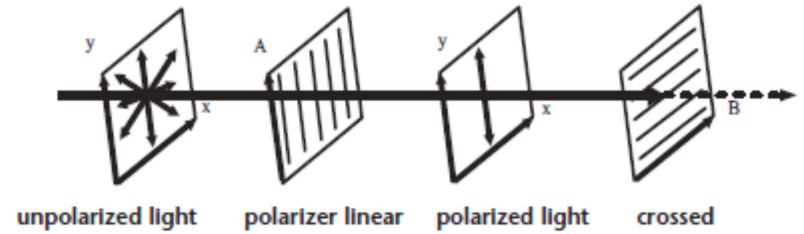
Polarization by Reflection

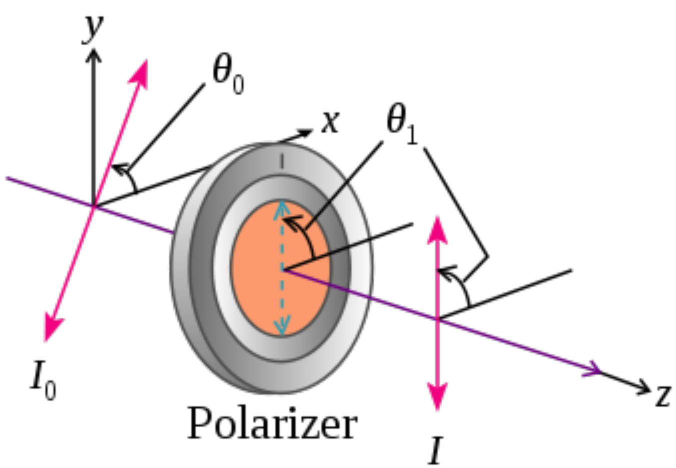
http://hyperphysics.phy-astr.gsu.edu/hbase/phyopt/polar.html



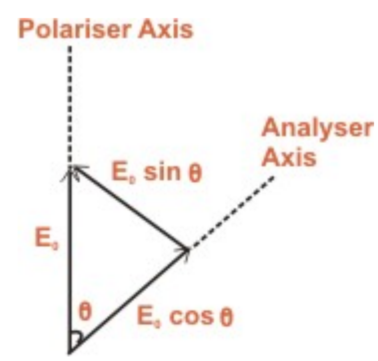
Malus’s Law





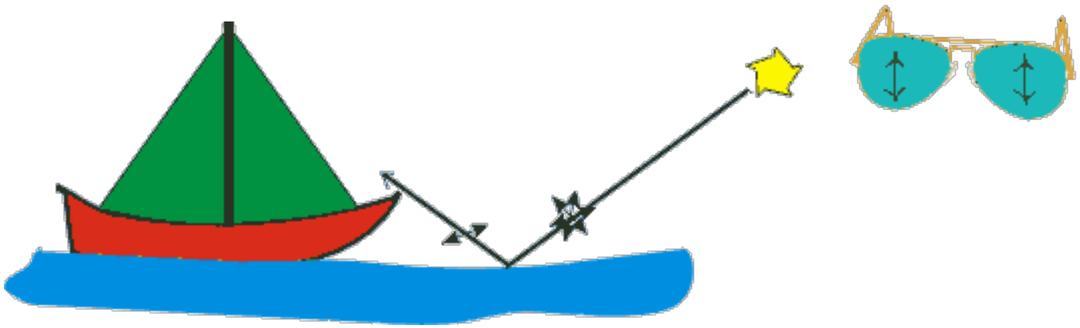


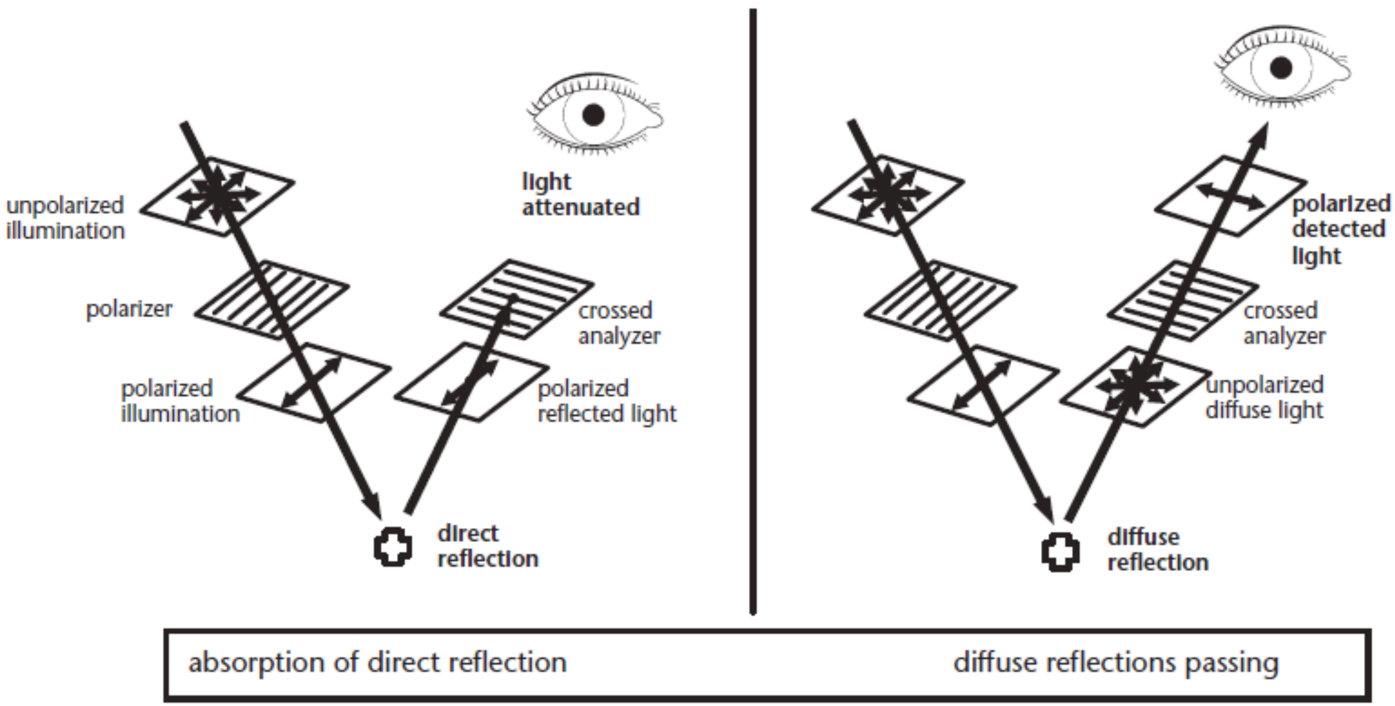
$$I = \frac{1}{2} c\epsilon_0 E_0^2 \cos^2\theta = I_0 \cos^2\theta,$$



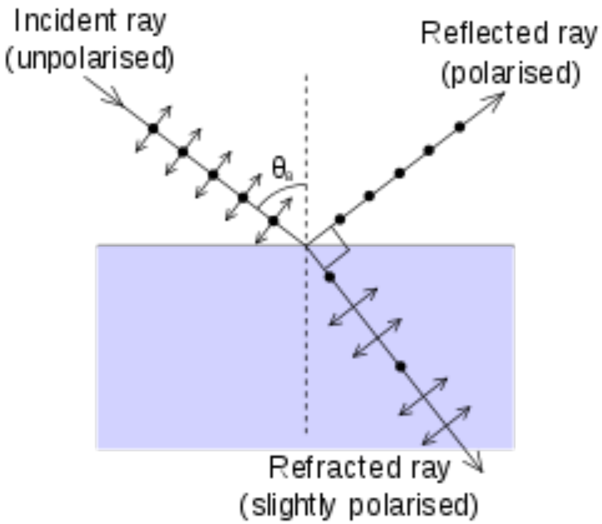
Where is the turtle?

Polarized sunglasses





Brewster Angle



$$\theta_1 + \theta_2 = 90^\circ,$$

$$n_1 \sin\left(\theta_1\right) = n_2 \sin\left(\theta_2\right),$$

$$n_1 \sin\left(\theta_B\right) = n_2 \sin\left(90^\circ - \theta_B\right) = n_2 \cos\left(\theta_B\right).$$

$$\theta_B = \arctan\left(\frac{n_2}{n_1}\right),$$

Polarization by scattering (Rayleigh scattering/Blue Sky)

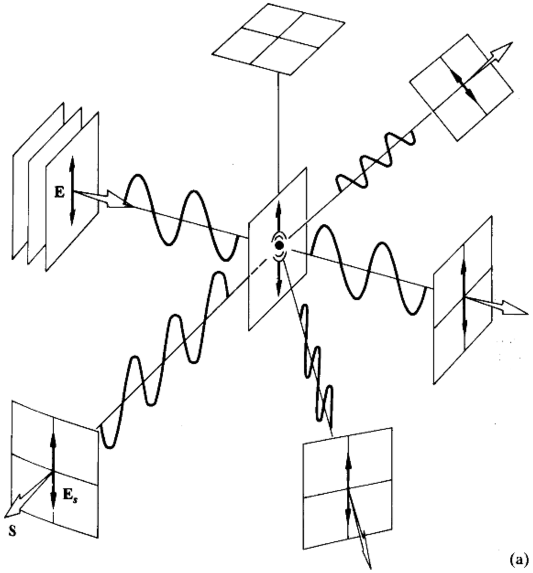


FIGURE 8.35*a* Scattering of polarized light by a molecule.

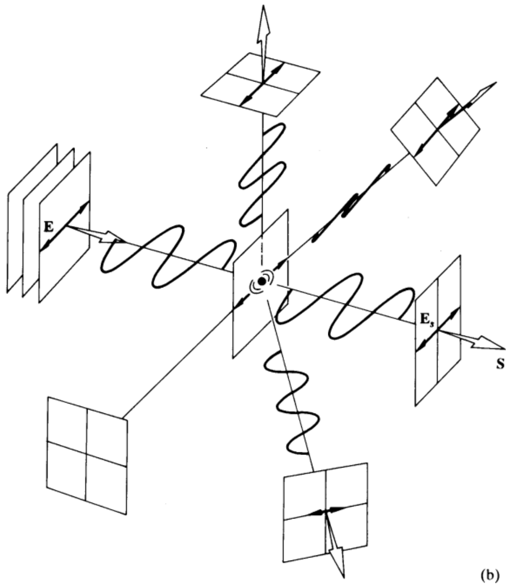


FIGURE 8.35*b*

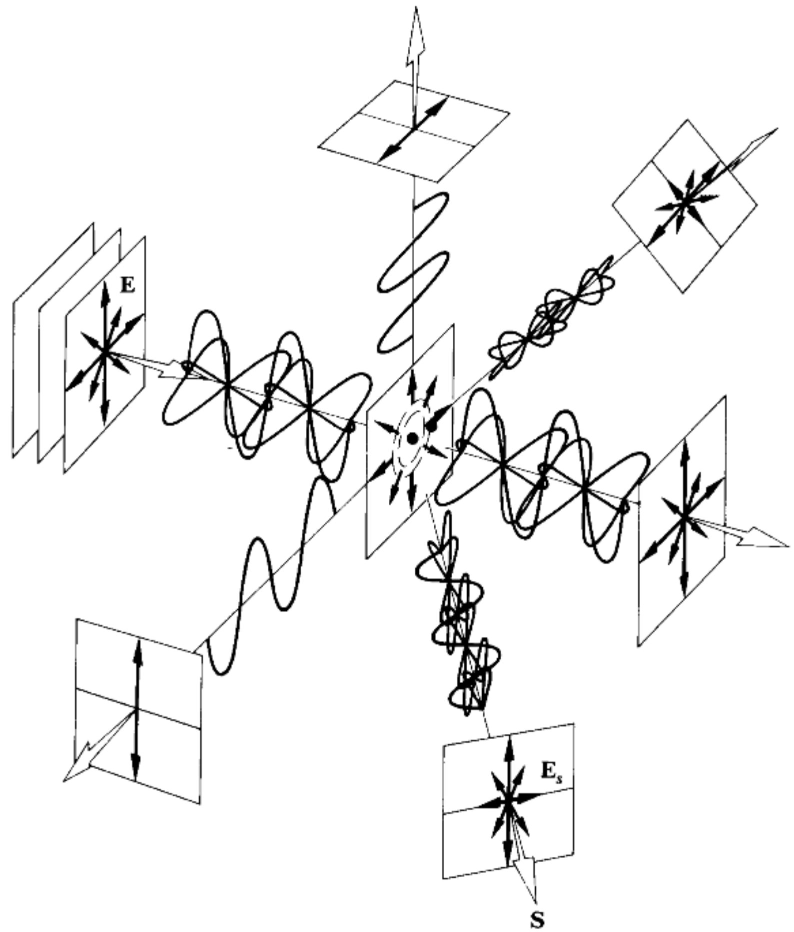
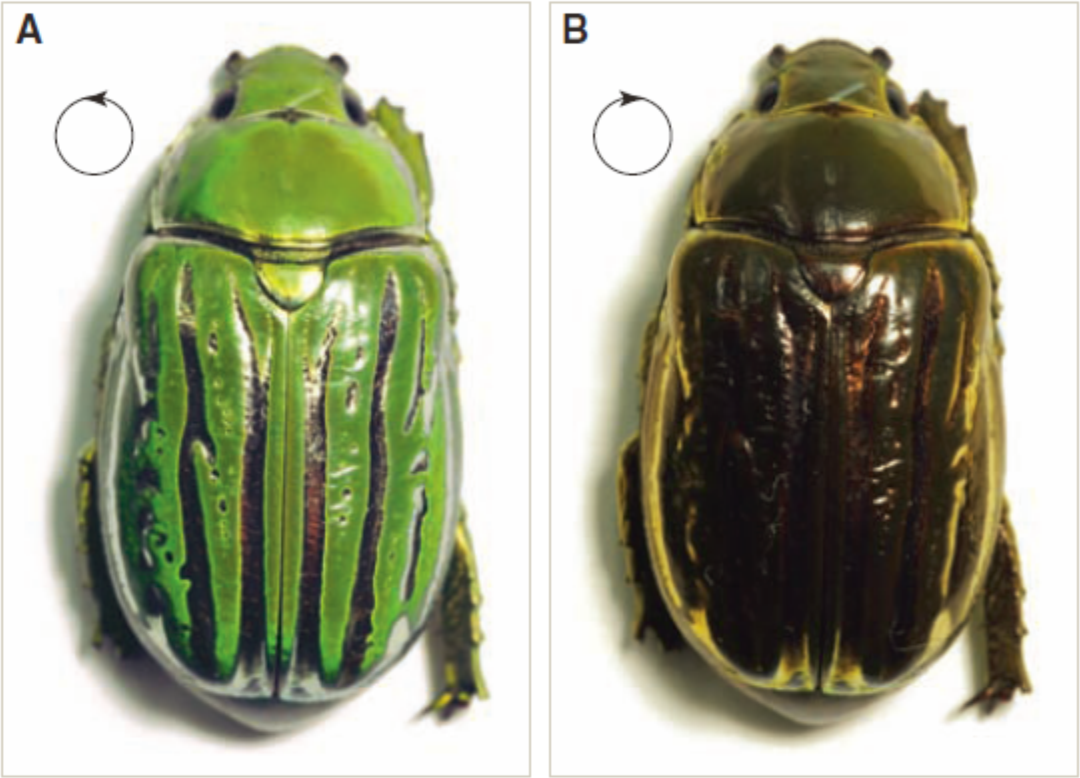


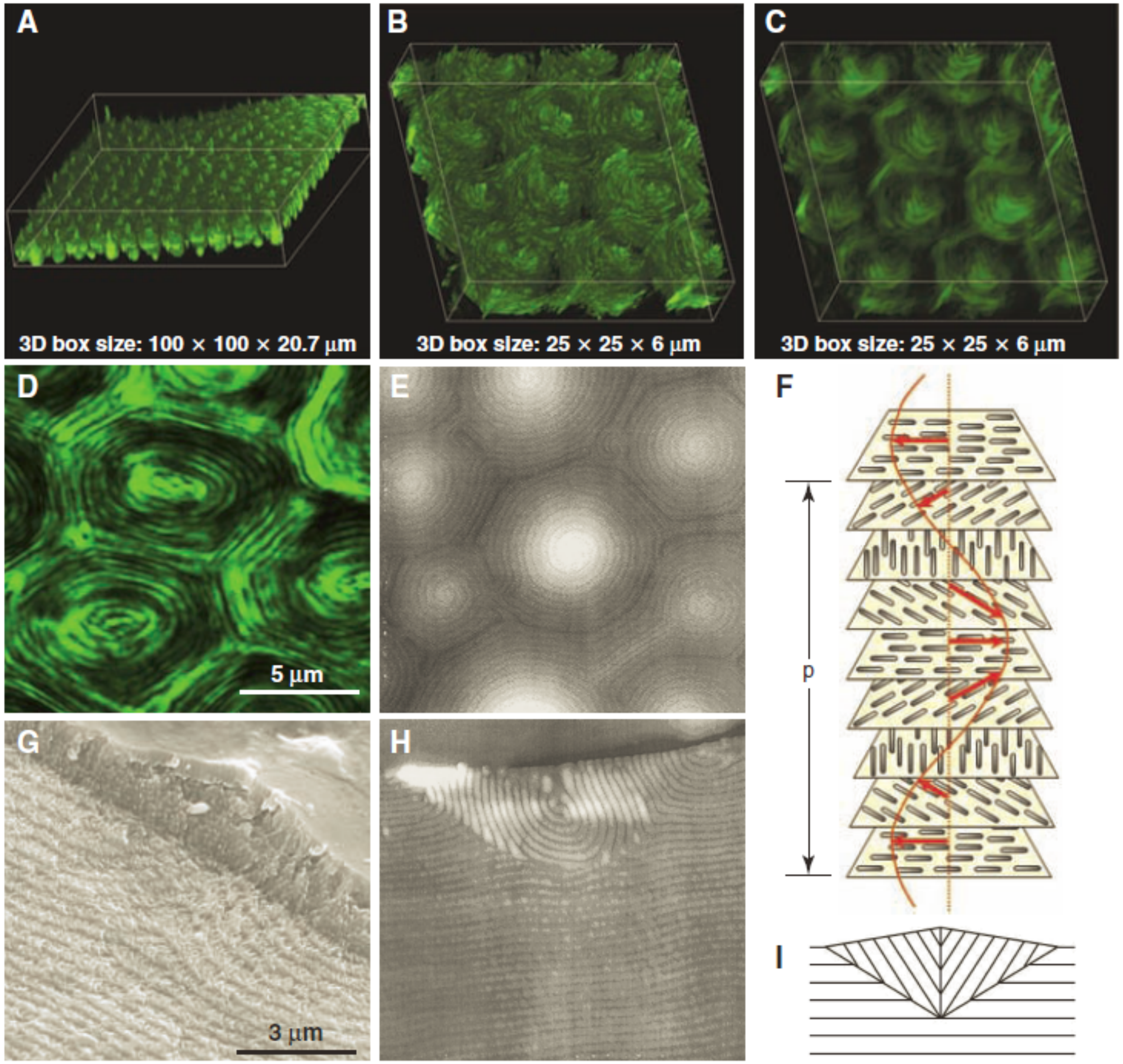
FIGURE 8.36 Scattering of unpolarized light by a molecule.

Circularly polarized light in nature

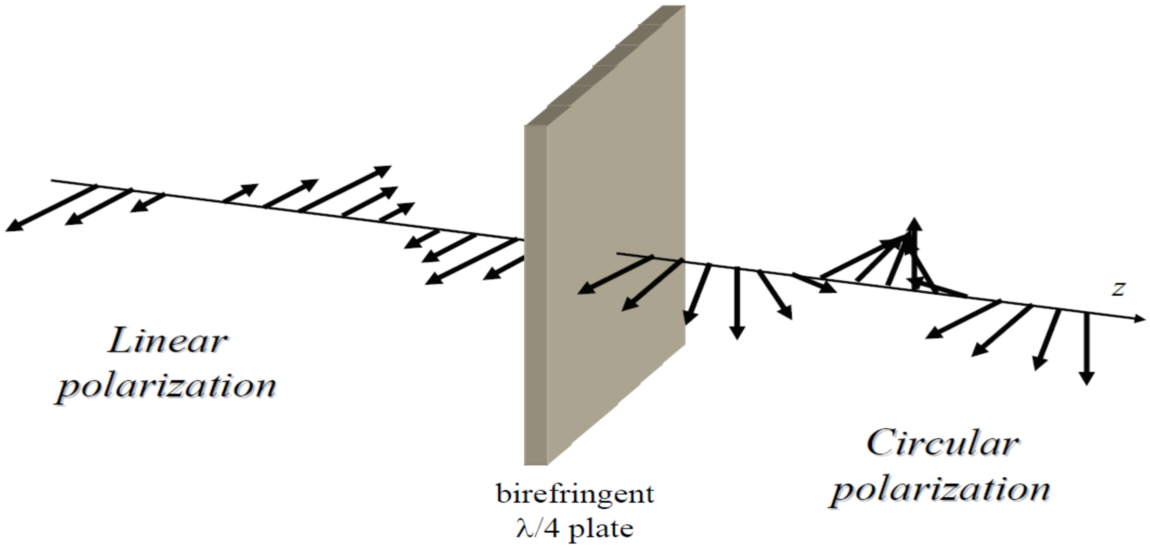
Fig. 1. Photographs of the beetle *C. gloriosa*. (**A**) The bright green color, with silver stripes as seen in unpolarized light or with a left circular polarizer. (**B**) The green color is mostly lost when seen with a right circular polarizer.

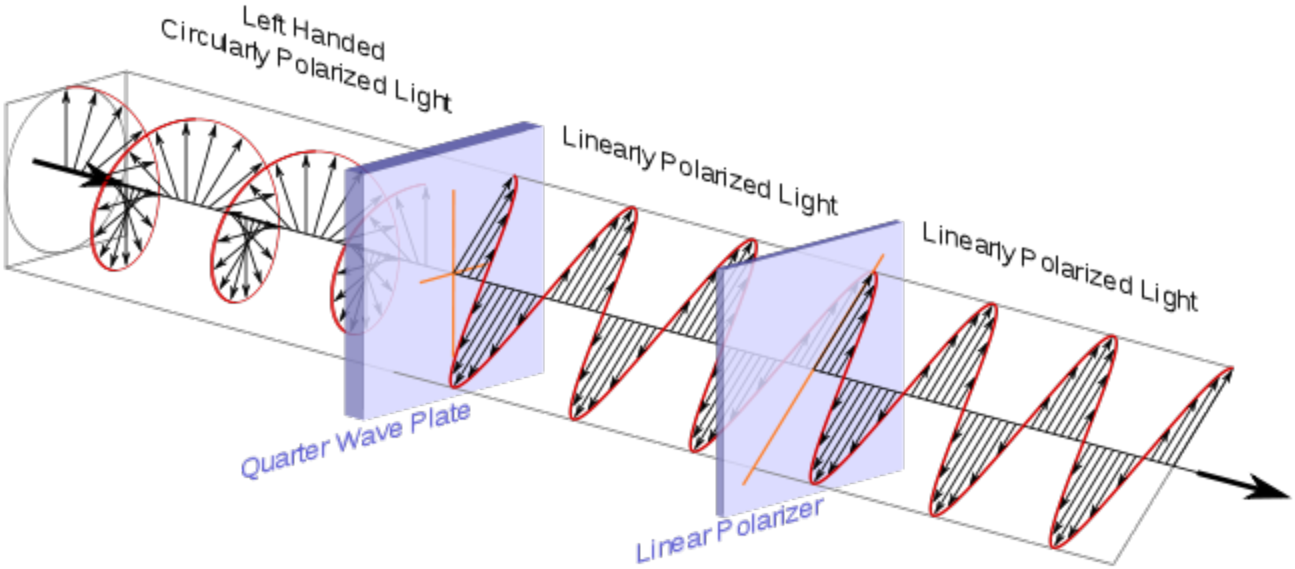


Morphology and microstructure of cellular pattern of C. gloriosa

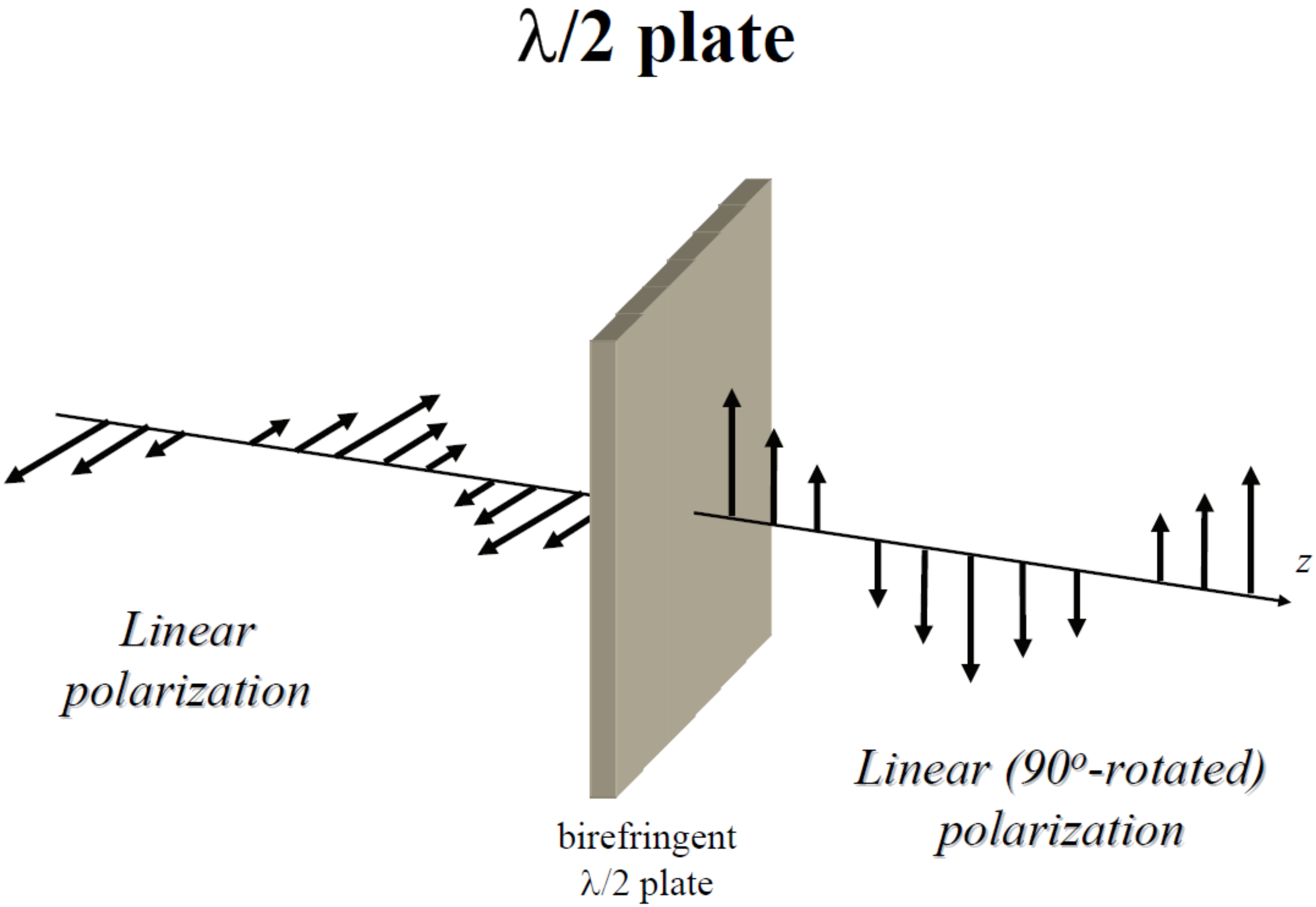


Quarter wave plate

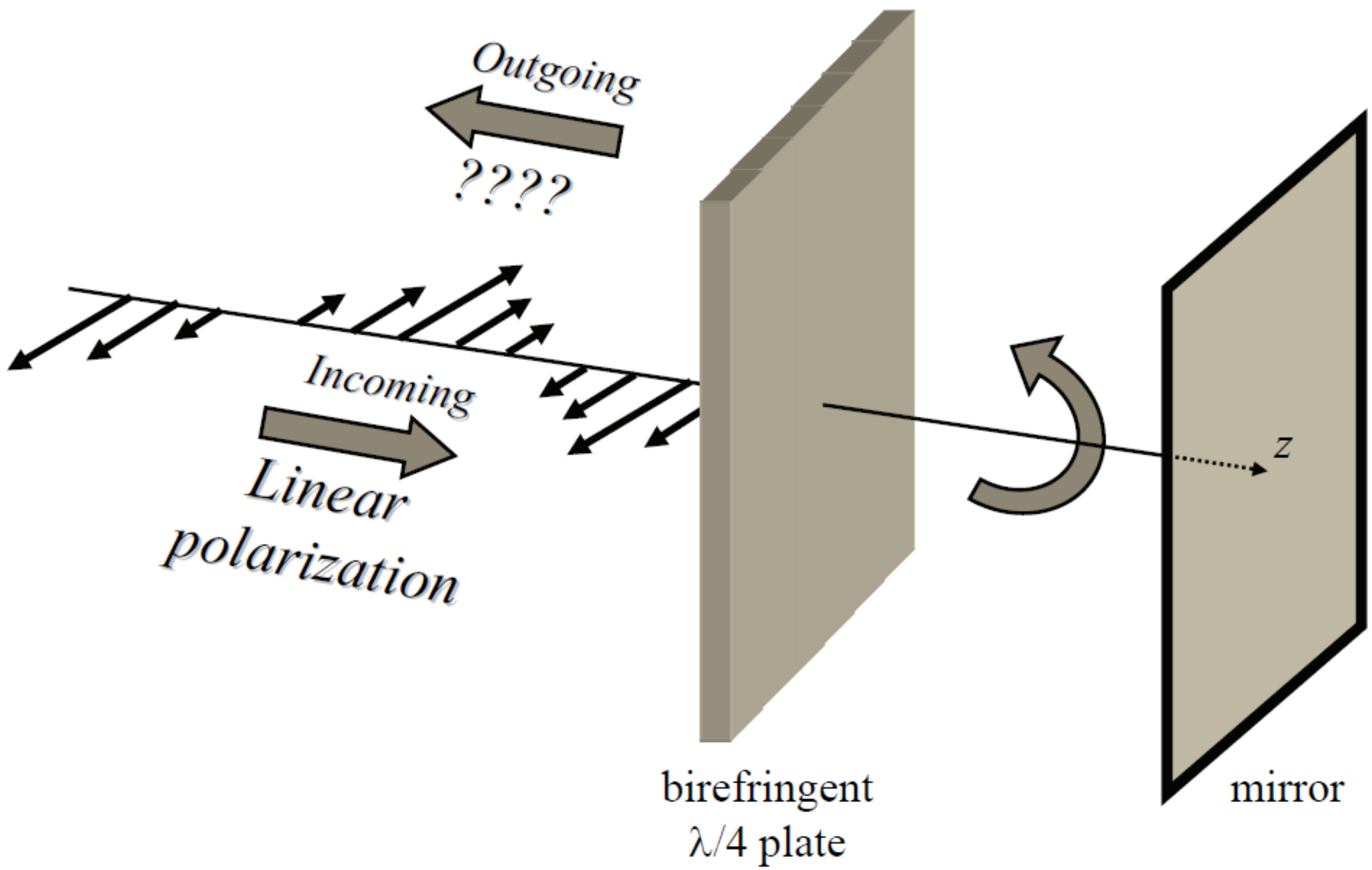




Half wave plate

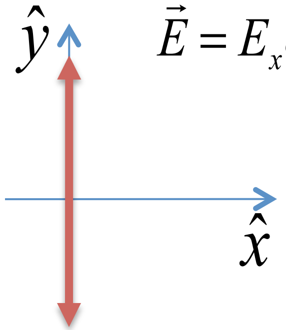


Quiz for the Lab – Bonus Credit 0.2 pts

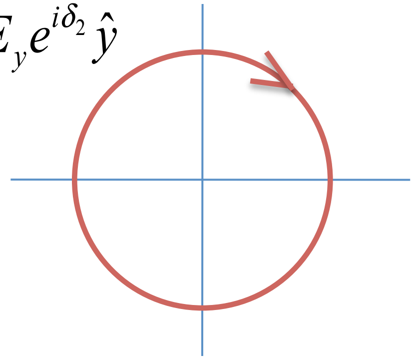


Polarization: Summary

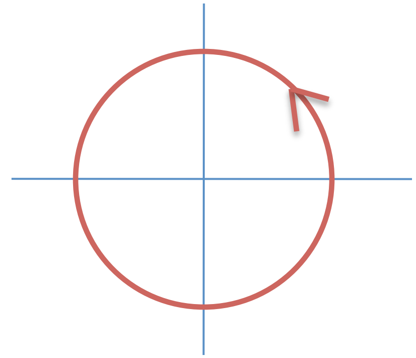
$$\vec{E} = E_x e^{i\delta_1}\hat{x} + E_y e^{i\delta_2}\hat{y}$$



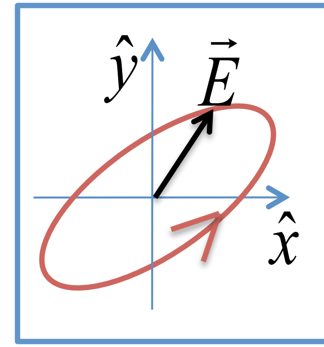
linear polarization
y-direction



right circular
polarization

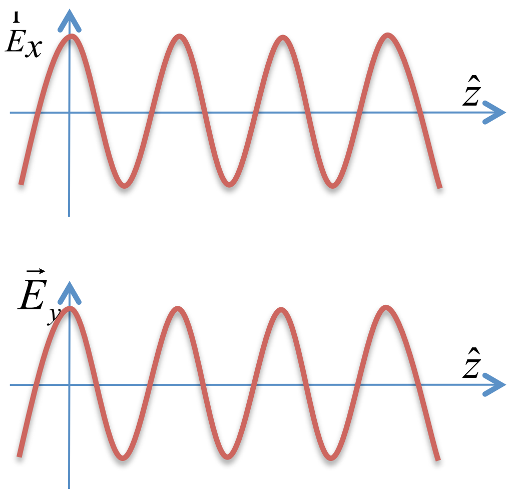


left circular
polarization

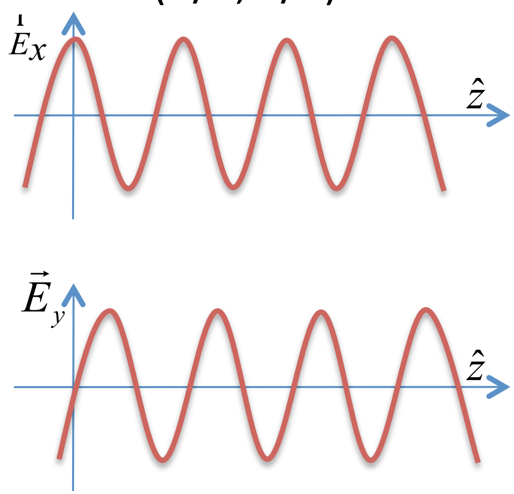


left elliptical
polarization

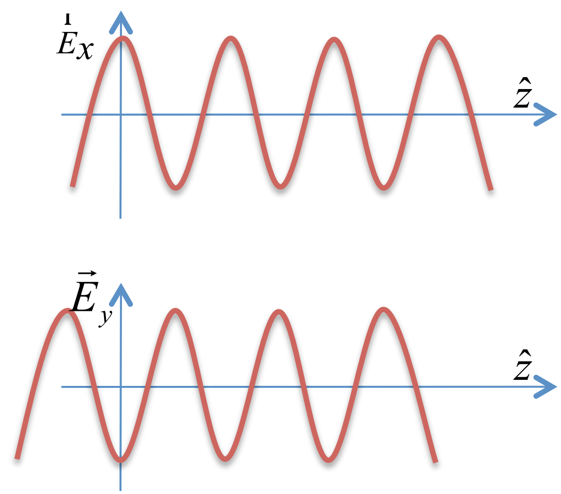
Phase difference = 0^0



Phase difference ➔
90^0 ($\pi/2$, $\lambda/4$)



Phase difference ➔
180^0 (π, $\lambda/2$)



Polarization Applets

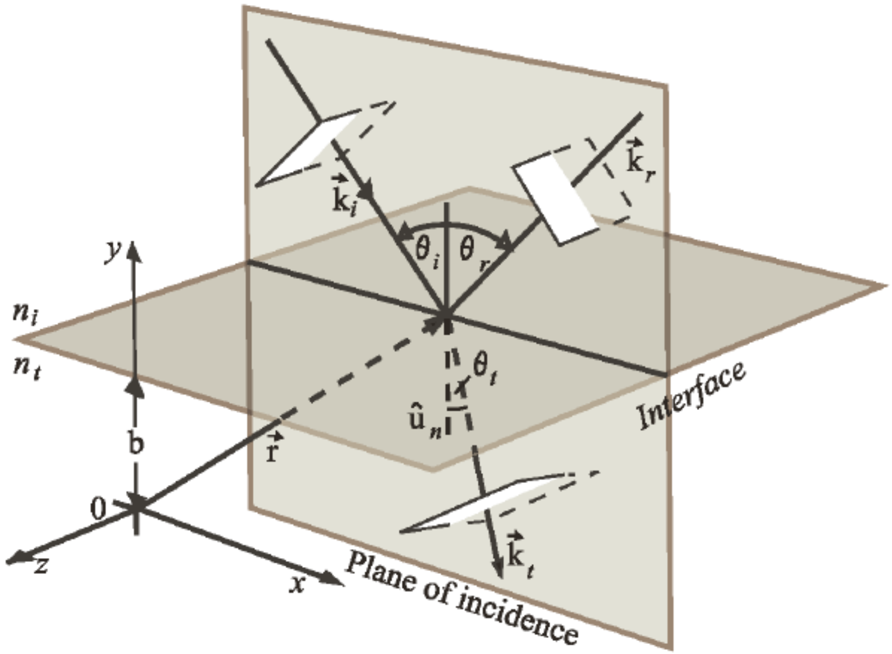
- [Polarization Exploration](http://webphysics.davidson.edu/physlet_resources/dav_optics/Examples/polarization.html)

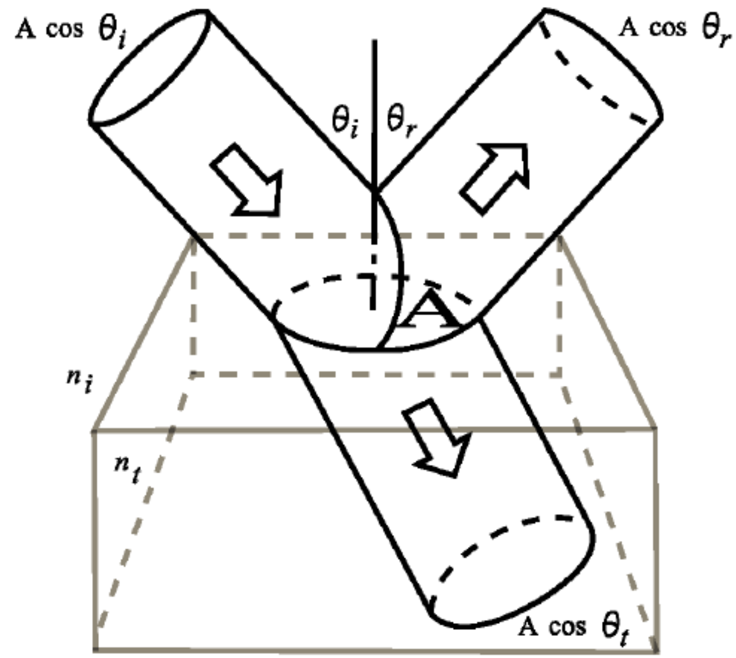
http://webphysics.davidson.edu/physlet_resources/dav_optics/Examples/polarization.html

- 3D View of Polarized Light

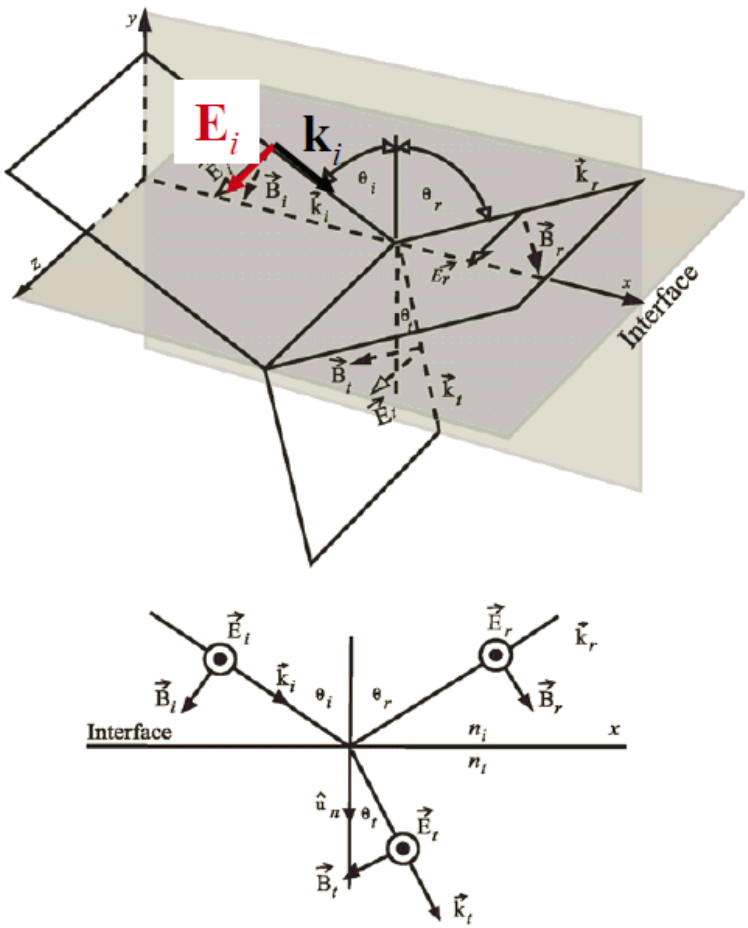
http://fipsgold.physik.uni-kl.de/software/java/polarisation/index.html

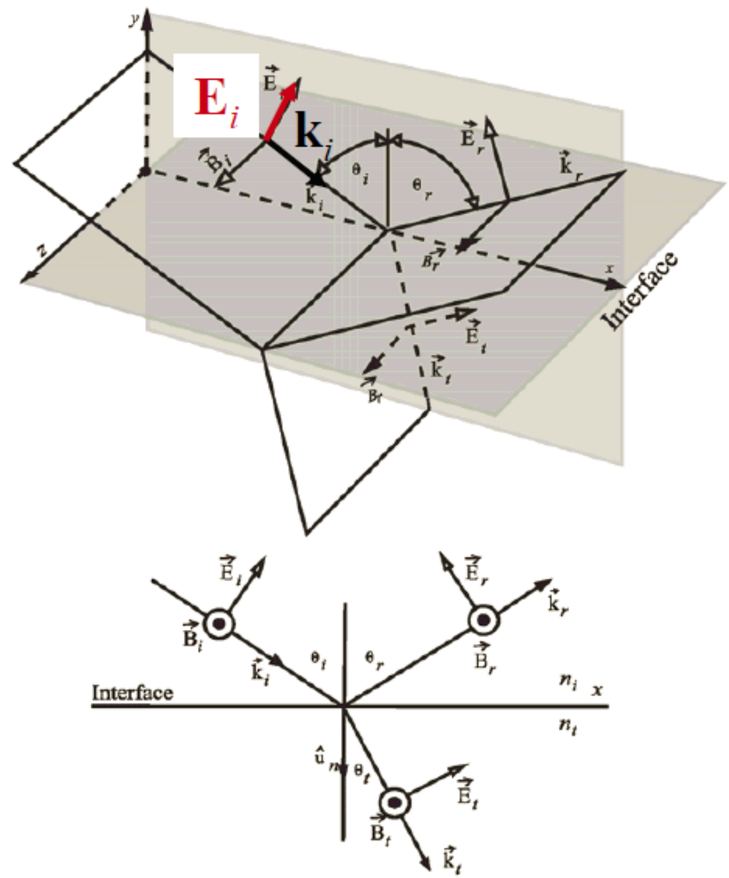
Reflection and Transmission @ dielectric interface





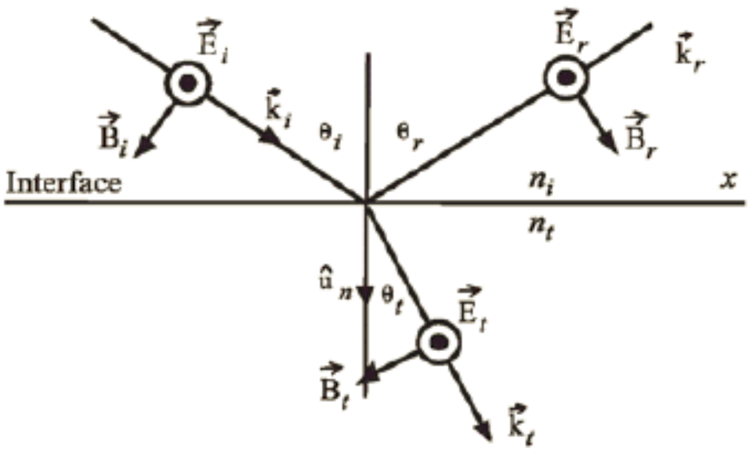
Beyond Snell's Law: Polarization?





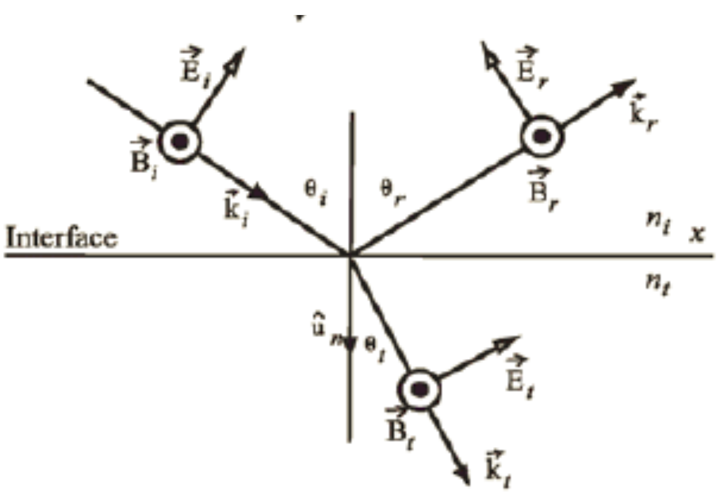
Reflection and Transmission (Fresnel's equations)

Can be deduced from the application of boundary conditions of EM waves. An online calculator is available at http://hyperphysics.phy-astr.gsu.edu/hbase/phyopt/freseq.html



$$r_\perp = \left(\frac{E_{0r}}{E_{0i}}\right)_\perp = \frac{n_i \cos\theta_i - n_t \cos\theta_t}{n_i \cos\theta_i + n_t \cos\theta_t}$$

$$t_\perp = \left(\frac{E_{0t}}{E_{0i}}\right)_\perp = \frac{2 n_i \cos\theta_i}{n_i \cos\theta_i + n_t \cos\theta_t}$$



$$r_\parallel = \left(\frac{E_{0r}}{E_{0i}}\right)_\parallel = \frac{n_t \cos\theta_i - n_i \cos\theta_t}{n_t \cos\theta_i + n_i \cos\theta_t}$$

$$t_\parallel = \left(\frac{E_{0t}}{E_{0i}}\right)_\parallel = \frac{2 n_i \cos\theta_t}{n_t \cos\theta_i + n_i \cos\theta_t}$$

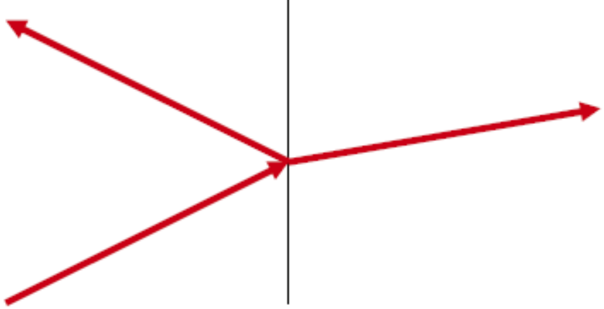
Reflection and Transmission of Energy @ dielectric interfaces

Recall Poynting vector definition:

$$\|\mathbf{S}\| = c\,\varepsilon_0 \|\mathbf{E}\|^2$$

different on the two sides of the interface

$$\frac{c_{\text{vacuum}}}{n_i} \qquad \frac{c_{\text{vacuum}}}{n_t}$$

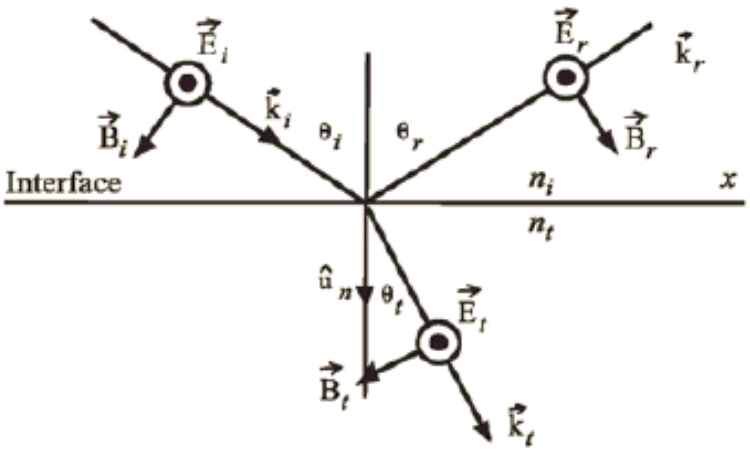


$$R = \left(\frac{E_{0r}}{E_{0i}}\right)^2 = r^2$$

$$T = \frac{n_t \cos\theta_t}{n_i \cos\theta_i}\left(\frac{E_{0t}}{E_{0i}}\right)^2 = \frac{n_t \cos\theta_t}{n_i \cos\theta_i} t^2$$

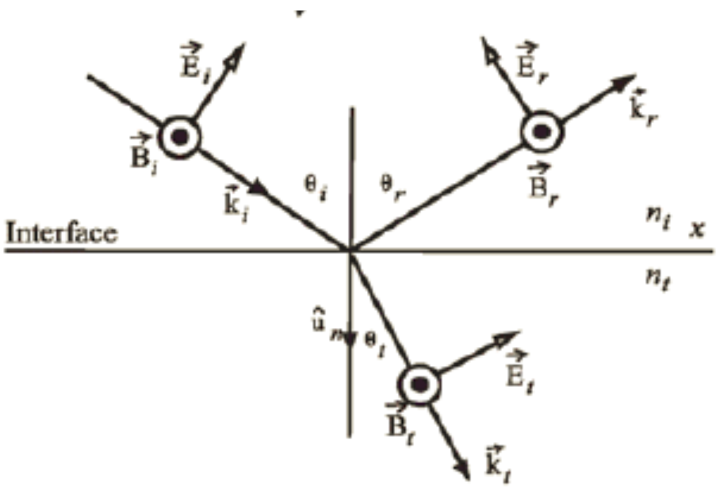
Reflection and Transmission (Fresnel's equations)

Can be deduced from the application of boundary conditions of EM waves.



$$r_\perp = \left(\frac{E_{0r}}{E_{0i}}\right)_\perp = \frac{n_i \cos\theta_i - n_t \cos\theta_t}{n_i \cos\theta_i + n_t \cos\theta_t}$$

$$t_\perp = \left(\frac{E_{0t}}{E_{0i}}\right)_\perp = \frac{2n_i \cos\theta_i}{n_i \cos\theta_i + n_t \cos\theta_t}$$



$$r_\parallel = \left(\frac{E_{0r}}{E_{0i}}\right)_\parallel = \frac{n_t \cos\theta_i - n_i \cos\theta_t}{n_t \cos\theta_i + n_i \cos\theta_t}$$

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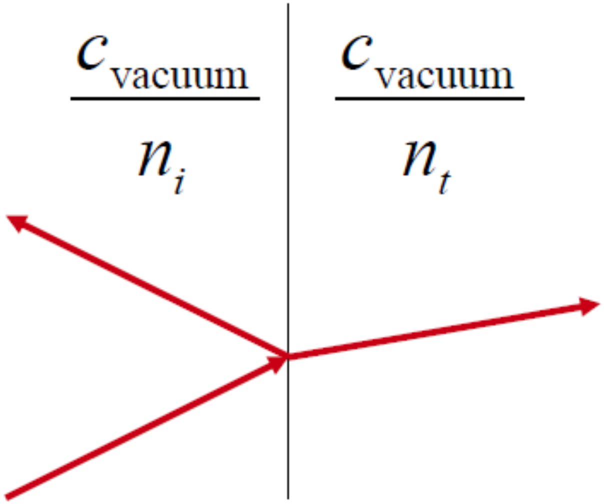
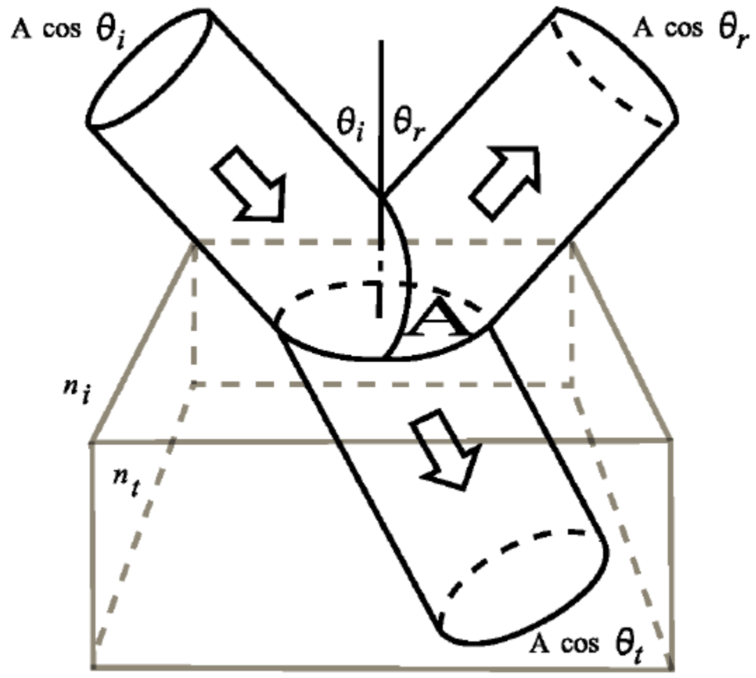


$$R = \left(\frac{E_{0r}}{E_{0i}}\right)^2 = r^2$$

$$T = \frac{n_t \cos\theta_t}{n_i \cos\theta_i}\left(\frac{E_{0t}}{E_{0i}}\right)^2 = \frac{n_t \cos\theta_t}{n_i \cos\theta_i} t^2$$

Energy Conservation

$$R + T = 1, \text{ i.e. } \quad r^2 + \frac{n_t \cos\theta_t}{n_i \cos\theta_i} t^2 = 1$$



Normal Incidence

$$r_\perp = \left(\frac{E_{0r}}{E_{0i}}\right)_\perp = \frac{n_i \cos\theta_i - n_t \cos\theta_t}{n_i \cos\theta_i + n_t \cos\theta_t}$$

$$t_\perp = \left(\frac{E_{0t}}{E_{0i}}\right)_\perp = \frac{2n_i \cos\theta_i}{n_i \cos\theta_i + n_t \cos\theta_t}$$

$$r_\parallel = \left(\frac{E_{0r}}{E_{0i}}\right)_\parallel = \frac{n_t \cos\theta_i - n_i \cos\theta_t}{n_t \cos\theta_i + n_i \cos\theta_t}$$

$$t_\parallel = \left(\frac{E_{0t}}{E_{0i}}\right)_\parallel = \frac{2n_i \cos\theta_t}{n_t \cos\theta_i + n_i \cos\theta_t}$$

Note: independent of polarization

$\theta_i = 0$ and $\theta_t = 0$ →

$$r_\perp = r_\parallel = \frac{n_t - n_i}{n_t + n_i}$$

$$t_\perp = t_\parallel = \frac{2n_i}{n_t + n_i}$$

$$R_\perp = R_\parallel = \left(\frac{n_t - n_i}{n_t + n_i}\right)^2$$

$$T_\perp = T_\parallel = \frac{4n_t n_i}{(n_t + n_i)^2}$$

Reflectance and Transmittance @ dielectric interfaces