PHY 491

HW Assignment #1, Sept 2-9, 2011

1. Using a hydrogenic model, estimate the 1st ionization energy of Li atom, assuming that the two electrons in the 1s state essentially screen the nuclear charge making the effective charge +1e. The actual value of this 1st ionization energy is 5.39 eV. Discuss possible physical reasons for this difference.

2. Calculate the 3rd ionization energy of the Li atom. Is your answer exact?

3. What is the probability of finding the 1s electron in Pb^{81+} inside the Pb nucleus? The nuclear radius $R = r_0 A^{1/3}$, where r_0=1.2 fermi and A is the atomic number of Pb.

4. Binding energies of excitons in quantum wells can be approximated by a 2-dimensional (2d) hydrogen atom model. Show that the radial part of the Schrodinger equation after separating the radial, R(r) and angular Y(θ) parts is given by (in atomic units)

$$\frac{1}{2}\left(R'' + \frac{1}{r}R'\right) - \frac{m^2}{2r^2}R + \left(E + \frac{1}{r}\right)R = 0$$

The angular part of the wave functions are $e^{im\theta}$, R' and R'' are the first and second derivatives of R with respect to r. Use the same scaling that was used for the 3d case in defining the variable $\rho = \kappa r;\ \kappa = \sqrt{-2E}$ to write down a second order differential equation for $R(\rho)$ in terms of the parameter $\rho_0 = \frac{2}{\kappa}$

How does $R(\rho)$ behave as ρ → 0 and ∞? Define a function $v(\varrho)$ following the same procedure as for the 3d case. Solve this equation for physical solutions and find the spectrum of the 2d hydrogen atom.