PHY 491
Home Work Assignment #4
September 26-October 3, 2011

4.1
Consider an atom whose ground state is $^3S_1$. What is the value of Lande g-factor? Find the magnetization $\boldsymbol{M}$ as a function of magnetic field B (oriented along the z axis), temperature T, and concentration $n = N/V$. Show that in the limit of very high temperature $\mu_B B << k_B T$, the susceptibility is given by $\chi = 8n\mu_B^2 / 3k_B T$.

4.2
An exotic proposal to get nuclear fusion between two deuterons is to to use the idea of muon catalysis. One constructs a "Hydrogen molecule ion", only with deuterons instead of protons and a muon in place of an electron. Use your knowledge of the H_2^+ ion to predict the equilibrium separation between the deuterons in the muonic molecule. Explain why the chance of getting fusion is better for muons rather than electrons.

4.3
The schrodinger equation for one electron in an attractive one-dimensional delta-function potential of the form $V(x) = -e^2\delta(x)$ is

$$-\frac{\hbar^2}{2m}\frac{d^2\psi(x)}{dx^2} - e^2\delta(x)\psi(x) = E\psi(x)$$

In atomic unit $\left(\hbar = m = e^2 = 1\right)$, the normalized ground state wave function is
$\psi_1(x) = e^{-|x|}$, with energy $E_1 = -1/2$

(i) Check that the above wave function and energy are correct.

(ii) Consider a one-dimensional H_2 molecule with a $\delta-$like both ion-electron (as above) and repulsive electron-electron interaction. The ions are fixed at a distance R, and neglect ion-ion repulsion.

(a) Write down the Schrodinger equation for this one-dimensional H_2 molecule.
(b) Construct a *gerade* molecular orbital (MO) for this molecule with correct normalization coefficient
(c) Calculate the ground state energy for the molecule using this MO.
(d) Construct a Heitler-London (HL) wave function for the molecule and calculate the energy.
(e) Compare the energies obtained using the two approaches and discuss the physics.
Use:

$$\int_{-\infty}^{+\infty} e^{-|x-R/2|} e^{-|x+R/2|} dx = (1+R)e^{-R}$$

$$\int_{-\infty}^{+\infty} e^{-3|x-R/2|} e^{-|x+R/2|} dx = (3e^{-R} - e^{-3R})/4$$

$$(a+b)^4 = a^4 + 4a^3b + 6a^2b^2 + 4ab^3 + b^4$$