PHY 491, Homework 6
October 17-22, 2011

Problem 6.1

Calculate the density of states of electron gas in 2 and 1 dimensions. Derive expressions for the Fermi energy in atomic units, where the energy is expressed in Hartree and the length is expressed in Bohr radius. For a 2-d electron gas the density is 1.5 x 10^{11} cm^{-2}. Express this density in atomic units. What is the Fermi energy for this 2d electron gas?

$$2-Dimesnion\ (2d)$$

$$D(\varepsilon)d\varepsilon = 2\bullet\frac{2\pi\, k\, dk}{\left(\frac{2\pi}{L}\right)^2}$$

$$\varepsilon = \varepsilon_{\vec{k}} = \frac{\hbar^2 k^2}{2m} \Rightarrow k\, dk = \frac{m}{\hbar^2}d\varepsilon$$

$$D(\varepsilon) = \frac{A}{2\pi}\left(\frac{2m}{\hbar^2}\right); independent\ of\ \varepsilon$$

$$1-Dimension\ (1d)$$

$$D(\varepsilon)d\varepsilon = 2\bullet\frac{2\,dk}{\left(\frac{2\pi}{L}\right)};\ 2\ for\ spin\ and\ 2\ for\ k\ and\ -k$$

$$k = \sqrt{\frac{2m}{\hbar^2}}\varepsilon^{1/2};\ dk = \frac{1}{2}\sqrt{\frac{2m}{\hbar^2}}\varepsilon^{-1/2}d\varepsilon$$

$$D(\varepsilon) = \frac{L}{\pi}\left(\frac{2m}{\hbar^2}\right)^{1/2}\varepsilon^{-1/2}$$

Using the above density of states calculate the Fermi energies in 1d and 2d. Now express the Fermi energy in Hartree (H) and length in Bohr radius (a_B).

(b)

$$2d:\ \bar{\varepsilon}_F = N\pi\frac{1}{\bar{A}};\ where\ \ \bar{\varepsilon}_F = \frac{\varepsilon_F}{H}\ and\ \bar{A} = \frac{A}{a_B^2}$$

$$1d:\ \bar{\varepsilon}_F = \frac{\pi^2}{2}\left(\frac{N}{\bar{L}}\right)^2;\ where\ \ \bar{\varepsilon}_F = \frac{\varepsilon_F}{H}\ and\ \bar{L} = \frac{L}{a_B}$$

(c)

$$\frac{N}{A} = 1.5 \bullet 10^{11} \frac{1}{cm^2};$$
$$\frac{N}{\overline{A}} = \frac{N}{A/a_B^2} = \frac{N}{A} a_B^2 = 1.5 \bullet 10^{11} \frac{1}{cm^2} \bullet \left(0.521 \bullet 10^{-8} cm\right)^2 = 0.4 \bullet 10^{-5}$$
$$\overline{\varepsilon}_F = \pi \frac{N}{\overline{A}} = \pi \bullet 0.4 \bullet 10^{-5}$$
$$\varepsilon_F = \pi \bullet 0.4 \bullet 10^{-5} \bullet H = \pi \bullet 0.4 \bullet 10^{-5} \bullet 27.2 eV = 34.2 \bullet 10^{-5} eV$$

Problem 6.2
The atom He^3 is a fermion with spin ½ (Why?). The density of He^3 liquid is 0.081 gm/cm^3 near T=0. Calculate the Fermi energy ε_F and the Fermi temperature T_F.

$$Mass\,density\ \rho = \frac{N}{V} M_{He^3} = 0.081\, gm/cm^3$$
$$Number\ density\ \frac{N}{V} = \frac{\rho}{M_{He^3}} = \frac{0.081\, gm/cm^3}{3x1.66x10^{-24}\, gm} = 1.626x10^{22} 1/cm^3 = 1.626x10^{28} 1/m^3$$
$$\varepsilon_F = \frac{\hbar^2}{2M_{He^3}} \left(3\pi^2 \frac{N}{V}\right)^{2/3} = 6.78x10^{-23} J = 4.24x10^{-4} eV$$
$$T_F = 4.24x10^{-4} x1.16x10^{+4} K = 4.91K$$

Problem 6.3
Assuming a free electron gas model for the valence electrons for the following metals, calculate the Fermi energy (in eV) and the zero point pressure (in Atmospheric pressure). Use Table 4 on page 24 of Kittel.

Li, Na, Cs, Cu, Mg, Al, In, Pb

This problem you can do easily. Make sure you do a couple of them (Na, Mg, In).