PHY 491, Homework 7
October 31-November 4, 2011

Problem 7.1

Use the following equation for the drift velocity of an electron in a constant electric field oscillating with a frequency ω

$$m\frac{dv}{dt}+\frac{v}{\tau}=-eE(t)$$

to show that the frequency dependent conductivity (ac conductivity) is given by

$$\sigma(\omega)=\sigma(0)\frac{1+\omega\tau}{1+(\omega\tau)^2}$$

where $\sigma(0)=\frac{ne^2\tau}{m}$ and n is the density of electrons.

Problem 7.2

Consider a square lattice with lattice constant a. Draw the 1^{st} BZ and give the coordinates of it's symmetry points (here use 4-fold rotation about the z-axis)

Now consider the states of an electron whose $\vec{k}$ vector corresponds to one of the corner points of the BZ.

How many plane waves have the same kinetic energy as the one corresponding to this $\vec{k}$? What are their $\vec{k}$ vectors?

(We will call these as degenerate set of plane waves, because in the absence of crystal potential these plane waves have the same kinetic energy)

Construct two stationary wave functions from the above degenerate set such that for one $[\psi_1(x,y)]$ the probability of finding the electron at the lattice sites is maximum and for the other $[\psi_2(x,y)]$ it is minimum.

Problem 7.3

Now consider the case when there is a periodic potential in this square lattice given by

$$U(x,y)=-U_0\left[\cos\left(\frac{2\pi}{a}x\right)+\cos\left(\frac{2\pi}{a}y\right)\right]$$

What is the energy gap between the two states $[\psi_1(x,y)]$ and $[\psi_2(x,y)]$ given in Problem 7.2. Explain your result physically by matching the probability distribution of the Electron in these two states and the attractive regions of the potential.