Physics 842 – Fall 2011  
Classical Electrodynamics II  

Problem Set #10 – due Tuesday December 6

1. Debye Relaxation: A simple model for the dielectric relaxation of polar molecules in a gas, liquid, or solid was given by Peter Debye nearly 100 years ago (1912). We divide the response of the molecules into a “fast” part describing the electronic response of the molecules, and a “slow” part describing the hindered molecular rotations. The response function \( g(t) \), defined by

\[
\mathcal{P}(t) = \int_{0}^{\infty} g(t') \mathcal{E}(t-t') dt'
\]

has the approximate form:

\[
g(t) = \chi_{\text{fast}} \delta(t) + \frac{\chi_{\text{slow}}}{\tau_D} e^{-t/\tau_D}
\]

where \( \tau_D \) is the typical time needed for a molecule to re-orient in the applied electric field. (You calculated \( \chi_{\text{slow}} \) for a system of magnetic dipoles, using statistical mechanics, in problem 2 of Problem Set #6. The answer is the same for electric dipoles, and is \( \chi_{\text{slow}} = \frac{np_0^2}{3k_B T} \), where \( p_0 \) is the magnitude of the permanent molecular dipole moment.)

a) Calculate the dielectric susceptibility, \( \varepsilon(\omega) \). Express \( \varepsilon(\omega) \) in terms of its value at very high frequency, \( \varepsilon_{\infty} \equiv 1 + 4\pi\chi_{\text{fast}} \), and at zero frequency, \( \varepsilon_0 \equiv 1 + 4\pi(\chi_{\text{fast}} + \chi_{\text{slow}}) \).

b) Plot \( \varepsilon' \) and \( \varepsilon'' \) vs. \( \omega \tau_D \), first on a linear frequency plot, and then on a log frequency plot. You are welcome to use Mathematica to make the plots, or you can do them by hand.

2. Kramers-Kronig relations: (This problem is taken from Jackson, Chapter 7.)

a) The imaginary part of the dielectric susceptibility for a material is given by:

\[
\varepsilon''(\omega) = \lambda \left[ \theta(\omega - \omega_1) - \theta(\omega - \omega_2) \right]
\]

with \( 0 < \omega_1 < \omega_2 \), where \( \theta(x) \) is the Heaviside step function. Calculate the real part of the susceptibility, \( \varepsilon'(\omega) \), from the Kramers-Kronig relations, given in equation (82.8) of Landau & Lifshitz.

b) Do the same thing for this case:

\[
\varepsilon''(\omega) = \frac{\lambda \gamma \omega}{(\omega_0^2 - \omega^2)^2 + \gamma^2 \omega^2}
\]

c) For both parts (a) and (b), make rough plots of \( \varepsilon'(\omega) \) and \( \varepsilon''(\omega) \) vs. \( \omega \).

Quiz #10

The quiz on Thursday, December 8, will consist of one of the above problems.