Physics 842 – Fall 2011

Classical Electrodynamics II

Problem Set #7 – due Thursday November 3

1. A right circular cylinder of radius R and length L is a uniformly-magnetized along its axis.

   a) Last week you calculated $\vec{B}$ along the axis of the cylinder (both inside and outside) using the surface current density J_S. This week I want you to calculate $\vec{B}$ along the axis using the magnetic scalar potential and the fictitious magnetic charge density $\rho_m = -\vec{\nabla} \bullet \vec{M}$. Solve the Poisson equation $\nabla^2 \phi_m = -4\pi\rho_m$ the same way you would do if this were an electrostatics problem:

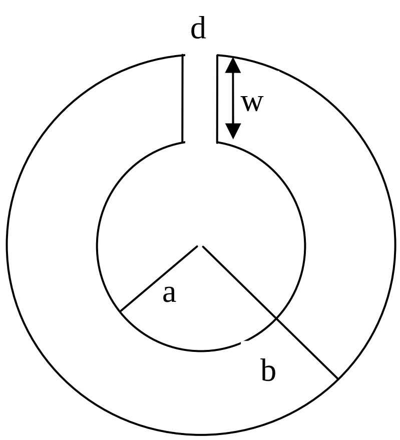
   $$\phi_m(\vec{r}) = \int \frac{\rho_m(\vec{r}\,')}{|\vec{r} - \vec{r}\,'|} d^3 r'$$

   where the magnetic charge is actually a surface charge density on the two ends of the cylinder: $\sigma_m = \vec{M} \bullet \hat{n}$. Compare your answer with what you obtained last week using the surface current density. Hint: Be careful with absolute values when you take square roots.

2. Find $\vec{B}$ everywhere in space for a uniformly magnetized sphere. For this problem, I suggest you also use ϕ_m, but solve the problem using Legendre polynomials and the correct boundary conditions for $\vec{B}$ and $\vec{H}$ at the surface of the sphere.

3. A toroidal electromagnet consists of a soft iron core of permeability μ >> 1 with N turns of copper wire wound around it, carrying current I. The gap between the pole pieces is d, the width of the pole pieces is w, and the average radius of the toroid is $R=(a+b)/2$.

   a) Assuming that $d \ll w \ll R$, calculate the magnetic field B in the gap. Use $\vec{\nabla} \times \vec{H} = \frac{4\pi}{c}\vec{j}$ and the boundary condition on $B_\perp$ at the pole pieces.



   b) Simplify your expression in the limit μ >> R/d.

   Note that this problem can be solved in two lines once you've figured out how to set it up.

(over)

4. Consider a spherical shell of material with magnetic permeability μ, with inner and outer radii *a* and *b*, respectively, placed in a uniform external magnetic field $\vec{B}_0$.

   a) Calculate $\vec{B}$ everywhere in space. (This shouldn't be too difficult, because you already did the dielectric equivalent of this problem on Problem Set #3.)

   b) Simplify your expression for the field inside the cavity in the limit $\mu >> 1$. You should find that the field is proportional to $1/\mu$. This is how magnetic shielding works. The field lines are concentrated inside the magnetic material, so very little field leaks into the cavity.

Quiz #7

The quiz on Tuesday, November 8, will consist of one of the following problems:

- Problems 1 - 4 on Problem Set #7
- Problems 1 - 4 at the end of Section 30 (problem 2 won't be on the quiz, but I want you to look at it.)