1. A right circular cylinder of radius \( R \) and length \( L \) is a uniformly-magnetized along its axis.
   
a) Last week you calculated \( \mathbf{B} \) along the axis of the cylinder (both inside and outside) using the surface current density \( J_s \). This week I want you to calculate \( \mathbf{B} \) along the axis using the magnetic scalar potential and the fictitious magnetic charge density \( \rho_m = -\mathbf{\nabla} \cdot \mathbf{M} \). Solve the Poisson equation \( \nabla^2 \phi_m = -4\pi \rho_m \) the same way you would do if this were an electrostatics problem:
   \[
   \phi_m(\mathbf{r}) = \int \frac{\rho_m(\mathbf{r}')}{|\mathbf{r} - \mathbf{r}'|} d^3r'
   \]
   where the magnetic charge is actually a surface charge density on the two ends of the cylinder: \( \sigma_m = \mathbf{M} \cdot \mathbf{n} \). Compare your answer with what you obtained last week using the surface current density. Hint: Be careful with absolute values when you take square roots.

2. Find \( \mathbf{B} \) everywhere in space for a uniformly magnetized sphere. For this problem, I suggest you also use \( \phi_m \), but solve the problem using Legendre polynomials and the correct boundary conditions for \( \mathbf{B} \) and \( \mathbf{H} \) at the surface of the sphere.

3. A toroidal electromagnet consists of a soft iron core of permeability \( \mu >> 1 \) with \( N \) turns of copper wire wound around it, carrying current \( I \). The gap between the pole pieces is \( d \), the width of the pole pieces is \( w \), and the average radius of the toroid is \( R=(a+b)/2 \).
   
a) Assuming that \( d << w << R \), calculate the magnetic field \( \mathbf{B} \) in the gap. Use \( \mathbf{\nabla} \times \mathbf{H} = \frac{4\pi}{c} \mathbf{j} \) and the boundary condition on \( B_\perp \) at the pole pieces.
   
   b) Simplify your expression in the limit \( \mu >> R/d \).

   Note that this problem can be solved in two lines once you’ve figured out how to set it up.

(over)
4. Consider a spherical shell of material with magnetic permeability $\mu$, with inner and outer radii $a$ and $b$, respectively, placed in a uniform external magnetic field $\vec{B}_0$.

a) Calculate $\vec{B}$ everywhere in space. (This shouldn’t be too difficult, because you already did the dielectric equivalent of this problem on Problem Set #3.)

b) Simplify your expression for the field inside the cavity in the limit $\mu >> 1$. You should find that the field is proportional to $1/\mu$. This is how magnetic shielding works. The field lines are concentrated inside the magnetic material, so very little field leaks into the cavity.

Quiz #7

The quiz on Tuesday, November 8, will consist of one of the following problems:

- Problems 1 - 4 on Problem Set #7
- Problems 1 - 4 at the end of Section 30 (problem 2 won’t be on the quiz, but I want you to look at it.)