Physics 842 – Fall 2011
Classical Electrodynamics II

Problem Set #8 – due Tuesday November 15

1. The magnetization of a ferromagnet is proportional to the total angular momentum of the electrons (spin and orbital). $\vec{m} = -\gamma\vec{J}$. If we apply a magnetic field $\vec{H}$, there will be a torque on the magnetic moment. Averaging over all the individual moments to get the macroscopic limit, we obtain the Landau-Lifshitz equation:
$$\frac{d\vec{M}}{dt} = -\gamma\vec{M} \times \vec{H}$$
Find the time evolution of $\vec{M}$ in a ferromagnet placed in a constant field $\vec{H}_0$. Hint: You should find oscillations at the Larmor frequency, $\omega_L = \gamma H_0$.

2. Consider an infinite ferromagnet in a uniform constant field $\vec{H}_0$ along the z-axis, and an additional weak field $\vec{h}\cos(\omega t)$ along the x-axis. Calculate the steady-state small oscillations of the magnetization $\vec{M}$ at frequency ω using the Landau-Lifshitz equation, to first order in $\vec{h}$. (Do <u>not</u> include the solution to the homogeneous differential equation – oscillations at the Larmor frequency -- which you found in problem 1.) Hints: Write the oscillating field as $\vec{h}\left(e^{i\omega t} + e^{-i\omega t}\right)/2$ and just treat the first term. Then write the total magnetization as $\vec{M} = \vec{M}_0 + \vec{m}e^{i\omega t}$, plug in and solve for $\vec{m}$. Notice that your solution diverges on resonance; that is because we have not included the damping terms in the Landau-Lifshitz equation.

3. Complete our class discussion of magnetization reversal for a ferromagnet with uniaxial anisotropy: Describe the evolution of the energy density *u* as a function of the angle θ between the magnetization $\vec{M}$ and the easy axis $\hat{z}$ as the magnetic field varies from large positive H_z to large negative H_z, with $0 < H_x < \beta M$. (See Figure 21 in Section 41 of Landau and Lifshitz. Note that L&L utilize the thermodynamic potential $\tilde{\Phi}$ rather than the energy density.) Make plots of *u* vs θ for several values of H_z; then make a plot of M_z vs. H_z. Explain what happens when H_z crosses into the interior of the astroid coming from large positive H_z, and when it leaves the astroid going toward large negative H_z.

Quiz #8

The quiz on Thursday, November 17, will consist of one of the following problems:

- Problems 1 - 3 on Problem Set #8

- Problems 4, 5, and 7 at the end of Section 34. (Don’t worry about the integrals in problem 4 – they are not easy!)