Physics 842 – Fall 2011
Classical Electrodynamics II

Problem Set #9 – due Tuesday November 29 (after Thanksgiving)

1. Critical dimension of single-domain magnetic particles. For this problem, you will need to first convert some of the equations used in class (and in Landau & Lifshitz) to SI units. In those units, $\vec{B} = \mu_0\left(\vec{H} + \vec{M}\right)$, and the magnetostatic energy is $U = \frac{-\mu_0}{2}\int \vec{M} \bullet \vec{H}\, dV$.

   The saturation magnetization of Ni is $M_0 = 5.1 \times 10^5$ A/m (in SI units). Calculate the magnetostatic energy of a uniformly-magnetized Ni sphere of radius R. Now assume that the magnetostatic energy will be reduced by half if the particle forms two hemispherical domains. Calculate the energy of the domain wall, given that its surface tension is $\gamma = 0.7 \times 10^{-3}$ J/m^2. From these calculations, what is the maximum size of a spherical Ni particle where the single-domain state is energetically favorable over the two-domain state.

   To check your answer, convert both M_0 and γ to cgs units and repeat the calculation in those units.

2. In Section 43 of Landau & Lifshitz and in class, the energy due to non-uniform magnetization is written in terms of the macroscopic magnetization in the form:
$$U_{non-uniform} = \frac{\alpha}{2}\left(\left|\nabla M_x\right|^2 + \left|\nabla M_y\right|^2 + \left|\nabla M_z\right|^2\right) \qquad (1)$$
   In class, I motivated this expression by writing down a microscopic model for the exchange interaction between nearby atomic spins:
$$H_{exchange} = -\sum_{i\neq j} J_{ij}\vec{S}_i \bullet \vec{S}_j = -\sum_{i\neq j} J_{ij} S^2 \cos\left(\theta_{ij}\right) \qquad (2)$$
   where S is a dimensionless spin variable. Assume that $S = 1$, that we have a simple cubic lattice and that J_{ij} is nonzero only for nearest-neighbor atoms. To show the equivalence of these two expressions, consider a Bloch domain wall, where the magnetization rotates by π in the plane perpendicular to the direction of magnetization change. In other words:
$$M_z(x) = M_0\cos(qx) \qquad\qquad M_y(x) = M_0\sin(qx)$$
   with $qa << 1$, where a is the lattice constant. Calculate the energy per unit area of the domain wall using both models. By comparing your results, you should come up with an expression for α in terms of J_{ij}, M_0, and a. Hint: Consider the angle of rotation between adjacent planes of atoms in the domain wall: $\theta = qa$,. Expand the cosine in Eqn (2) for very small θ. Remember to consider only the difference in exchange energy between the situation in the domain wall and the situation with uniform magnetization. At the end, check that your expression for α has the right units.

3. Stoner-Wohlfarth model of magnetization switching in single-domain particles. (This problem has some similarity with problem 3 on the previous Homework, but that is OK.)

   Consider a magnetic particle in the shape of a prolate ellipsoid with a = b << c, initially magnetized along its +c axis.) Assume that the particle is too small to form domains, and that there is no magnetocrystalline anisotropy. In that case, the only two terms in the energy are the magnetostatic energy (also known as shape anisotropy) and the dipole energy in the external magnetic field. You can express your answers in terms of the demagnetizing factors, n_c and $n_a = n_b$.

   a) The particle is aligned with its +c axis along the +z axis, and we apply an external magnetic field first in the -z direction to switch the magnetization, then in the +z direction to switch it back. Make a plot of M_z vs. H. To do this, find the minima in the total energy as you change H, just as you did last week.

   b) Now the particle is aligned with its c axis along the x axis, and we do the same experiment with H along the z axis. Make a plot of M_z vs. H in this case.

   c) Finally, the particle is oriented with its c-axis at some angle between the x and z axes. (You can choose $\theta = 45$ degrees if you want to be precise.) We do the same experiment, always with H along the z axis. Make a plot of M_z vs. H in this case.

   If you had an ensemble of particles oriented in random directions, your overall plot of M vs. H would be a superposition of these plots for individual particles. This is one way to get the interesting shapes one finds in plots of M vs. H for real ferromagnetic systems.

4. Eddy Current Heating: In class we discussed a semi-infinite conductor with conductivity σ filling the half-space z > 0. Outside the conductor, the magnetic field varies in time as $\vec{H}(t) = \mathrm{Re}\{\vec{H}_0 e^{-i\omega t}\}$. Inside the conductor, the field has the form $\vec{H}(z,t) = \mathrm{Re}\{\vec{H}_0 e^{-z/\delta} e^{iz/\delta} e^{-i\omega t}\}$, where $\delta = c/\sqrt{2\pi\sigma\omega}$ is the skin depth.

   a) Choose $\vec{H}_0 = H_0\hat{y}$ for convenience. Calculate $\vec{E}(z,t)$ inside the conductor using the Maxwell equation $\vec{\nabla} \times \vec{E} = -\frac{1}{c}\frac{\partial \vec{B}}{\partial t}$.

   b) Calculate the Joule heat dissipated inside the conductor, per unit area of the surface, from the equation: $Q = \int \vec{j} \bullet \vec{E} dV = \int \sigma E^2 dV$. Note that Landau & Lifshitz calculate Q a different way, so please do it this way to see if you get the same answer: equation (59.10).

Quiz #9

The quiz on Thursday, December 1, will consist of one of the above problems.