Source: “Optical Physics”, Lipson^3

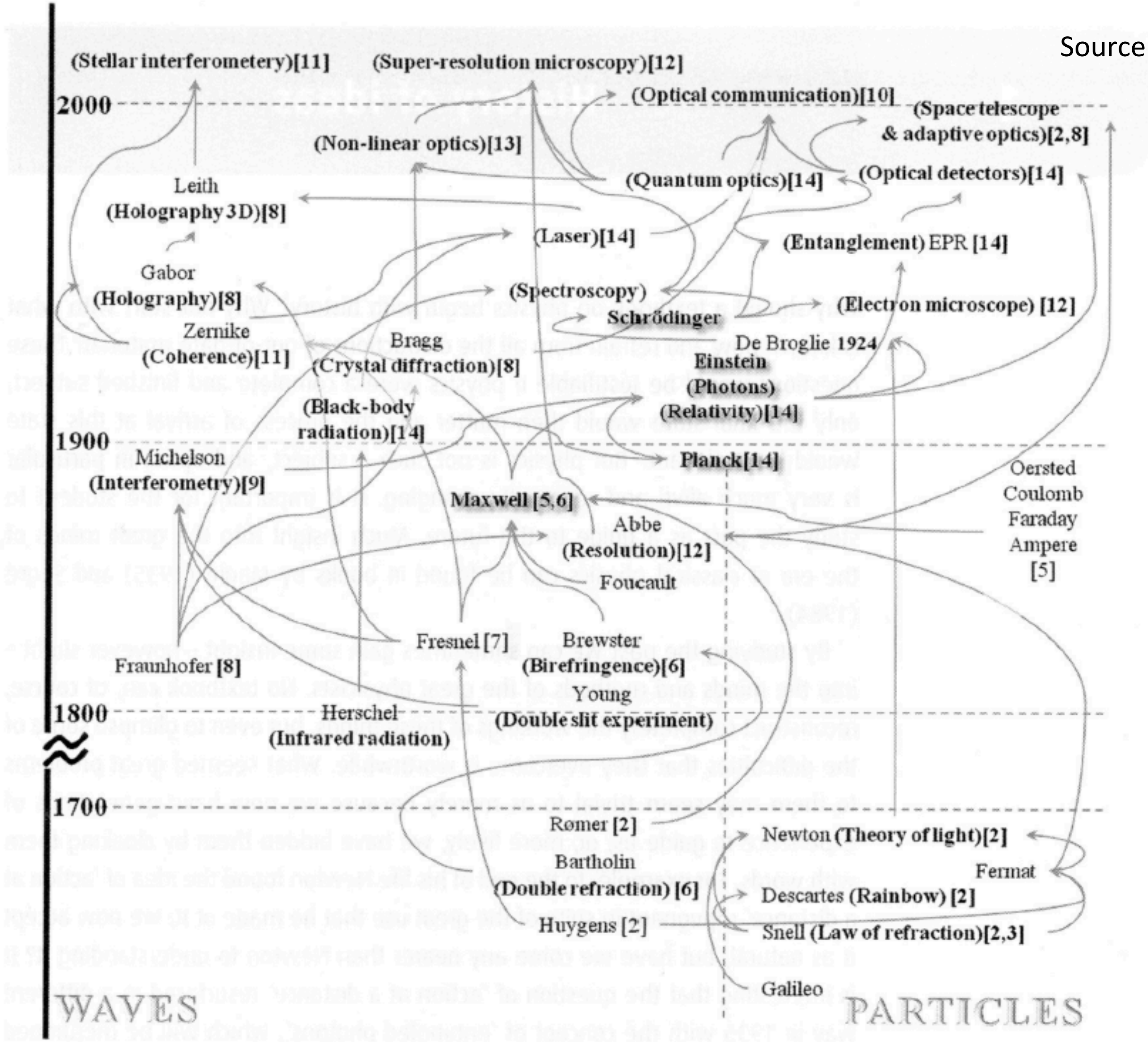
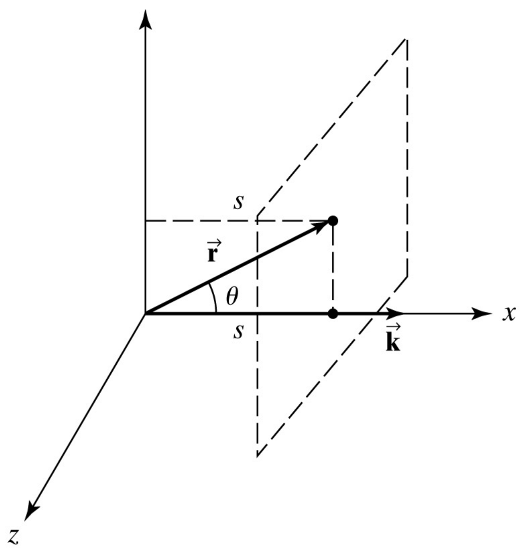


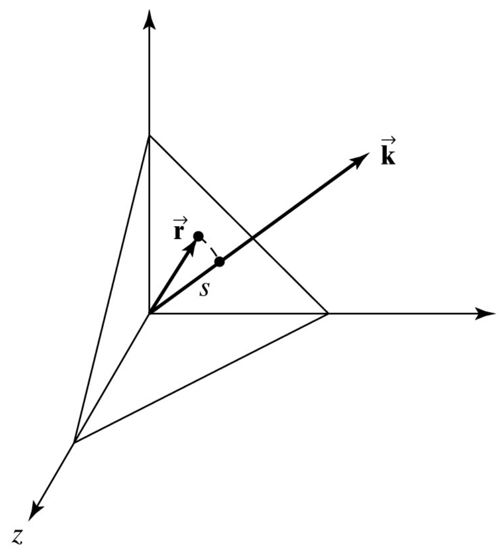
Figure 1.1 The development of optics, showing many of the interactions. Notice that there was little development in the eighteenth century, mainly because of Newton’s erroneous idea of light particles. The numbers in square brackets indicate the chapters where the topics are discussed.

Wave Number, Wave Vector, and Momentum

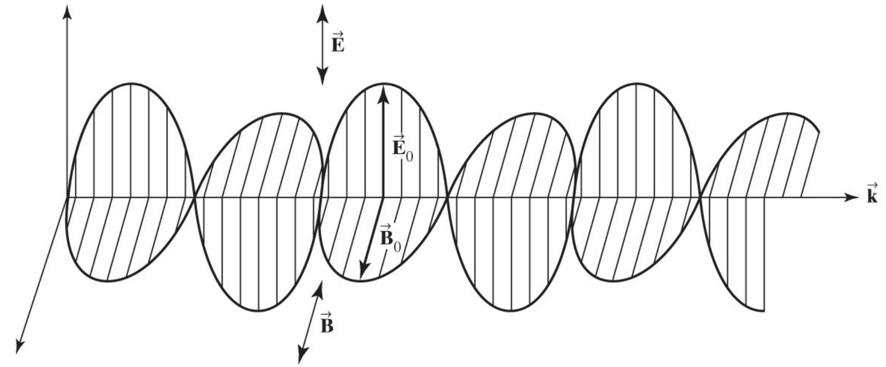
Chapter 4, Pedrotti^3)



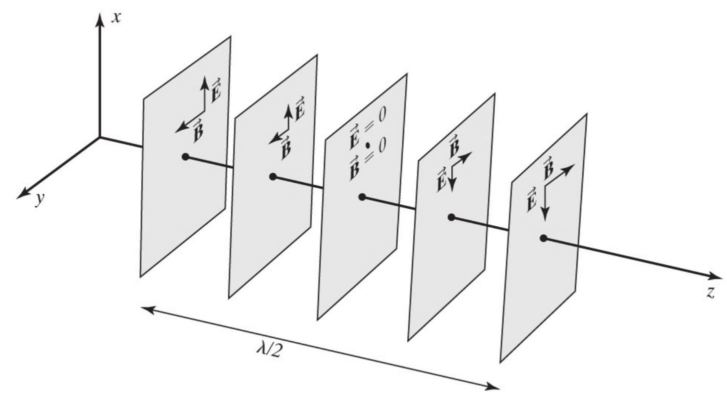
(a)



(b)



(a)



(b)

Index of refraction and 'speed' of light

The speed of light in vacuum is a physical constant.

$c = 299\ 792\ 458$ m/s (exact) ~ $3\text{x}10^8$ m/s

In a medium, light *generally* propagates more slowly.

- in air: $v = c/1.0003$ $n_{air} = 1.00$
- in water: $v = c/1.33$ $n_{water} = 1.33$
- in glass: $v = c/1.52$ $n_{glass} = 1.52$

In general:

$v = c/n$ is the "***phase velocity***"

wavelength/n

frequency is the same (in linear optics)

n also depends on the wavelength ➔ **dispersion**.

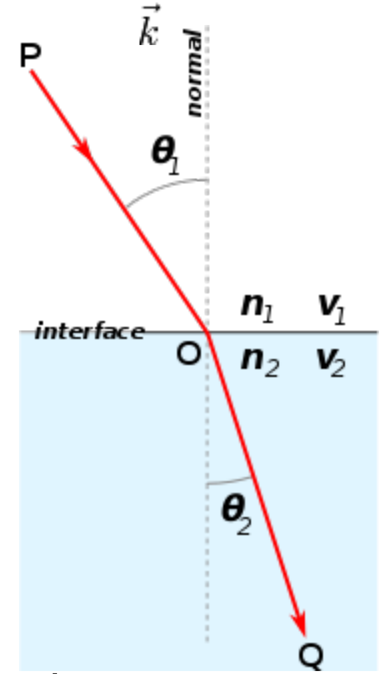
Snell's Law

$$n_1 \sin\theta_1 = n_2 \sin\theta_2 \ .$$

$$\frac{\sin\theta_1}{\sin\theta_2} = \frac{v_1}{v_2} = \frac{n_2}{n_1}$$

$$\frac{\sin\theta_1}{\sin\theta_2} = \frac{v_1}{v_2} = \frac{\lambda_1}{\lambda_2}$$

Frequency is the same.



1. Huygens' principle
2. Fermat's principle
3. Interference of all possible paths of light wave from source to observer

— it results in destructive interference everywhere except extrema of phase (where interference is constructive)—which become actual paths.

4. Application of the general **boundary conditions of Maxwell equations** for electromagnetic radiation. ➔ amplitude of reflected and refractive waves [Chapter 23]

5. Conservation of momentum based on translation symmetry considerations

Derive Snell's Law by Translation Symmetry

A *homogeneous* surface can not change the transverse momentum.
The propagation vector is proportional to the photon's momentum.

$$E = pc$$

$$p = \hbar k$$

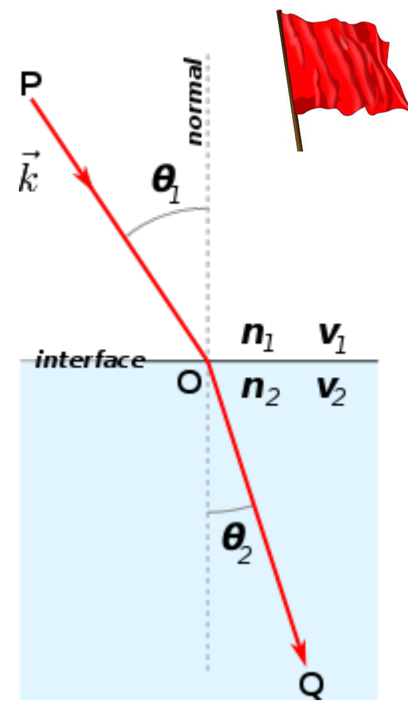
The transverse wave number must remain the same.

$$\vec{k}_1 \bullet \hat{x} = \vec{k}' \bullet \hat{x} = \vec{k}_2 \bullet \hat{x}$$

$$k_1 \sin\theta_1 = k' \sin\theta' = k_2 \sin\theta_2$$

$$k_1 = k' = \frac{2\pi}{\lambda_1} = \frac{2\pi}{\lambda_0 / n_1} = \frac{2\pi}{\lambda_0} n_1 = k_0 n_1$$

$$k_2 = \frac{2\pi}{\lambda_2} = \frac{2\pi}{\lambda_0} n_2$$

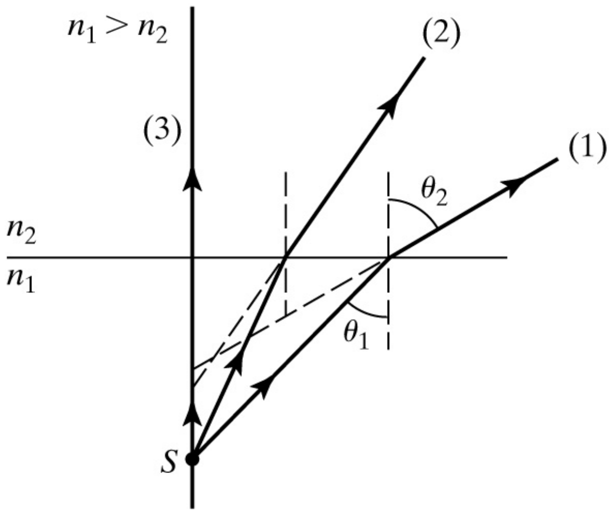


$$\hat{x}$$

$n_1 k_0 \sin\theta_1 = n_2 k_0 \sin\theta_2$
$n_1 \sin\theta_1 = n_2 \sin\theta_2$

$$k_0 = \frac{2\pi}{\lambda_0} = \frac{\omega}{c}$$

Refraction through Plane Surfaces

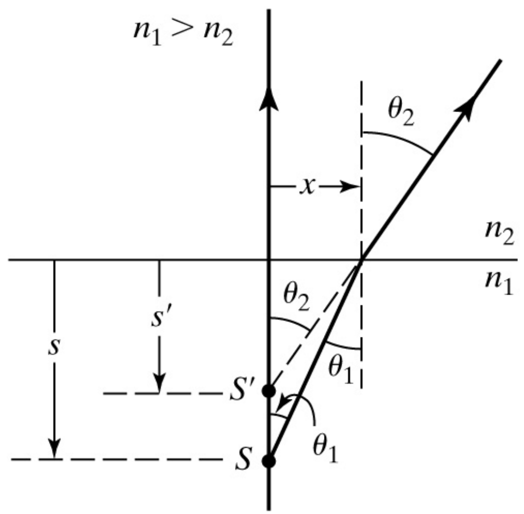


(a)

$n_1>n_2$: The refracted rays bend away from the normal.

$n_1<n_2$: The refracted rays bend toward the normal

A source point S below an interface emerge into an upper medium of lower refractive index.
➔ No unique image point is determined. These rays have no common intersection of virtual image point below the surface.



(b)

Small angle approximation, i.e. consider only
Paraxial Rays (making small angles with the optical axis)

$$\sin\theta \cong \tan\theta \cong \theta \quad (\text{in radians})$$

Snell's law can be approximated by

$$n_1 \tan\theta_1 \cong n_2 \tan\theta_2$$

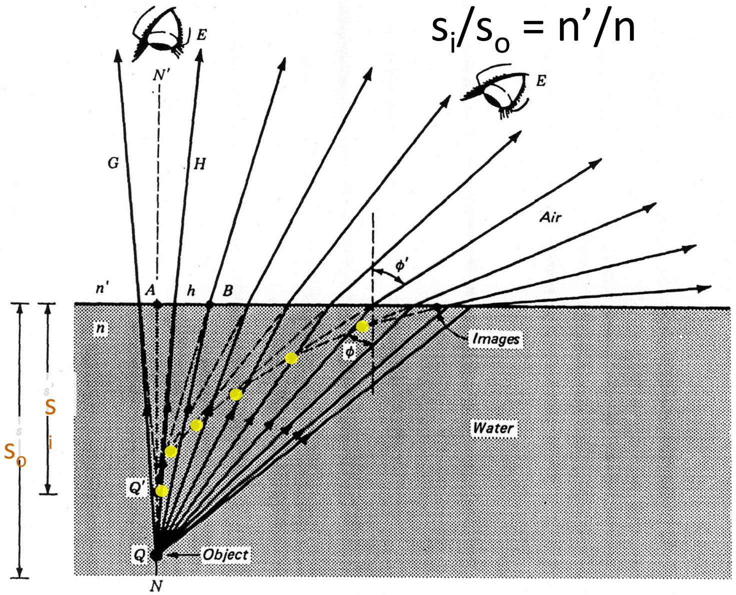
$$n_1\left(\frac{x}{s}\right) = n_2\left(\frac{x}{s'}\right)$$

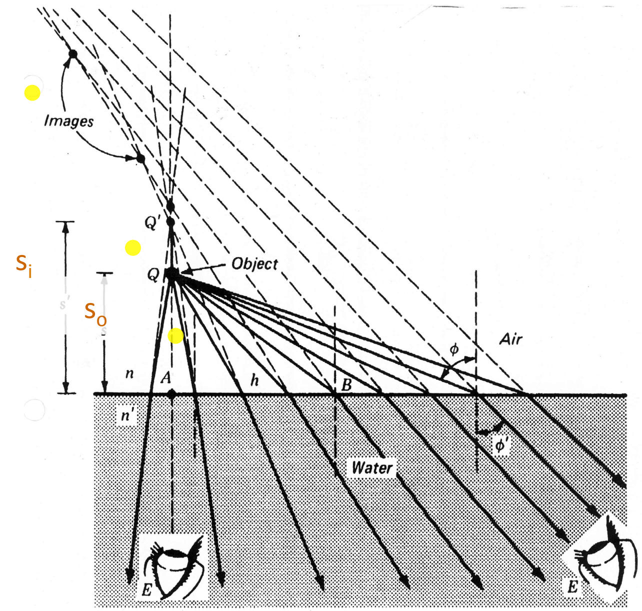
s': the image distance
s : the depth of the object

$$s' = \left(\frac{n_2}{n_1}\right)s \quad \text{for a small viewing angle } \theta_2\text{; vary with the angle of viewing}$$

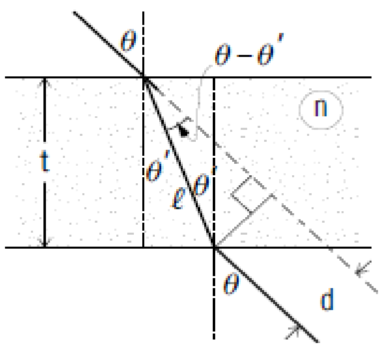
The fish problem …

$s_i/s_o = n'/n$



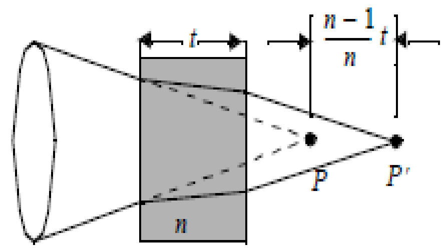


Displacement by a glass plate



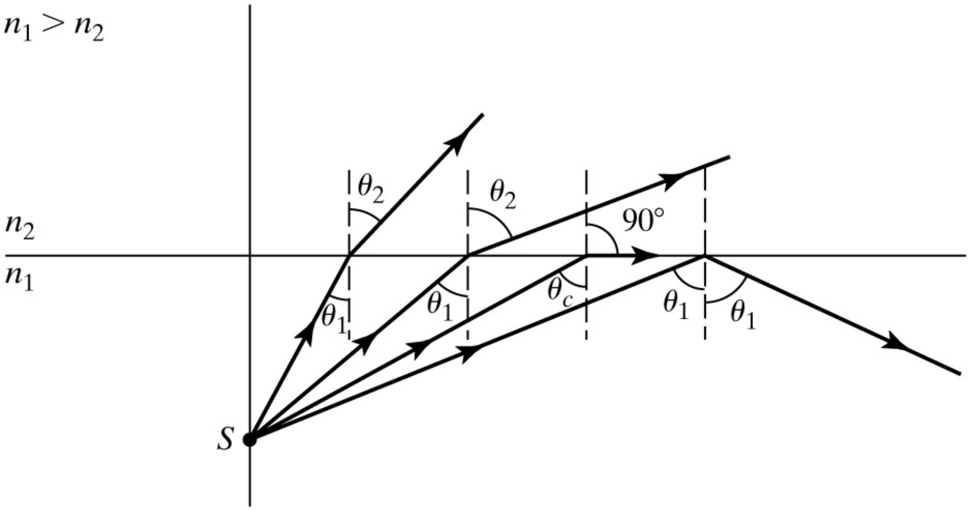
$$d = \frac{t\sin\left(\theta - \theta'\right)}{\cos\left(\theta'\right)}$$

Plane parallel plate placed in between a lens and its focus:



A simple calculation based on the paraxial approximation shows that the focus is displaced by amount $\boxed{\frac{n-1}{n}t}$. However, at steeper incidence angles, the focal shift becomes a function of the incidence angle, which leads to spherical aberration.

Total Internal Reflection



(c)

The critical angle θ_{crit} is the value of θ_1 for which θ_2 equals 90°:

Example : Water n_2 =1.33 (=4/3)
Air n_1=1.00

$$\theta_{\rm crit} = \arcsin\left(\frac{n_2}{n_1}\sin\theta_2\right) = \arcsin\frac{n_2}{n_1} = 48.6°.$$

Assignments:
Glass n = 1.52
Diamond n = 2.42

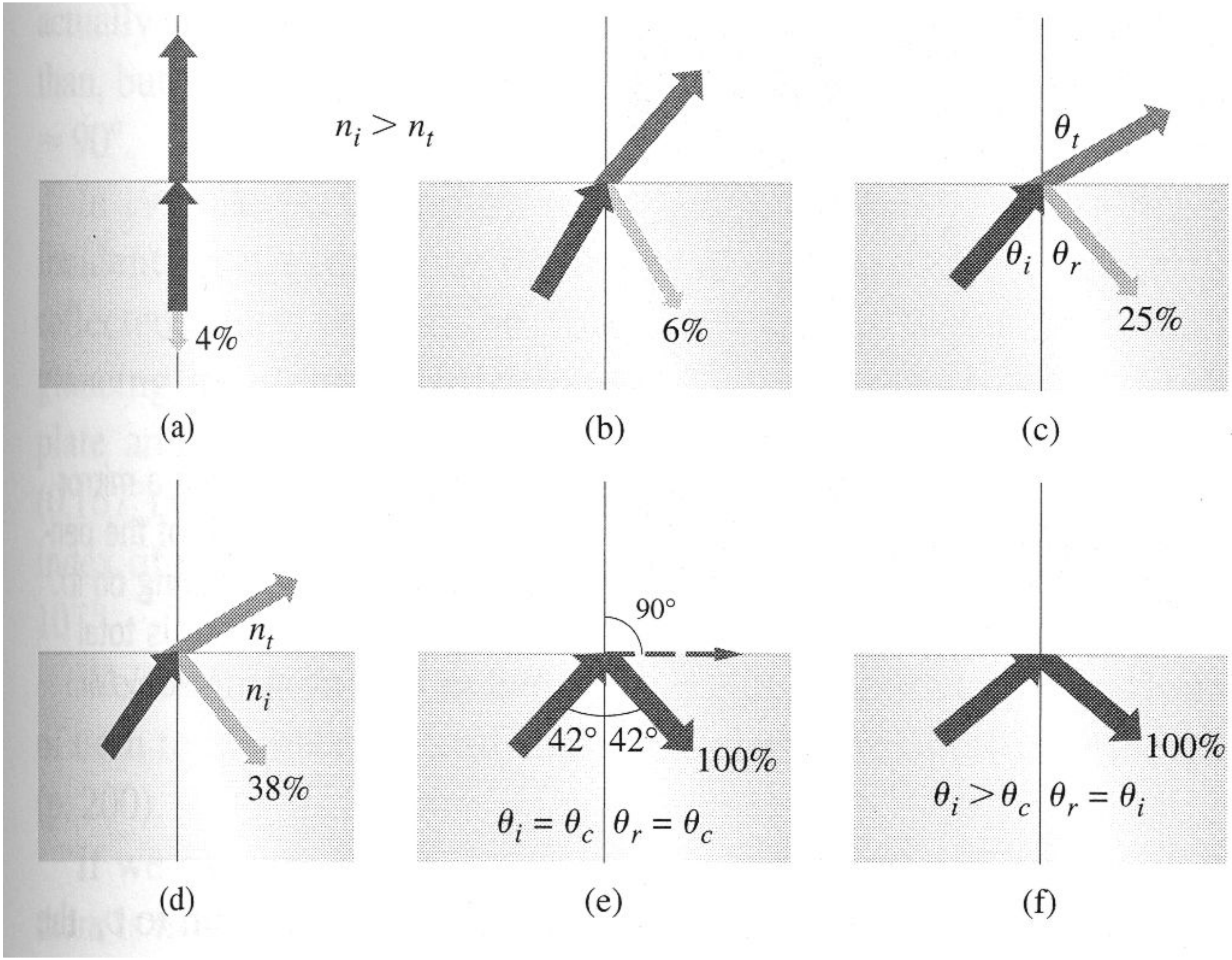
The largest possible angle of incidence which still results in a refracted ray is called the **critical angle**; in this case the refracted ray travels along the boundary between the two media.

For example, consider a ray of light moving from water to air with an angle of incidence of 50°.
The refractive indices of water and air are approximately 1.333 and 1, respectively,
so Snell's law gives us the relation

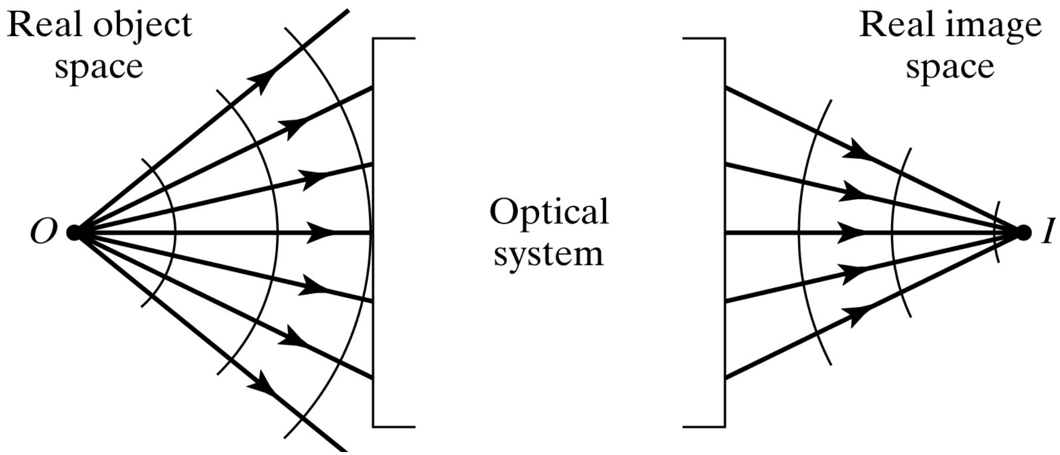
$$\sin\theta_2 = \frac{n_1}{n_2}\sin\theta_1 = 1.333 \cdot 0.766 = 1.021,$$

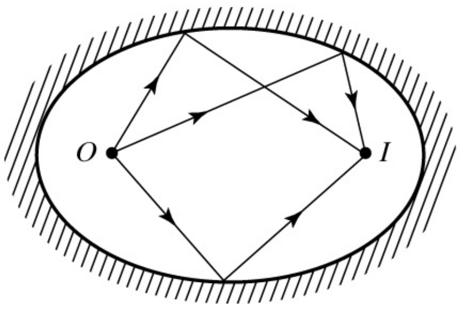
which is impossible to satisfy.

Reflection and refraction at a flat dielectric interface

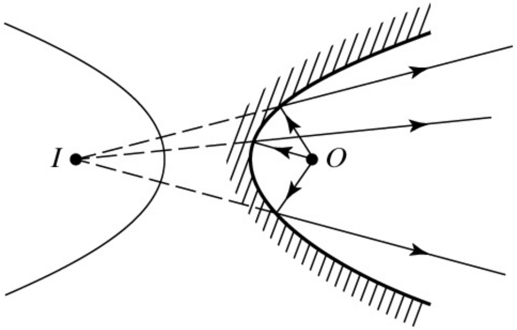


Imaging by an Optical System

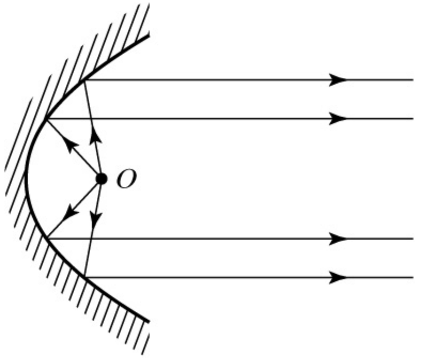




(a) Ellipsoid



(b) Hyperboloid



(c) Paraboloid

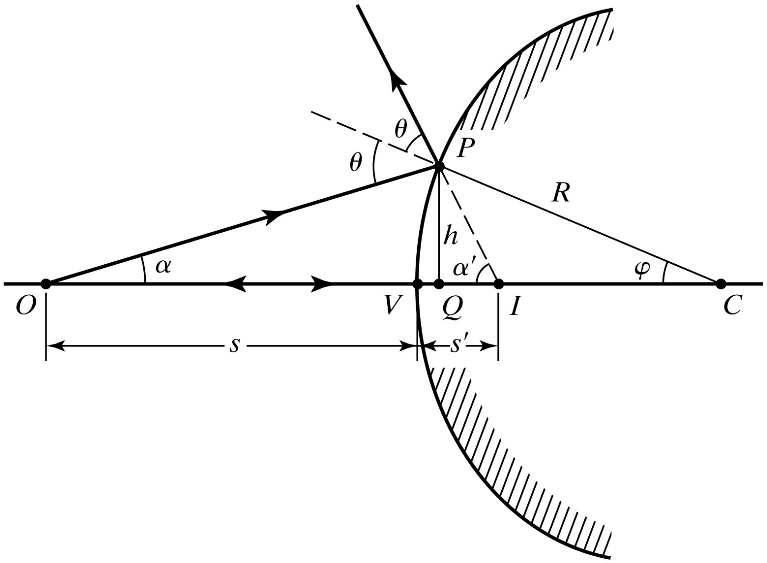
Key words: object point, image point, object space, image space, wavefronts.
Rays spread out radially in all directions from object point O. A Ray diagram deals with selected major rays.

Nonideal images are formed in practice because of
(1) light scattering,
(2) aberrations, and
(3) diffraction.

Trading off *fabrication (grinding)* and *spherical aberrations*.

Molded aspheric lenses are more commonly nowadays.

Reflection at a Spherical Surface



$$\frac{1}{s}+\frac{1}{s'}=-\frac{2}{R}$$

Sing Convention
(assuming the light propagates from left to right)

$$\sin\varphi=\varphi-\frac{\varphi^3}{3!}+\frac{\varphi^5}{5!}-\cdots$$

$$\cos\varphi=1-\frac{\varphi^2}{2!}+\frac{\varphi^4}{4!}-\cdots$$

$$\varphi\to 0\,(\text{small angle})$$

$$\sin\varphi\cong\varphi \quad \text{and} \quad \cos\varphi\cong 1$$

The object distance

s is positive when O is to the left of V (real object).

s is negative when O is to the right of V (virtual object).

The image distance

s' is positive when I is to the left of V (real image).

s' is negative when I is to the right of V (virtual image).

The radius of curvature

R is positive when C is to the right of V (a convex mirror).

R is negative when C is to the left of V (a concave mirror).

positive object and image distances ➔ real objects and real images
convex mirrors ➔ positive radii of curvature (eg. a silver spoon)

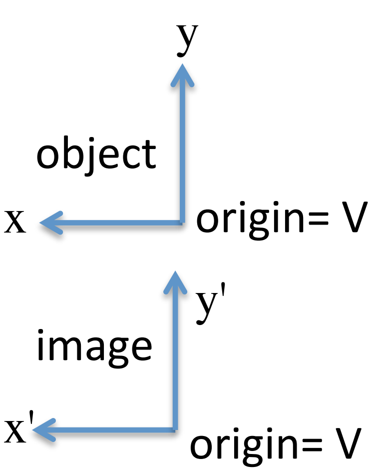
Ray Diagrams for Spherical Mirrors

$$\frac{1}{s}+\frac{1}{s'}=\frac{1}{f}, \quad f=-\frac{R}{2}$$

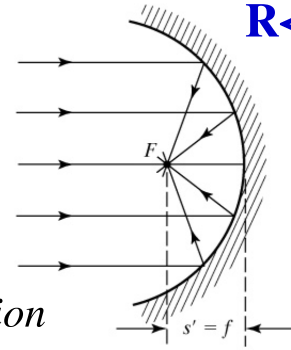
$$m=-\frac{s'}{s} \quad \textit{lateral magnification}$$

$$\frac{1}{s_o}+\frac{1}{s_i}=\frac{1}{f}, \quad f=-\frac{R}{2}$$

$$m=\frac{h_i}{h_o}=-\frac{s_i}{s_o}$$

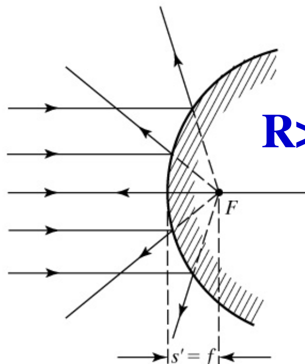


R<0

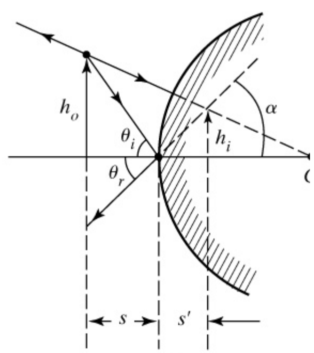


(a)

R>0

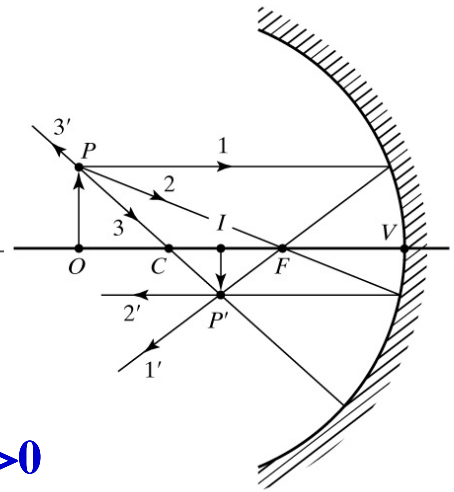


(b)

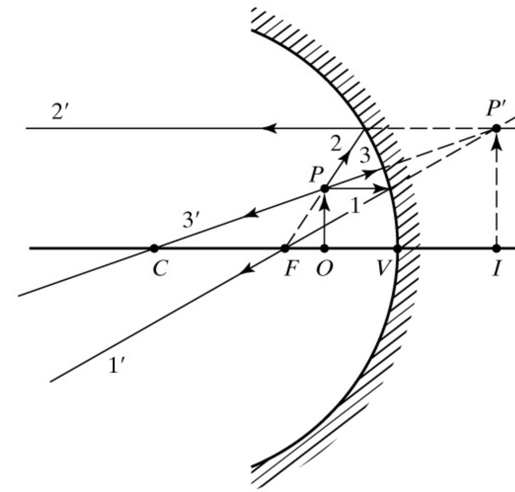


(c)

Ray Diagrams



(a)



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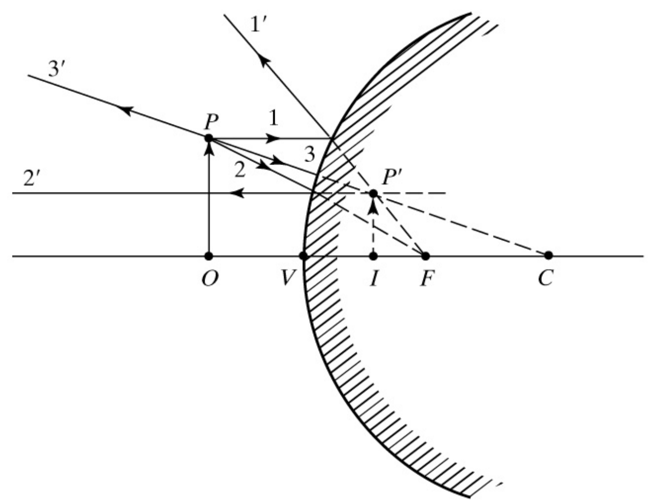
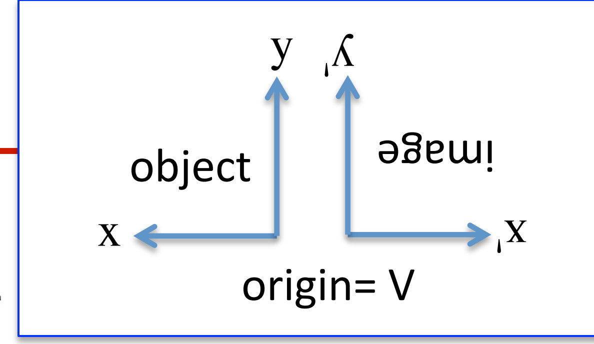
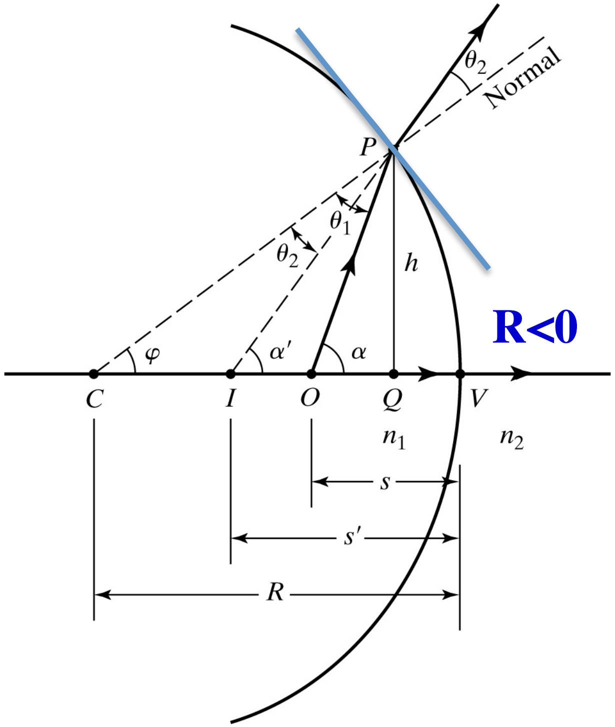
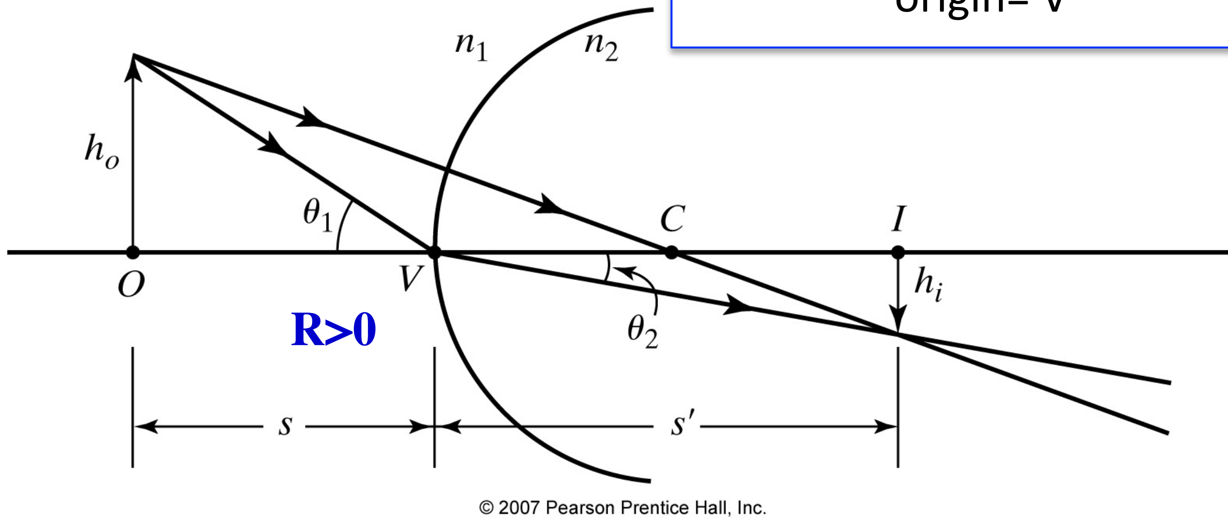


(c)

Refraction at a Spherical Surface







$$\frac{n_1}{s} + \frac{n_2}{s'} = \frac{n_2 - n_1}{R}$$

$$m = \frac{h_i}{h_o} = -\frac{n_1 s'}{n_2 s}$$

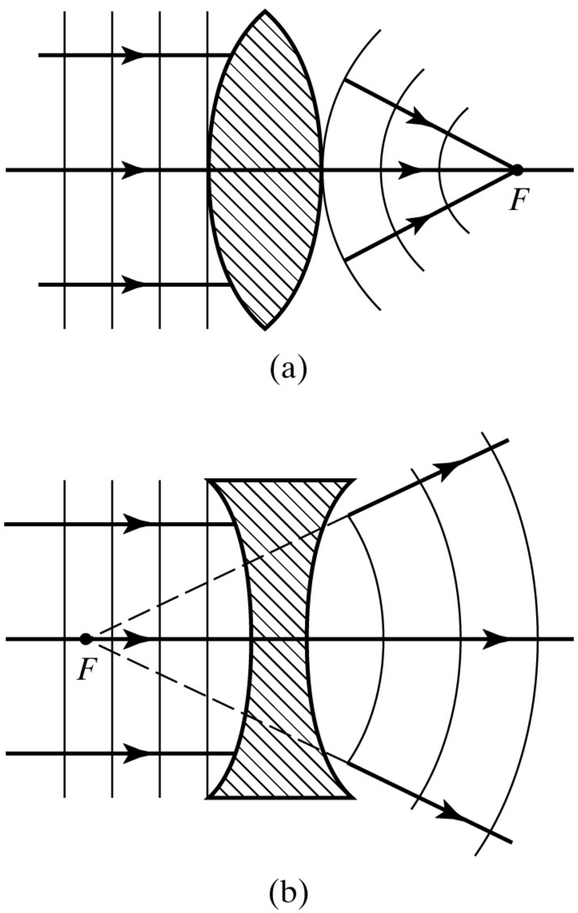
When $R \to \infty$ (i.e. a plane surface)

$$s' = -\left(\frac{n_2}{n_1}\right) s$$

$$m = +1$$

Thin Lenses

Lens action on plane wave-fronts of light.



Ray Diagrams

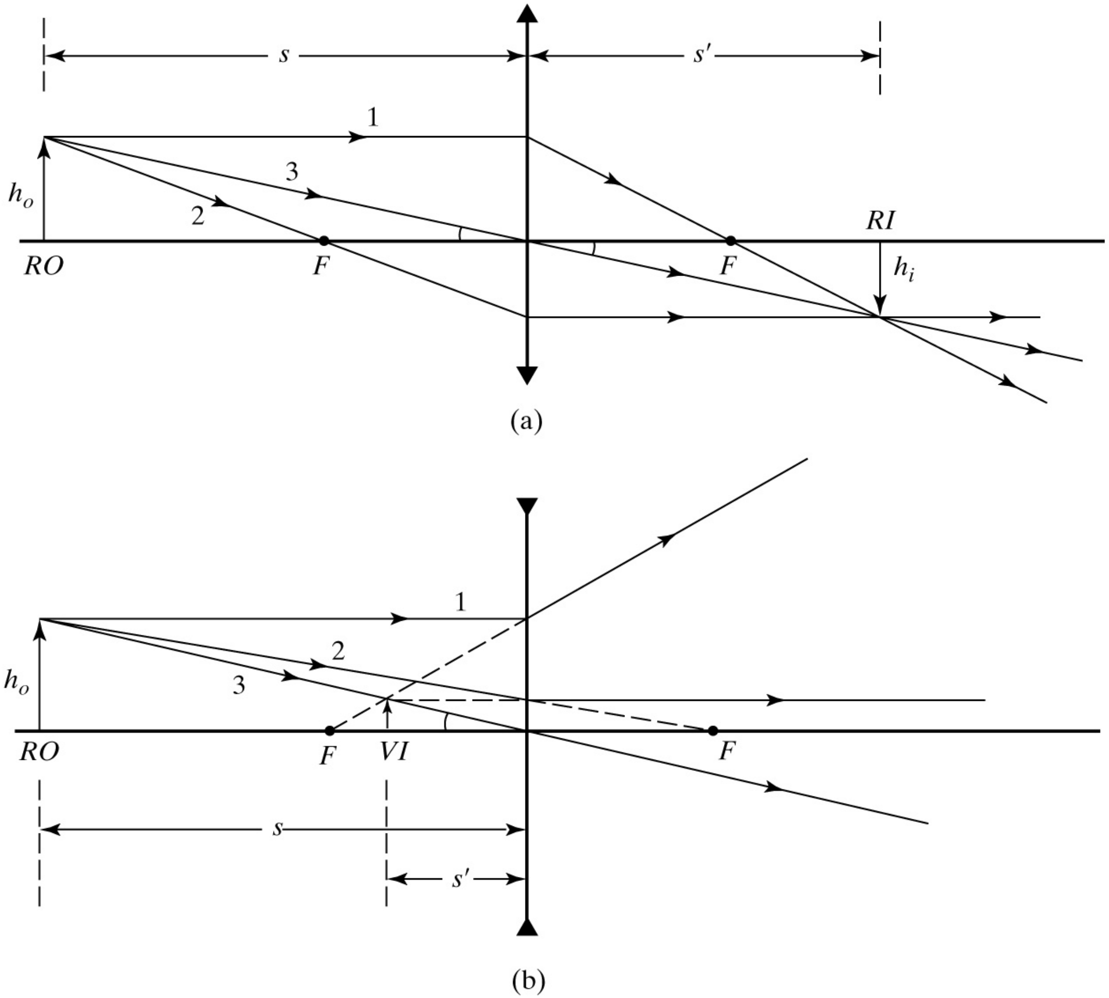
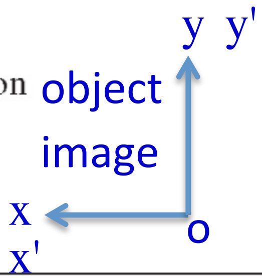
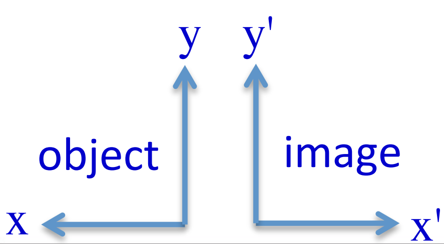
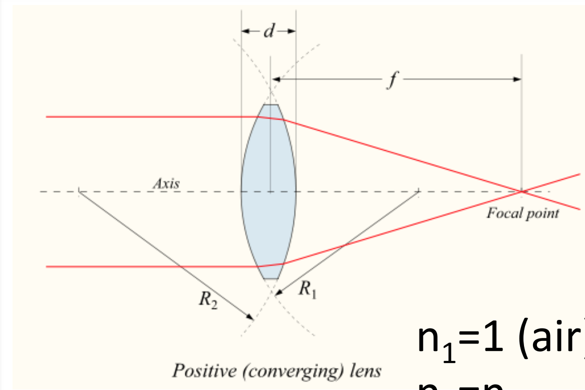


TABLE 2-1 SUMMARY OF GAUSSIAN MIRROR AND LENS FORMULAS

	Spherical surface	Plane surface
Reflection	$\frac{1}{s} + \frac{1}{s'} = \frac{1}{f}, f = -\frac{R}{2}$	$s' = -s$
	$m = -\frac{s'}{s}$	$m = +1$
	Concave: $f > 0, R < 0$	
	Convex : $f < 0, R > 0$	
Refraction Single surface	$\frac{n_1}{s} + \frac{n_2}{s'} = \frac{n_2 - n_1}{R}$	$s' = -\frac{n_2}{n_1}s$
	$m = -\frac{n_1 s'}{n_2 s}$	$m = +1$
	Concave: $R < 0$	
	Convex : $R > 0$	
Refraction Thin lens	$\frac{1}{s} + \frac{1}{s'} = \frac{1}{f}$	
	$\frac{1}{f} = \frac{n_2 - n_1}{n_1}\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$	the lensmaker's formula
	$m = -\frac{s'}{s}$	
	Concave: $f < 0$	
	Convex : $f > 0$	







n_1=1 (air)
n_2=n

The lensmaker's formula

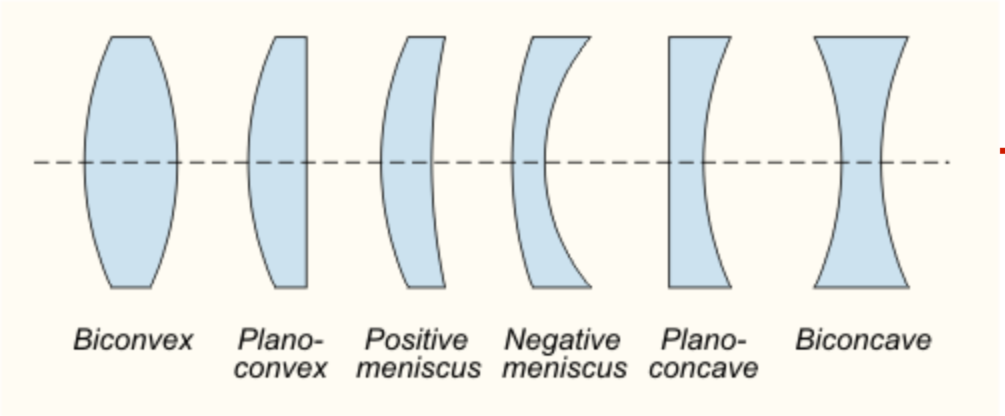
$$\frac{1}{f} = (n-1)\left[\frac{1}{R_1} - \frac{1}{R_2} + \frac{(n-1)d}{nR_1R_2}\right]$$

A "Thin" lens➔ d is negligible

$$\frac{1}{f} \approx (n-1)\left[\frac{1}{R_1} - \frac{1}{R_2}\right].$$

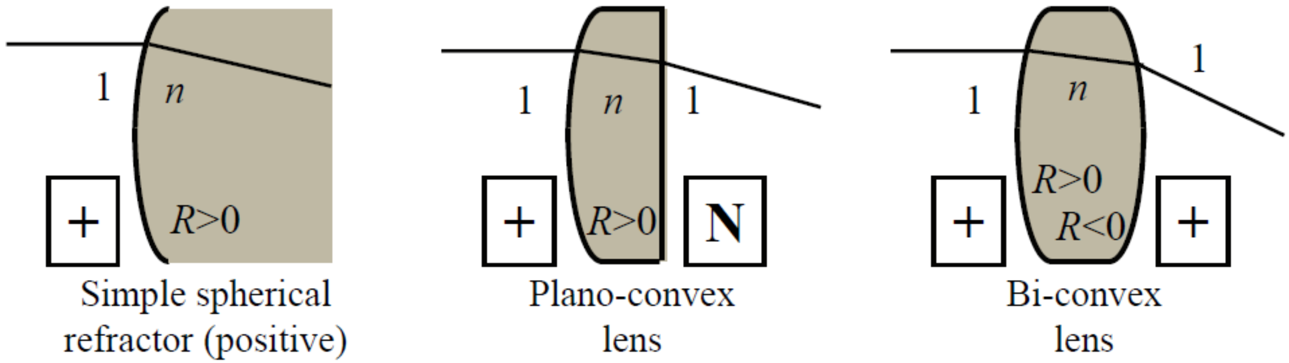
The refractive power of a lens of focal length f

$$D\,[\text{diopters}] = \frac{1}{f\,[\text{in meters}]}$$

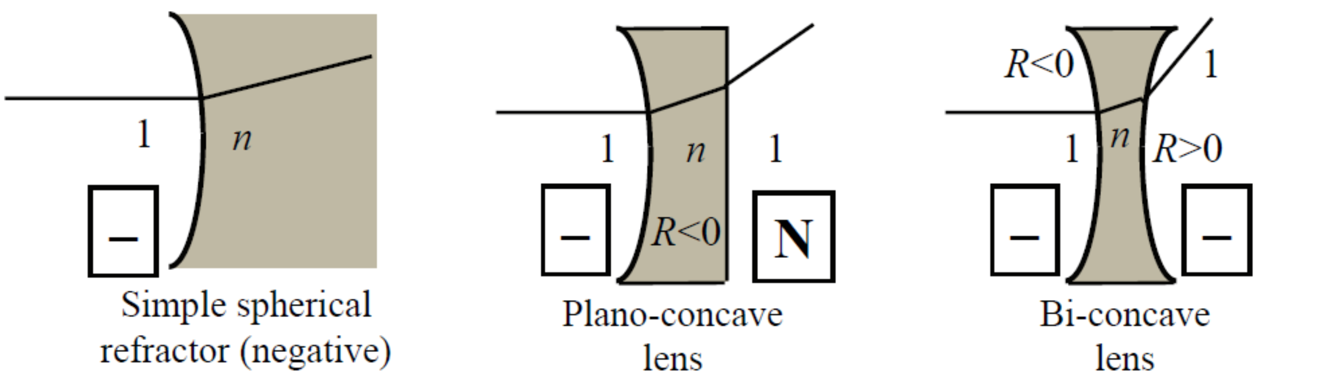


$$\frac{1}{f} \approx (n-1)\left[\frac{1}{R_1} - \frac{1}{R_2}\right].$$

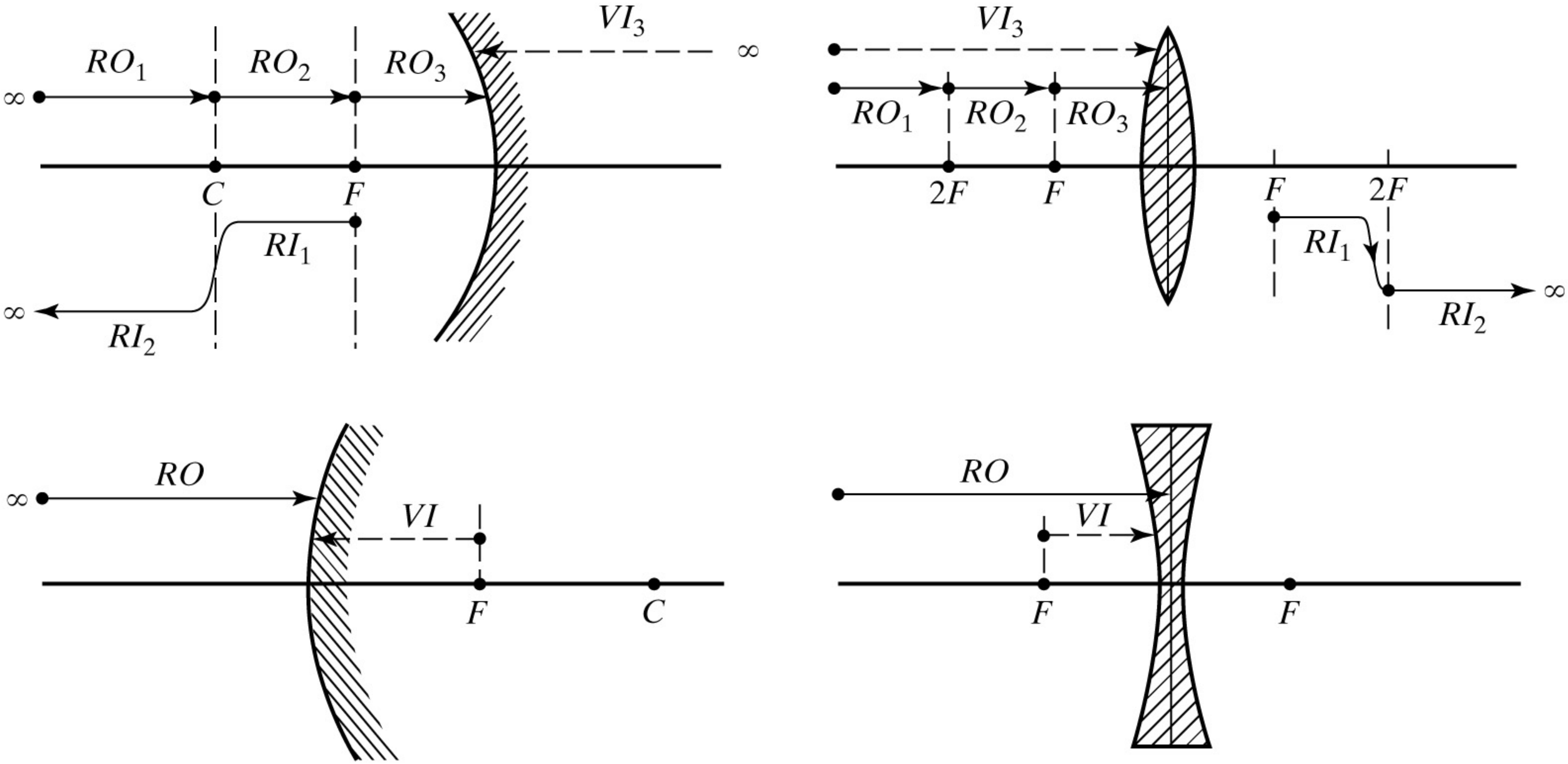
Positive power : exiting rays converge



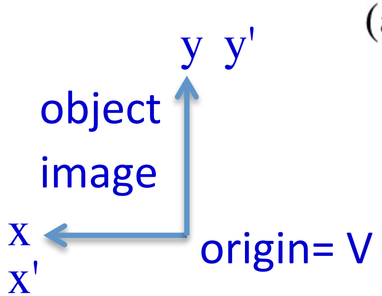
Negative power : exiting rays diverge

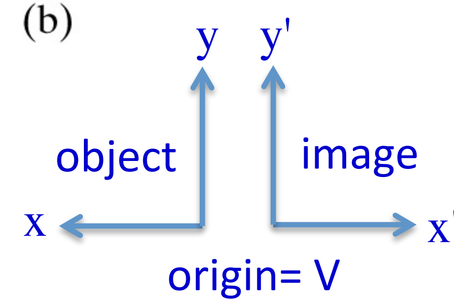


Summary of Image Formation: Spherical Mirrors & Thin Lenses



(a) (b)





Summary : Real and Virtual Images

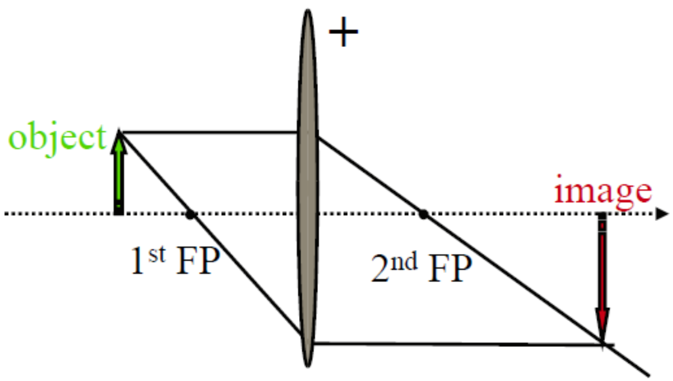


image: real & inverted; M_T<0

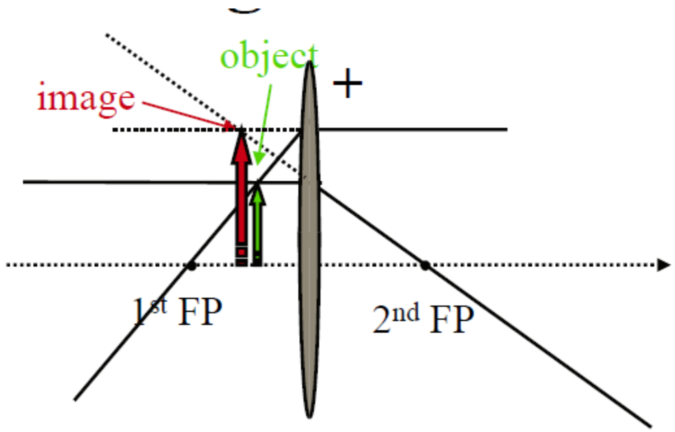


image: virtual & erect; M_T>1

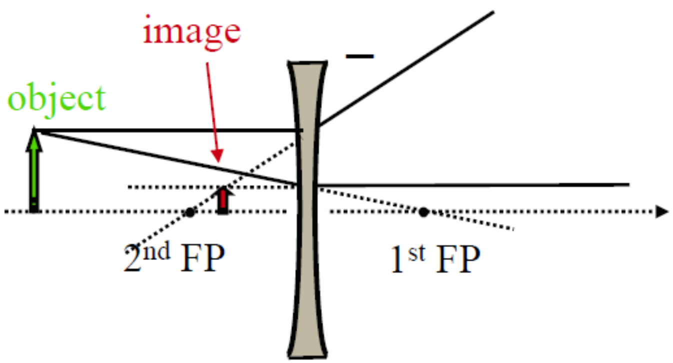


image: virtual & erect; 0<M_T<1

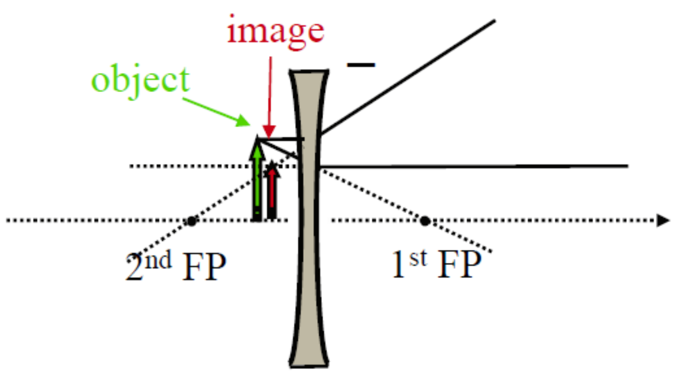
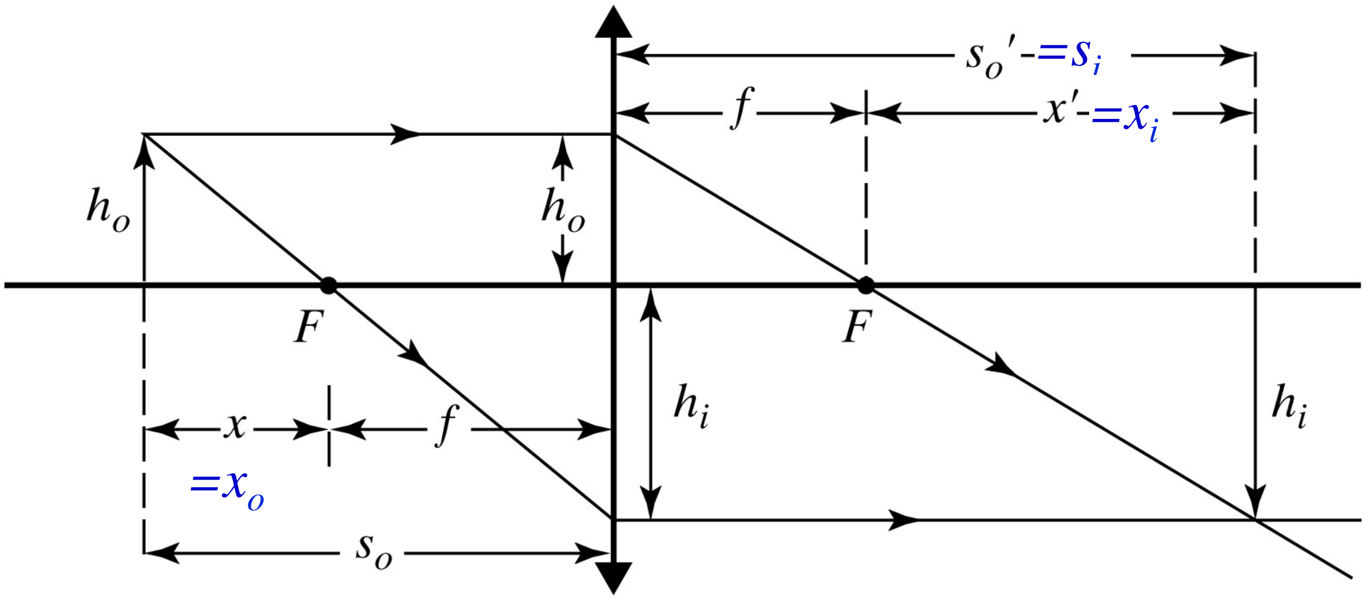


image: virtual & erect; 0<M_T<1

Newtonian Equation for the Thin Lens



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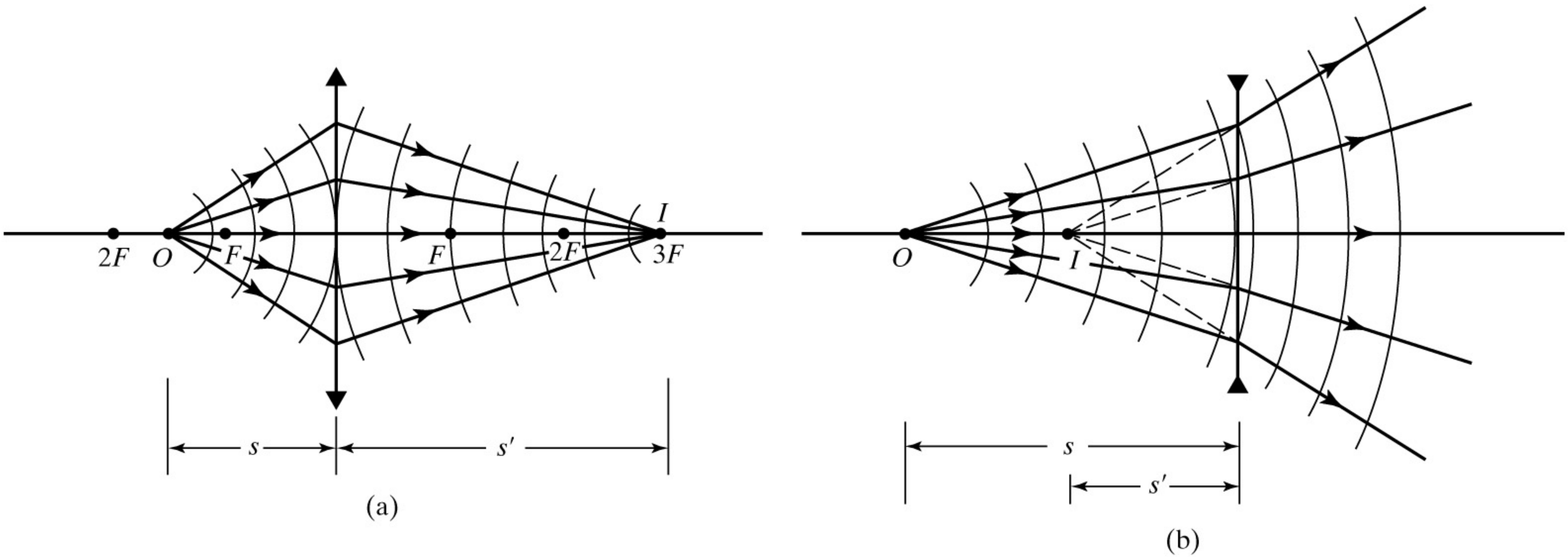
$$\frac{1}{f} = \frac{1}{s_0} + \frac{1}{s_i}$$

$$x_0 x_i = f^2$$

$$M_L \equiv \frac{y_i}{y_0} = -\frac{s_i}{s_o}$$ *lateral magnification*

$$M_T \equiv \frac{dx_i}{dx_0} = -\frac{f^2}{x_0^2}$$ *transverse magnification*

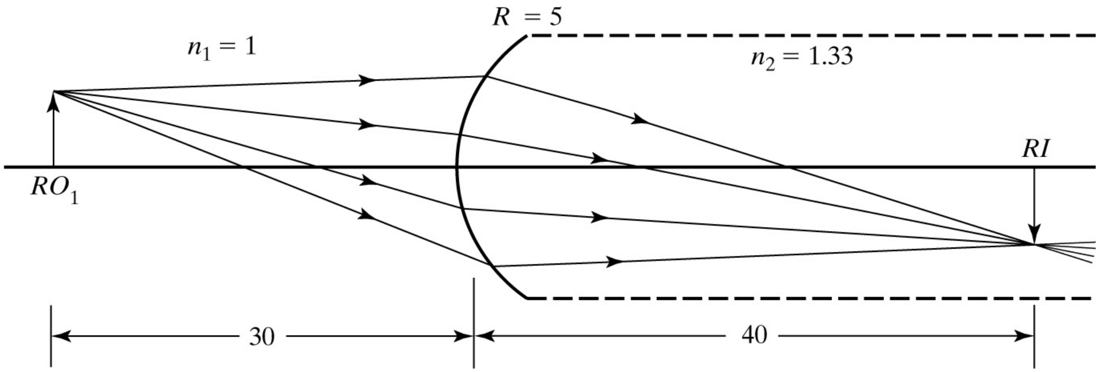
Change in curvature of wavefronts by a thin lens



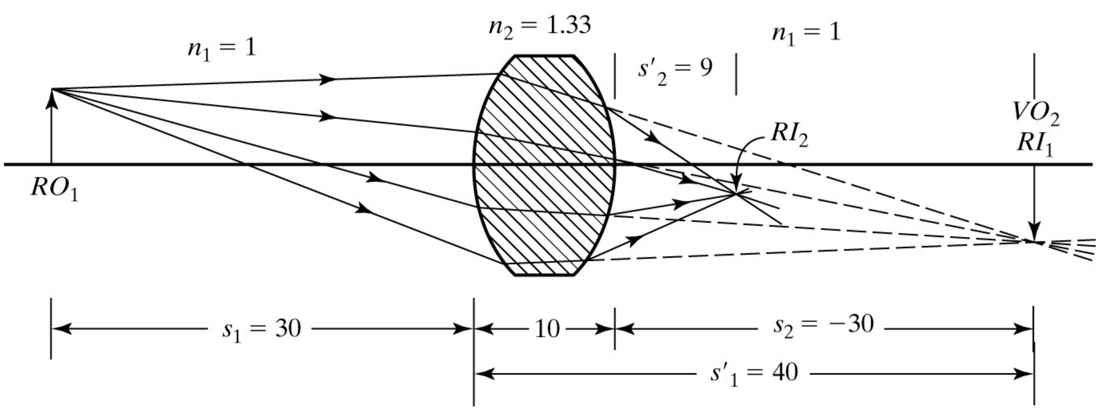
Example of refraction by spherical surfaces

Extra Credit (0.25 pt)

Explain the imaging formation by a cylinder filled with water.
Be specific (i.e. use 'real' values'), See Example 2-2 (page 34)

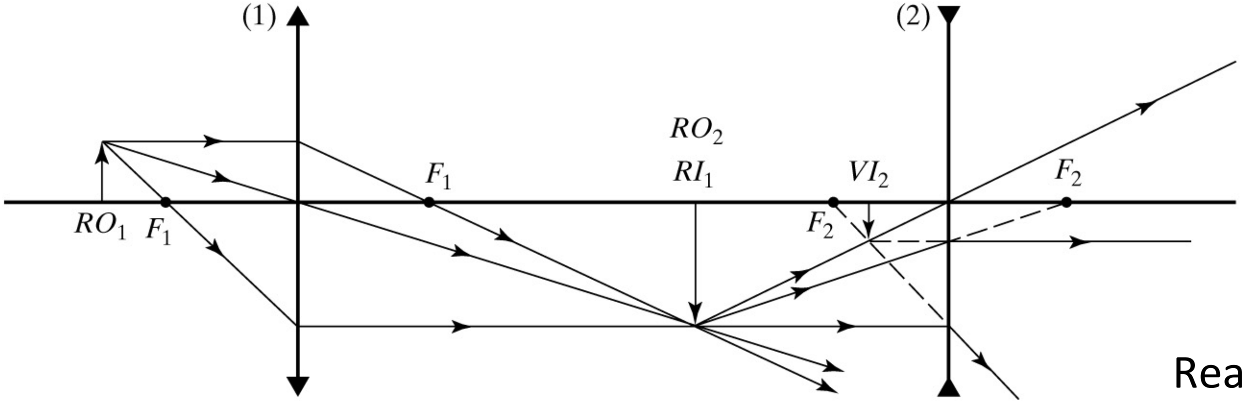


(a)

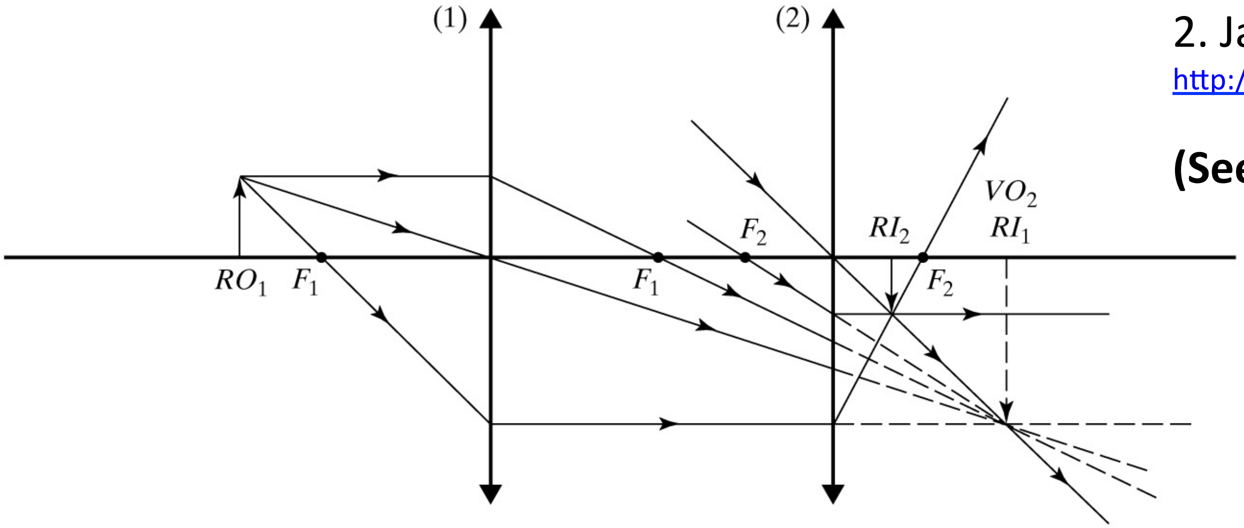


(b)

Thin Lens Combination → Sequential Imaging



(a)



(b)

Reading Assignments

1. Example 2-3 (page 38)

2. Java Applets
http://silver.neep.wisc.edu/~shock/tools/ray.html

(See also Lab #1 Appendix)