

Capacitors/Inductors in series and in parallel handout

From Cook: Electronics A Complete Course

7-2 CAPACITORS IN COMBINATION

Like resistors, capacitors can be connected in either series or parallel. As you will see in this section, the rules for determining total capacitance for parallel- and series-connected capacitors are opposite to series- and parallel-connected resistors.

7-2-1 Capacitors in Parallel

In Figure 7-9(a), you can see a $2\ \mu\text{F}$ and $4\ \mu\text{F}$ capacitor connected in parallel with one another. As the top plate of capacitor *A* is connected to the top plate of capacitor *B* with a wire, and a similar situation occurs with the bottom plates, you can see that this is the same as if the top and bottom plates were touching one another, as shown in Figure 7-9(b). When drawn so that the respective plates are touching, the dielectric constant and plate separation is the same as shown in Figure 7-9(a), but now we can easily see that the plate area is actually increased. Consequently, if capacitors are connected in parallel, the effective plate area is increased; and since capacitance is proportional to plate area [$C \propto A/d$], the capacitance will also increase. Total capacitance is actually calculated by adding the plate areas, so total capacitance is equal to the sum of all the individual capacitances in parallel.

$$C_T = C_1 + C_2 + C_3 + C_4 + \dots$$

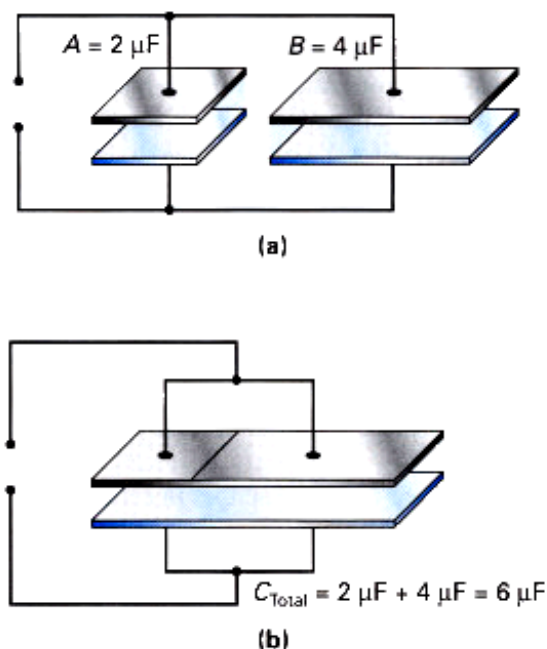


FIGURE 7-9 Capacitors in Parallel.

EXAMPLE:

Determine the total capacitance of the circuit in Figure 7-10(a). What will be the voltage drop across each capacitor?

Solution:

$$\begin{aligned}C_T &= C_1 + C_2 + C_3 \\ &= 1 \mu\text{F} + 0.5 \mu\text{F} + 0.75 \mu\text{F} \\ &= 2.25 \mu\text{F}\end{aligned}$$

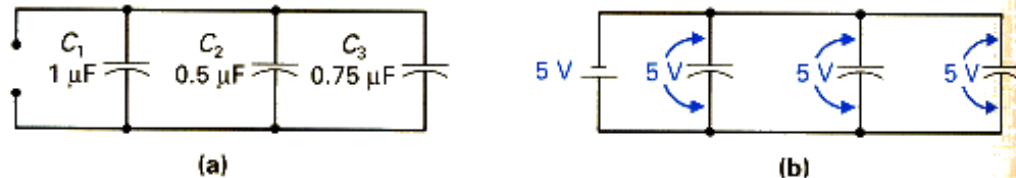


FIGURE 7-10 Example of Parallel-Connected Capacitors.

As with any parallel-connected circuit, the source voltage appears across all the components. If, for example, 5 V is connected to the circuit of Figure 7-10(b), all the capacitors will charge to the same voltage of 5 V because the same voltage always exists across each section of a parallel circuit.

7-2-2 Capacitors in Series

In Figure 7-11(a), we have taken the two capacitors of 2 μF and 4 μF and connected them in series. Since the bottom plate of the A capacitor is connected to the top plate of the B capacitor, they can be redrawn so that they are touching, as shown in Figure 7-11(b).

The top plate of the A capacitor is connected to a wire into the circuit, and the bottom plate of B is connected to a wire into the circuit. This connection creates two center plates that are isolated from the circuit and can therefore be disregarded, as shown in Figure 7-11(c). The first thing you will notice in this illustration is that the dielectric thickness ($d \uparrow$) has increased, causing a greater separation between the plates. The effective plate area of this capacitor has decreased, as it is just the area of the top plate only. Even though the bottom plate extends out further, the electric field can only exist between the two plates, so the surplus metal of the bottom plate has no metal plate opposite for the electric field to exist.

Consequently, when capacitors are connected in series the effective plate area is decreased ($A \downarrow$) and the dielectric thickness increased ($d \uparrow$), and both of these effects result in an overall capacitance decrease ($C \downarrow \downarrow = (8.85 \times 10^{-12}) \times K \times A \downarrow / d \uparrow$).

The plate area is actually decreased to the smallest individual capacitance connected in series, which in this example is the plate area of A. If the plate area were the only factor, then capacitance would always equal the smallest capacitor value. However, the dielectric thickness is always equal to the sum of all the capacitor dielectrics, and this factor always causes the total capacitance (C_T) to be less than the smallest individual capacitance when capacitors are connected in series.

The total capacitance of two or more capacitors in series therefore is calculated by using the following formulas: For two capacitors in series,

$$C_T = \frac{C_1 \times C_2}{C_1 + C_2}$$

(product-over-sum formula)

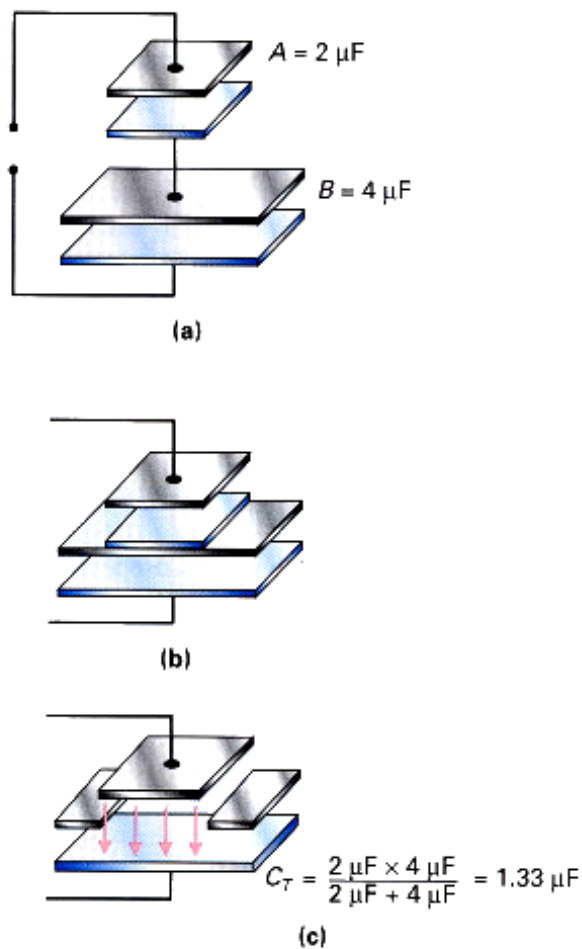


FIGURE 7-11 Capacitors in Series.

For more than two capacitors in series,

$$C_T = \frac{1}{(1/C_1) + (1/C_2) + (1/C_3) + \dots}$$

(reciprocal formula)

EXAMPLE:

Determine the total capacitance of the circuit in Figure 7-12.

Solution:

$$\begin{aligned} C_T &= \frac{1}{(1/C_1) + (1/C_2) + (1/C_3)} \\ &= \frac{1}{(1/4 \mu\text{F}) + (1/2 \mu\text{F}) + (1/1 \mu\text{F})} \\ &= \frac{1}{1.75 \times 10^6} = 5.7143 \times 10^{-7} \\ &= 0.5714 \mu\text{F} \text{ or } 0.6 \mu\text{F} \end{aligned}$$

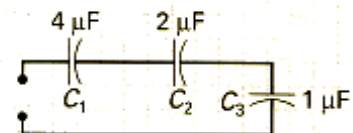


FIGURE 7-12 Example of Series-Connected Capacitors.

The total capacitance for capacitors in series is calculated in the same way as total resistance when resistors are in parallel.

As with series-connected resistors, the sum of all of the voltage drops across the series-connected capacitors will equal the voltage applied (Kirchhoff's voltage law). With capacitors connected in series, the charged capacitors act as a voltage divider, and therefore the voltage-divider formula can be applied to capacitors in series.

$$V_{cx} = \frac{C_T}{C_x} \times V_T$$

where V_{cx} = voltage across desired capacitor
 C_T = total capacitance
 C_x = desired capacitor's value
 V_T = total supplied voltage

■ **EXAMPLE:**

Using the voltage-divider formula, calculate the voltage dropped across each of the capacitors in Figure 7-12 if $V_T = 24$ V.

■ **Solution:**

$$V_{C1} = \frac{C_T}{C_1} \times V_T = \frac{0.5714 \mu\text{F}}{4 \mu\text{F}} \times 24 \text{ V} = 3.4 \text{ V}$$

$$V_{C2} = \frac{C_T}{C_2} \times V_T = \frac{0.5714 \mu\text{F}}{2 \mu\text{F}} \times 24 \text{ V} = 6.9 \text{ V}$$

$$V_{C3} = \frac{C_T}{C_3} \times V_T = \frac{0.5714 \mu\text{F}}{1 \mu\text{F}} \times 24 \text{ V} = 13.7 \text{ V}$$

$$V_T = V_{C1} + V_{C2} + V_{C3} = 3.4 + 6.9 + 13.7 = 24 \text{ V}$$

(Kirchhoff's voltage law)

If the capacitor values are the same, as seen in Figure 7-13(a), the voltage is divided equally across each capacitor, as each capacitor has an equal amount of charge and therefore has half of the applied voltage (in this example, 3 V across each capacitor).

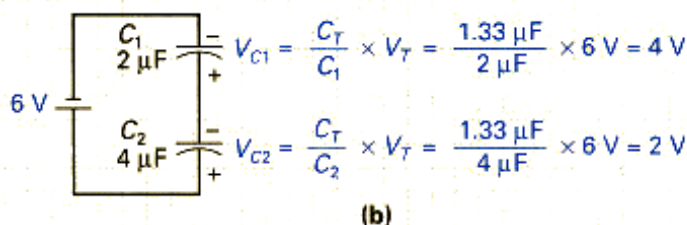
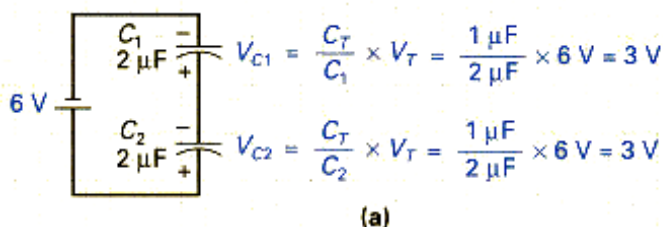


FIGURE 7-13 Voltage Drops across Series-Connected Capacitors.

When the capacitor values are different, the smaller value of capacitor will actually charge to a higher voltage than the larger capacitor. In the example in Figure 7-13(b), the smaller capacitor is actually half the size of the other capacitor, and it has charged to twice the voltage. Since Kirchhoff's voltage law has to apply to this and every series circuit, you can easily calculate that the voltage across C_1 will equal 4 V and is twice that of C_2 , which is 2 V. To understand this fully, we must first understand that although the capacitance is different, both capacitors have an equal value of coulomb charge held within them, which in this example is 8 μC .

$$\begin{aligned} Q_1 &= C_1 \times V_1 \\ &= 2 \mu\text{F} \times 4 \text{ V} = 8 \mu\text{C} \\ Q_2 &= C_2 \times V_2 \\ &= 4 \mu\text{F} \times 2 \text{ V} = 8 \mu\text{C} \end{aligned}$$

This equal charge occurs because the same amount of current flow exists throughout a series circuit, so both capacitors are being supplied with the same number or quantity of electrons. The charge held by C_1 is large with respect to its small capacitance, whereas the same charge held by C_2 is small with respect to its larger capacitance.

If the charge remains the same (Q is constant) and the capacitance is small, the voltage drop across the capacitor will be large, because the charge is large with respect to the capacitance:

$$V \uparrow = \frac{Q}{C \downarrow}$$

On the other hand, for a constant charge, a large capacitance will have a small charge voltage because the charge is small with respect to the capacitance:

$$V \downarrow = \frac{Q}{C \uparrow}$$

We can apply the water analogy once more and imagine two series-connected buckets, one of which is twice the size of the other. Both are being supplied by the same series pipe, which has an equal flow of water throughout, and are consequently each holding an equal amount of water, for example, 1 gallon. The 1 gallon of water in the small bucket is large with respect to the size of the bucket, and a large amount of pressure exists within that bucket. The 1 gallon of water in the large bucket is small with respect to the size of the bucket, so a small amount of pressure exists within this bucket. The pressure within a bucket is similar to the voltage across a capacitor, and therefore a small bucket or capacitor will have a greater pressure or voltage associated with it, while a large bucket or capacitor will develop a small pressure or voltage.

To summarize capacitors in series, all the series-connected components will have the same charging current throughout the circuit, and because of this, two or more capacitors in series will always have equal amounts of coulomb charge. If the charge (Q) is equal, the voltage across the capacitor is determined by the value of the capacitor. A small capacitance will charge to a larger voltage ($V \uparrow = Q/C \downarrow$), whereas a large value of capacitance will charge to a smaller voltage ($V \downarrow = Q/C \uparrow$).

SELF-TEST EVALUATION POINT FOR SECTION 7-2



Use the following questions to test your understanding of Section 7-2.

1. If 2 μF , 3 μF , and 5 μF capacitors are connected in series, what will be the total circuit capacitance?
2. If 7 pF, 2 pF, and 14 pF capacitors are connected in parallel, what will be the total circuit capacitance?
3. State the voltage-divider formula as it applies to capacitance.
4. True or false: With resistors, the large value of resistor will drop a larger voltage, whereas with capacitors the smaller value of capacitor will actually charge to a higher voltage.

EXAMPLE:

Refer to Figure 8-19(a) and (b) and calculate the inductance of each.

Solution:

$$\text{a. } L = \frac{5^2 \times 0.01 \times (6.28 \times 10^{-5})}{0.001} = 15.7 \text{ mH}$$

$$\text{b. } L = \frac{10^2 \times 0.1 \times (1.1 \times 10^{-4})}{0.1} = 11 \text{ mH}$$

TABLE 8-2 Permeabilities of Various Materials

MATERIAL	PERMEABILITY (μ)
Air or vacuum	1.26×10^{-6}
Nickel	6.28×10^{-5}
Cobalt	7.56×10^{-5}
Cast iron	1.1×10^{-4}
Machine steel	5.65×10^{-4}
Transformer iron	6.9×10^{-4}
Silicon iron	8.8×10^{-3}
Permalloy	0.126
Superalloy	1.26

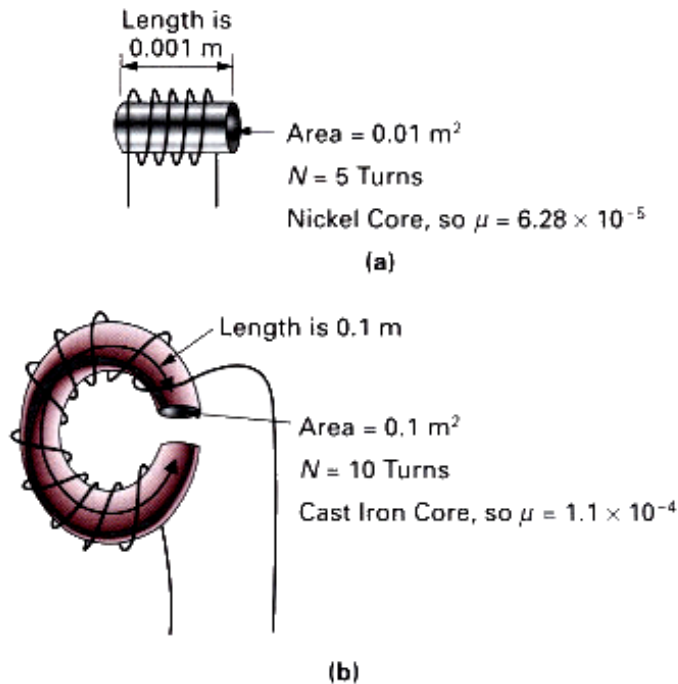


FIGURE 8-19 Inductor Examples.

8-3-2 Inductors in Combination

Inductors oppose the change of current in a circuit and so are treated in a manner similar to resistors connected in combination. Two or more inductors in series merely extend the coil length and increase inductance. Inductors in parallel are treated in a manner similar to resistors, with the total inductance being less than that of the smallest inductor's value.

1. Inductors in Series

When inductors are connected in series with one another, the total inductance is calculated by summing all the individual inductances.

$$L_T = L_1 + L_2 + L_3 + \dots$$

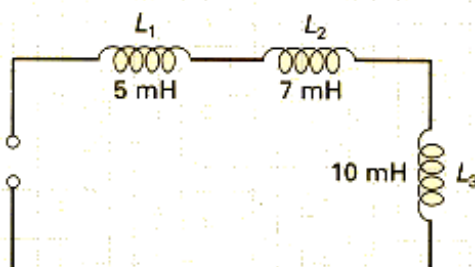
EXAMPLE:

Calculate the total inductance of the circuit shown in Figure 8-20.

Solution:

$$\begin{aligned} L_T &= L_1 + L_2 + L_3 \\ &= 5 \text{ mH} + 7 \text{ mH} + 10 \text{ mH} \\ &= 22 \text{ mH} \end{aligned}$$

FIGURE 8-20 Inductors in Series.



2. Inductors in Parallel

When inductors are connected in parallel with one another, the reciprocal (two or more inductors) or product-over-sum (two inductors) formula can be used to find total inductance, which will always be less than the smallest inductor's value.

$$L_T = \frac{1}{(1/L_1) + (1/L_2) + (1/L_3) + \dots}$$

$$L_T = \frac{L_1 \times L_2}{L_1 + L_2}$$

EXAMPLE:

Determine L_T for the circuits in Figure 8-21(a) and (b).

Solution:

a. Reciprocal formula:

$$\begin{aligned} L_T &= \frac{1}{(1/L_1) + (1/L_2) + (1/L_3)} \\ &= \frac{1}{(1/10 \text{ mH}) + (1/5 \text{ mH}) + (1/20 \text{ mH})} \\ &= 2.9 \text{ mH} \end{aligned}$$

b. Product over sum:

$$\begin{aligned} L_T &= \frac{L_1 \times L_2}{L_1 + L_2} \\ &= \frac{10 \mu\text{H} \times 2 \mu\text{H}}{10 \mu\text{H} + 2 \mu\text{H}} \\ &= \frac{20 \times 10^{-12} \text{ H}^2}{12 \mu\text{H}} = 1.67 \mu\text{H} \end{aligned}$$

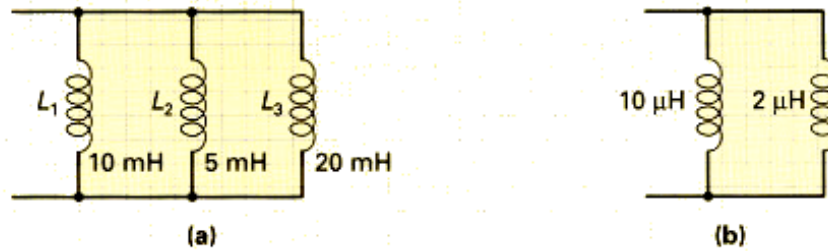


FIGURE 8-21 Inductors in Parallel.

8-3-3 *Types of Inductors*

As with resistors and capacitors, inductors are basically divided into the two categories of fixed-value and variable-value inductors, as shown by the symbols in Figure 8-22(a) and (b). Within these two categories, inductors are generally classified by the type of core material used. Figure 8-22(c) shows a variety of different types.

8-3-4 *Inductive Time Constant*

Inductors will not have any effect on a steady value of direct current (dc) from a dc voltage source. If, however, the dc is changing (pulsating), the inductor will oppose the change

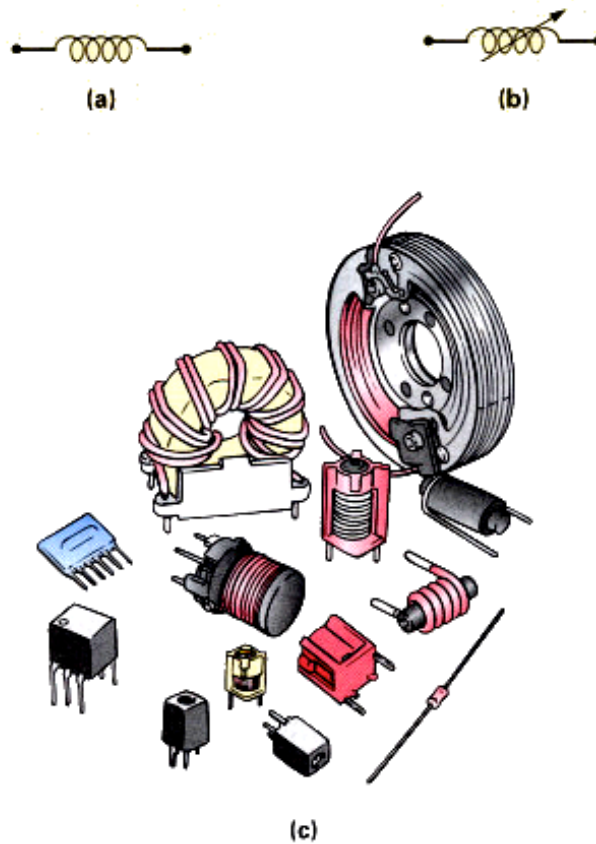


FIGURE 8-22 Inductor Types. (a) Fixed-Value Inductor Symbol. (b) Variable-Value Inductor Symbol. (c) Physical Appearance of Inductors.