**The Compton Effect**

**Introduction**

In this experiment we will study two aspects of the interaction of photons with electrons. The first of these is the Compton effect named after Arthur Holly Compton who received the Nobel Prize for physics in 1927 for its discovery. The other deals with the radiation emitted when a tightly bound electron from a heavy element is kicked out by a photon. This gives rise to “characteristic” X-rays that can be used to identify the element.

This experiment uses material from the Introduction to Nuclear Radiation.

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**Theory of the Compton Effect**

**Kinematics of the Compton Effect**

The Compton effect is based on treating light as consisting of particles of a given energy related to the frequency of the light wave. In this context, the particle of light is given the name “photon”. An energetic photon with energy of 0.1 MeV (million electron volts) or larger is also often referred to as a gamma ray. An MeV is an energy unit, equal to the kinetic energy an electron would gain by being accelerated through a voltage difference of 1 MV (10^6 volts). Photons whose energy is in the range of 0.1 to 100 keV are usually referred to as X-rays.

If a photon with energy $E_0$ strikes a stationary electron, as in Figure 1, then the energy of the scattered photon, $E$, depends on the scattering angle, $\Theta$, that it makes with the direction of the incident photon according to the following equation:

$$\cos \Theta = 1 - m_e c^2 \left[ \frac{1}{E} - \frac{1}{E_0} \right]$$

where $m_e$ is the mass of the electron and $m_e c^2 = 511$ keV = .511 MeV. Now in real matter, electrons are not stationary, but if the initial kinetic energy of the electron is small compared to the energy of the incoming photon, Eq. 1 will describe the situation well. This will be the case for gamma rays ($E > 10^5$ eV) scattering off outer electrons of atoms (typical kinetic energy of a few eV).

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**Fig. 1**: Schematic diagram of Compton Effect kinematics.
The derivation of this equation is based on applying special relativity and kinematics to the photon as a quantum of light, but in the form we use, requires little or no reference to the wave nature of light! It’s all about energy of the electron and photon, and the angle of the outgoing photon compared to the initial direction.

The total energy of the electron $E_e$ is the sum of its kinetic energy $T_e$ and its rest energy $m_e c^2$, i.e. $E_e = T_e + m_e c^2$. The total energy of the recoiling electron can be computed from energy conservation in the reaction and is given by:

$$E_e = E_0 + m_e c^2 - E$$

or equivalently:

$$T_e = E_0 - E$$

Clearly the electron energy achieves its maximum value in this scattering where the photon has its minimum energy. The lowest energy for the scattered photon results when it is back-scattered, i.e., it emerges at 180 degrees with respect to its original direction, in which case Eq. 1 shows that the incident and $E_{min}$, the minimum scattered photon energies are related as:

$$\frac{1}{E_{min}} - \frac{1}{E_0} = \frac{2}{m_e c^2}$$

The Compton Edge

These formulas do not tell us the relative probability of the possible scattering angles (and, equivalently, the relative probability of various recoiling electron energies. In Figure 2 we plot the energy dependence.

![Figure 2](image-url)

Fig. 2: The relative probability of finding an electron with “reduced” kinetic energy $t = T_e/m_e c^2$. This varies somewhat with $E_0$; $E_0 = .511$ MeV for this plot.
The important thing to take away from this plot is the rise in the probability with increasing kinetic energy up to the kinematic limit, where it abruptly falls to zero. In our experiment we will be looking for this shape, referred to as the Compton edge.

We have chosen to plot this for an incident photon with energy equal to the rest mass of an electron, \( E_0 = 0.511 \text{ MeV} = m_e c^2 \), but if we had chosen a different incident energy, the shape would be similar, just with a different end point. A more detailed explanation of how this plot was calculated is given in Appendix 2.

**Characteristic X-Ray spectra**

When electrons or photons scatter from atoms, they sometimes impart sufficient energy to atomic electrons to free them from their bound states. If this happens in a multi-electron atom, then a hole is created which is rapidly filled by an electrons cascading down from higher levels emitting the lost potential energy in the form of photons. When the level filled is the innermost atomic level, then X-rays are produced, which uniquely identify the element, are called characteristic K (or K-shell) X-rays. This can happen in our experiment in \( ^{133}\text{Cs} \rightarrow ^{133}\text{Ba} \) decays if the Ba decay product ejects an \( n=1 \) electrons, so the X-ray is characteristic of Ba. Alternatively, the energetic \( ^{133}\text{Cs} \) or \( ^{60}\text{Co} \) gammas may hit atoms in the detector or elsewhere and produce characteristic X-rays associated with the object they hit. These characteristic X-rays may form a peak in the observed spectrum if enough of them which have their entire energy absorbed by the NaI detector.

The K X-ray energy varies with the atomic number (Z) of the substance as \((Z - 1)^2\) where the subtractive constant arises due to the shielding effect of the other inner shell electron. The energies of these X-rays can be substantial e.g. for Pb they are about 80 keV. We can make a rough calculation of this quantity if we recall that the ionization potential of hydrogen (Z = 1) is about 13.6 eV which multiplied by \((82 - 1)^2\) gives a number of the right order of magnitude. These X-rays help identify the substance because they are measuring something related to the atomic number Z: in essence, the energy level of the most tightly bound electron.

**The Experiment**

**The Apparatus**

The apparatus in this experiment consists of a NaI (Tl) crystal attached to a photo multiplier tube. While the Geiger counter is useful in indicating the presence of ionizing radiation, it cannot give us any information about its energy, it has low efficiency for counting gamma rays and it has somewhat poor time response. A class of counters that overcome these difficulties are scintillation counters in which the incident radiation creates photons in proportion to the energy it loses and then these photons are amplified with a photomultiplier and converted into a voltage pulse whose amplitude is proportional to the energy deposited. One of the best of this kind of counters is an inorganic crystal of NaI doped with Tl. Some of its properties are summarized in the table below:
Sodium Iodide Scintillator

<table>
<thead>
<tr>
<th>Property</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Density (g/cm²)</td>
<td>3.67</td>
</tr>
<tr>
<td>Radiation length (cm)</td>
<td>2.59</td>
</tr>
<tr>
<td>Moliere Radius (cm)</td>
<td>4.5</td>
</tr>
<tr>
<td>dE/dx (MeV/cm) for MIP</td>
<td>4.8</td>
</tr>
<tr>
<td>Nucl. Int. Length (cm)</td>
<td>41.4</td>
</tr>
<tr>
<td>Decay time (ns)</td>
<td>250</td>
</tr>
<tr>
<td>Peak emission λ (nm)</td>
<td>410</td>
</tr>
<tr>
<td>Refractive Index</td>
<td>1.85</td>
</tr>
<tr>
<td>Light Output (photons/MeV)</td>
<td>40,000</td>
</tr>
</tbody>
</table>

The Radiation length and Moliere Radius are length scales associated with electromagnetic radiation. Electrons and photons deposit their energy in the NaI detector over a finite distance, not just at a single point. For high energy electrons (E >> a few MeV), for example, the Radiation length (longitudinal, i.e., along the incident direction) is the distance in which an electron loses all but 1/e of its energy and the Moliere radius is the transverse radius containing about 90% of the energy. In this lab we shall be dealing with sources of a few MeV or less in which case the radiation length and Moliere radius are even shorter, assuring us that the particles (electrons or photons) deposit their full energy in our detector. So the detector can capture all the energy—if the particle hits near enough to the center of the detector.

The decay time of a pulse is the time it takes to decay to 1/e of its original amplitude. Two pulses will not be confused if they come a few decay times (5 or 6) separated from one another.

The light created by energy deposition in the crystal is funneled by plastic light guides to the photocathode of a photomultiplier tube where it undergoes amplification on the way to being turned into a voltage pulse. Since the light collection process is not perfectly efficient (a typical number is 10%) and since a typical photocathode efficiency for converting photons into electrons is 25%, the 40,000 photons per MeV created in the crystal appear as about 1,000 electrons/MeV after the first stage of the photomultiplier tube. If the energy resolution of the detector were dominated by counting statistics only then we would expect a fractional resolution of 1/\sqrt{1000} or about 3% at 1 MeV. In practice, other effects tend to make this number somewhat larger.

The photomultiplier tube amplifies the initial signal by creating cascades of new electrons at several stages in proportion to the number of incident electrons. Typical gains in PM tubes can be 10⁶ or more. The gain of the tube is proportional to the high voltage applied and thus we must take care that the voltage is constant during a measurement.

After amplification in the photomultiplier, the signal undergoes some shaping and is fed to an analog-to-digital (ADC) converter residing in the PC. There it is turned into a number proportional to its amplitude, suitable for computer display and manipulation.

NaI can measure the energy deposition due to electrons or photons (which may be gamma rays or lower-energy X-rays). This is a more sophisticated detector than the Geiger counter. Instead of just a count (on or off), the output of the NaI counter is a voltage pulse proportional to the energy deposited in the counter, which is fed into a Multi Channel Analyzer (MCA) box read by the PC. Thus you will record a spectrum of energies, where the output recorded by the MCA is calibrated to match the amount of energy deposited in the detector. The output is a histogram of counts vs “channel number” (= histogram bin number. An instruction manual comes with each device and computer (PC). You should take a part of the first laboratory session to become familiar...
with the operation of the detector and the PC with the MCA card. Learn how to record and erase spectra, how to store spectra on your disk and how to subtract background spectra from spectra containing interesting characteristics. You should also learn how to make hard copies of your plots for inclusion into your formal write-up. Below you will find basic instructions for using the MCA; in an Appendix you will find more detailed information about the MCA.

**Calibration**

Calibration is the process of turning the horizontal scale of the histogram (“channel number”) into an energy scale in keV, by using the facts that a) the channel number is proportional to energy and b) certain features in the histogram occur at specific energies. We will rely on four lines as standards: (1) the photo-peak of the 661.6 keV gamma ray, which is the highest energy peak in the $^{137}$Cs spectrum, (2) a smaller X-ray peak at 32 keV which is the characteristic Barium K X-ray emitted by the $^{137}$Cs source, (3) the photo-peak of the 1.33 MeV gamma ray of the $^{60}$Co source, and (4) the photo-peak of the 1.17 MeV gamma ray of the $^{60}$Co source. The “photo-peaks” are caused by photons of specific energy being fully absorbed by the detector.

First use the $^{137}$Cs source. Put the source near the top of the source holder. Use the AutoCalibrate feature to find an appropriate high voltage and amplifier gain. The result will place the photopeak at 662 keV (AutoCalibrate expects a Cs source!).

Next add the $^{60}$Co source and take data with both sources side by side. You will probably find that the Co peaks (and 1300 keV) are not on scale. Go to Settings | Amp/HV/.. and lower the fine gain until the Co peaks are on scale (should find the Cs peak a bit below middle of x axis). Then (taking more data if necessary) perform a 2-point calibration, selecting 2 appropriate lines from the 4 available. **Check that the other two values are accurate!** From now on carefully refrain from changing the high voltage or gain. After calibration, the program gives energy corresponding to the cursor position instead of channel number. Check the energies by putting the cursor on your peaks. Save.

**If you find these instructions too sketchy, refer to Appendix 1.**

You will need to repeat this process the second week of the lab before you go on to do the remainder of your measurements. If you have time left, start your measurements (read on!).

**Measurements**

**General Overview**

We will not measure the angular Compton scattering distribution given by the Klein-Nishina formula. That would need a very powerful source of gamma rays. The safe handling of these sources would not be practical in this laboratory setting. We will, however, measure the end point for Compton scattering by measuring the energy of the photon that is back-scattered from a stationary electron. The photon is back-scattered when its final direction is roughly 180 degrees from its incoming direction (see Equation 1). The stationary electron from which it scattered might be in the detector, or in other nearby materials—electrons are in all matter. We will also measure the energy of the outgoing electron in the case that the incoming photon is back-scattered. We will do these measurements using first the $^{137}$Cs source and then the $^{60}$Co source, recalling that the former emits one photon and the latter two, at different energies.

In separate measurements we will measure the characteristic K X-rays of Pb and an “unknown” metal and use the energy of the characteristic spectra to identify the unknown.
The Compton Plateau

Measure the spectrum of the Cs source. Put it on a shelf just below the Nal detector. Choose a time sufficiently long so that statistics are not a problem. Save and print your spectrum.

**Question 1)** Is printing in linear or log better? Why?

Repeat for Co source. Do you need to perform background subtraction? If so, see detailed directions in Appendix 1.

On the spectra, label (as appropriate) the $^{137}$Cs and $^{60}$Co photo-peaks, the Barium X-ray, the Compton plateaus, and the Compton edge(s). Write the energy corresponding to each of these features next to the feature on the plot. Use the spectrum software to help with the energy measurements. Estimate your uncertainty in the peak position by trying to re-measure the peaks with the software. The Compton plateau should show up in your spectrum as a region of higher counting rate due to either electrons or photons in or near the most probable energy configurations, according to the theory of Compton scattering (remember that the detector measures either photons or electrons equally well).

**Q2)** Calculate the expected energy of each Compton edge from Compton effect kinematics.

**Q3)** Use the calculations to identify the Compton edges on the plots and label them according to the photo-peak from which they originate. Are they consistent?

The Backscattered Photon

Put the $^{60}$Co source in the lowest position in the holder. Elevate your NaI(Tl) detector assembly above the table on the 2x4 blocks to reduce the scattering material directly under the source. Take a 15 minute (Live time) measurement in this configuration and store it as background. Put six thick aluminum plates under the source holder (but on top of the blocks) and repeat the 15 minute measurement. Using the strip function of the MCA program, subtract the stored spectrum from the new one to see the spectrum of photons scattered from the aluminum.

Plot the difference spectrum.

**Q4)** Measure, identify and explain the new source of gamma line(s) How does this line (or lines) relate to the measurements you made on the Compton Plateau?

**Q5)** Calculate a predicted energy for this peak and compare to your measurement.

**Q6)** Can you demonstrate that energy is conserved in the Compton scattering process, using your measured values?

**Q7)** Why is it important to put the source in the lowest position for this measurement?

The K X-rays of Pb

This measurement is performed in a similar detector configuration to the measurement (identification) of the backscattered photon, but now our goal is to concentrate on identifying a characteristic X-ray (much lower energy) a substance. Note that back-scattering itself comes from (nearly) free electrons, which are in all atoms, so don’t really help identify a substance.

Count and store a background run. Repeat the previous measurement with the lead plate under the detector instead of Al. After background subtraction, identify a photon line near 80 keV.

**Q8)** Note its energy and provide an explanation for its existence.

**EC)** Why would a characteristic X-ray from Al be harder to find than one from Pb?
Identification of Unknown Metal

Count and store a background run. Place the “unknown” metal sample under your counter and count for the same period of time.

Q9) Note the appearance of an X-ray line in the subtracted spectrum.
Q10) From the measurement of its energy, try to identify the “unknown” element. A good resource is the web site

http://Hyperphysics.phy-astr.gsu.edu/hbase/Tables/kxray.html

which derives its data from a comprehensive database maintained by NIST, the National Institute of Standards and Technology. This will also list an energy for the Pb X-rays. With our apparatus, we won’t be able to distinguish between the lines corresponding to the two closely spaced X-ray lines listed, which are less than 1-2 keV apart for most elements.

The Report

Your report should contain a brief description of the experiment. It should also contain an orderly exposition of your measurements including computer printouts and graphs where appropriate. And of course, a summary table. Show examples of needed calculations and state clearly your conclusions: what did you measure? Did it agree with expectations?

Extra credit:
Derive formula (6) from Eq 1-3.
Derive (explaining as you go) formula (1); you’ll need relativistic particle kinematics.
There are a number of possible Extra Credit measurements as well; ask your TA for suggestions.

Appendix 1: Multichannel Analyzer Detailed Instructions

Basic commands for software:
Turn on analyzer box’s power switch before clicking on the little spectrum icon on the PC.

There may be issues with starting up the MCA after computers have been down for a while. Some computers claiming to not find any such USB hardware, or the claiming the USB is off, when it’s not. Sometimes just trying again does the trick, or logging out and back in. Sometimes it required a full reboot *with power off*, then once you were logged back in, turn the power on, then try again. At worst you may have to possibly agree to a series of screens to install software: say Accept, Continue Anyway, etc through at least 2 cycles of install. Remember, you certainly need power on the box *before* clicking on the MCA icon.

You start in Live mode (see title bar).
Settings | Energy Calibration | Autocalibrate for Cs to do initial setup. Print and it will preserve settings.
Eraser Icon: deletes spectrum
File | Print prints spectrum, along with additional settings
Now put *both* Cs and Co sources in top drawer.
Click Go and keep going till you like it, then Stop
File | Save to save spectrum
( or Save Setup and later Load Setup to snag the settings you used later)
Timed spectrum: Settings | Presets | Live Time then pick some big number like 1000 sec
Region of Interest Settings | ROI | Set ROI
Drag cursor from lower to upper region, fairly symmetrical around peak
Gives you total counts in peak, and calculates Centroid (peak) & full width at half-maximum
Can also do Settings | Presets | Integral Counts to run until desired counts in ROI
Ba X-ray: it may appear cut off or hard to see. If so, try the following: on the bottom of the x axis there are two little triangles. These are ULD and LLD (upper and lower discriminator level). The default LLD is 6%; lower it by clicking and dragging all the way to the left (0%).

Calibration
Don't be bothered by the report that the detector is at 500V after Autocalibrate: there is no strong reason for the NaI voltage to be the same as a GM voltage! The important issue is whether it puts the Cs peak at 662 like it should.

Next do 2-point calibration with Co and Cs sources side by side. Autocalibrate doesn’t work because it will put the Co gammas off scale (> 1024) after having put the Cs at channel 662.

Pick keV for units then click on peaks you choose, giving energy in KeV for each
If both Co peaks aren’t on scale, or 1300 keV isn’t on the x axis, go to Settings | Amp/HV/.. and lower the fine gain until the Co peaks are on scale.

Be sure to check all four calibration points to see that they have good energies. This will almost certainly force you to pick Ba X-ray as one of the calibration points. The peak of the Pb X-ray is at the K alpha line. K alpha is a close-spaced doublet, usually the strongest of the K X-rays, from 2p to ground.

Save your spectra! Then you can go back and remeasure! But print in live mode to capture settings. Just print from the MCA program—no need to export to Kgraph or Excel this week.

Background subtraction
1) The software has the smarts to scale background by live time.
2) Procedure: save real and bkg spectrum via File | Save File | Open (you are now in file mode—see Title bar; by default, same size as original screen!)
   Open spectrum file
   Then Strip Background | Load Background | open background file
   | Strip subtracts it. You can also display side by side.
For whatever reason, the term commonly used in Nuclear Physics for background subtraction is Stripping the Spectrum.

Appendix 2: Some Details of the Compton Effect

The Klein-Nishina Angular Distribution Formula
While Equations 1-3 tell us how to compute the energies of the scattered photon and electron in terms of the photon's angle, they do not tell us anything about the likelihood of finding a scattered photon at one angle relative to another. For this we must analyze the scattering process in terms of the interactions of electrons and photons.

The electron-photon interaction in the Compton effect can be fully explained within the context of our theory of Quantum Electrodynamics or QED for short. This subject is beyond the scope of this course and we shall simply quote some results. We are interested particularly in the angular dependence of the scattering or the differential cross-section and the total cross-section
both as a function of the energy of the incident photon. If you haven’t seen the term “cross section” before, for now you can just think of it as a quantity proportional to the relative probability of an interaction with a particular value of the variables.

First we write the differential cross-section, also known as the Klein-Nishina formula:

$$\frac{d\sigma}{d\Omega} = \frac{1}{r_0^2} \frac{\alpha^2}{[1 + 2 \varepsilon \sin^2 \frac{\Theta}{2}]^2} \left\{ 1 + \frac{4 \varepsilon^2 \sin^4 \frac{\Theta}{2}}{[1 + \cos^2 \Theta][1 + 2 \varepsilon \sin^2 \Theta/2]} \right\}$$  \hspace{1cm} (5)

where $\varepsilon = E_0/m_ec^2$ and $r_0$ is the "classical radius of the electron" defined as $e^2/m_ec^2$ and equal to about $2.8 \times 10^{-13}$ cm. The formula gives the probability of scattering a photon (integrated over $\phi$) into the solid angle element $d\Omega = 2\pi |d(\cos \Theta)| = 2\pi |\sin \Theta| d\Theta$ when the incident energy is $E_0$. We illustrate this angular dependence in Figure 3 for one particular photon energy.

Note that the most likely scattering is in the forward direction and that the probability of scattering backward is relatively constant with angle. On this plot and fig 2 we illustrate the dependence on initial photon energy $E_0$ by choosing some arbitrary values. Since the gamma ray energies of interest are around 1 MeV, we choose $E_0=0.5, 1.0, 2.0 m_ec^2$ as sample energies, that is for roughly $0.25, 0.5,$ and $1$ MeV (since $m_ec^2 = 0.511$ MeV).

Since we will be measuring energy, it is of interest to rewrite this to give the probability of measuring electrons with a given kinetic energy $T = E_e - m_ec^2$. We can get the expression plotted in figure 2 by substituting for the angle $\Theta$ in Eq. (5) via Equations (2) and (3), and further by using the solid angle definition given below Eq. (5), and applying the relationship:
\[
\frac{d\sigma}{dT_e} = \frac{d\sigma}{d\Omega} \cdot \frac{d\Omega}{dT_e} = \frac{d\sigma}{d\Omega} \cdot \frac{2\pi m_e c^2}{(E_0 - T_e)^2} = \frac{d\sigma}{d\Omega} \cdot \frac{2\pi}{m_e c^2 (\varepsilon - t)^2}
\]  \hspace{1cm} (6)

The final form on the right uses the definitions \( \varepsilon = E_0/m_e c^2 \) and \( t = T_e / m_e c^2 \).

**Energy dependence**

The Klein-Nishina formula can be integrated over photon scattering angle (or electron recoil energy) to yield the total cross-section, which displays the dependence on the incident energy (shown in Fig 4) for the Compton process:

\[
\sigma = 2\pi \cdot r^2 \left\{ \frac{1+\varepsilon}{\varepsilon^2} \left[ \frac{2+2\varepsilon}{1+2\varepsilon} - \frac{\ln(1+2\varepsilon)}{\varepsilon} \right] + \frac{\ln(1+2\varepsilon)}{2\varepsilon} - \frac{1+3\varepsilon}{(1+2\varepsilon)^2} \right\}
\]  \hspace{1cm} (7)

![Fig. 4: Energy dependence of Compton scattering, vs. \( x = \varepsilon/E_0 \cdot m_e c^2 \).](image)

The Compton process is weakly energy dependent up until about .1 MeV, when it begins to decrease significantly; other processes become more important at higher energies.