PHY321 Homework Set 1

1. [5 pts] Expand $\sin x$ about the point $x = \pi/4$. Hint: Represent the function as $\sin x = \sin(y + \pi/4)$ and assume y to be small.

2. [5 pts] For the two vectors

$$\vec{A} = \hat{i} - \hat{j} + 2\hat{k}\,, \quad \text{and} \quad \vec{B} = -2\hat{i} + \hat{j} + 3\hat{k}\,,$$

find

   (a) $\vec{A} - \vec{B}$ and $|\vec{A} - \vec{B}|$,

   (b) component of $\vec{B}$ along $\vec{A}$,

   (c) angle between $\vec{A}$ and $\vec{B}$,

   (d) $\vec{A} \times \vec{B}$,

   (e) $(\vec{A} + \vec{B}) \times (\vec{A} - \vec{B})$.

3. [10 pts] Coordinates of a particle moving in the x-y plane change with time t according to:

$$x(t) = 2\,\mathrm{m} + 2\,\mathrm{m/s} \cdot t\,, \qquad y(t) = 3\,\mathrm{m} + 4\,\mathrm{m/s^3} \cdot t^3\,,$$

   where t is in seconds and coordinates are in meters.

   (a) Obtain the equation of the trajectory $y = y(x)$ for the particle. Draw the trajectory within the region $0 \le x \le 4$ and $-1 \le y \le 7$.

   (b) Obtain the velocity components $v_x(t)$ and $v_y(t)$ as well as the magnitude of the velocity $v(t)$. For $t = -0.5$, 0 and 0.5 s, mark particle locations on the trajectory and the velocity vectors as arrows.

   (c) Obtain the components of the acceleration $a_x(t)$ and $a_y(t)$ as well as the magnitude of the acceleration $a(t)$. Indicate the acceleration vectors with arrows by the trajectory, for $t = -0.5$, 0 and 0.5 s.

   (d) Obtain the components of the acceleration along $a_\parallel(t)$ and perpendicular $a_\perp(t)$ to the trajectory. Hint: Using $\vec{v}$, construct unit vectors tangential $\hat{u}_\parallel$ and perpendicular $\hat{u}_\perp$. The parallel and perpendicular components of the acceleration may be then found from $a_\parallel = \vec{a} \cdot \hat{u}_\parallel$ and $a_\perp = \vec{a} \cdot \hat{u}_\perp$.

   (e) Discuss the behavior of the components $a_\parallel$ and $a_\perp$ with time in the context of the shape of the trajectory.

4. [10 pts] For a hill the elevation in meters is given by $z = 10 + 0.5\,x + 0.25\,y + 0.5\,x\,y - 0.25\,x^2 - 0.5\,y^2$, where x is the distance east and y is the distance north of the origin.

   (a) Where is the top of the hill and how high is it?

   (b) How steep is the hill at $x = y = 1$, i.e. what is the angle between a vector perpendicular to the hill and the z axis?

(c) In which compass direction is the slope at $x = y = 1$ steepest? Indicate whether the angle you provide is the angle measured in the standard way in the counter-clockwise direction from x-axis (east) or whether it is the compass azimuth. The compass azimuth is normally measured in the *clockwise* direction from *north*.

5. [10 pts] Consider action of two forces, $\vec{F}^A(\vec{r})$ and $\vec{F}^B(\vec{r})$, on a particle. These forces depend on particle position $\vec{r}$ and their components are given by

$$F_x^A = F_x^B = y^2\,, \quad F_y^A = 2xy\,, \quad F_y^B = xy\,, \quad F_z^A = F_z^B = 0\,,$$

where the force components are in newtons and coordinates are in meters.

(a) Compute the work in joules done by the two forces, $W^A = \int \vec{F}^A \cdot \mathrm{d}\vec{r}$ and $W^B = \int \vec{F}^B \cdot \mathrm{d}\vec{r}$, on the particle moving within the x-y plane along a parabolic trajectory $y = x^2$ from $(x, y) = (0, 0)$ to $(x, y) = (1, 1)$. The coordinates in the equation for the trajectory are in meters. Hint: Under the integral write $\vec{F}^A \cdot \mathrm{d}\vec{r} = F_x\,\mathrm{d}x + F_y\,\mathrm{d}y = \left(F_x + F_y\,\frac{\mathrm{d}y}{\mathrm{d}x}\right)\mathrm{d}x$ and carry out integration over x.

(b) Compute the work in joules done by the two forces, on the particle moving along another trajectory joining $(0, 0)$ and $(1, 1)$ and consisting of two straight at right angles to each other, first at constant y to $(x, y) = (1, 0)$ and then at constant x to $(1, 1)$. Compare the results to those obtained in 5a. Hint: Sketch the trajectory.

(c) Calculate the *curl* of the two forces.

(d) Discuss difference in the nature of the two forces in the context of the results in 5a, 5b and 5c. Can you find potential energy associated with either of the forces?