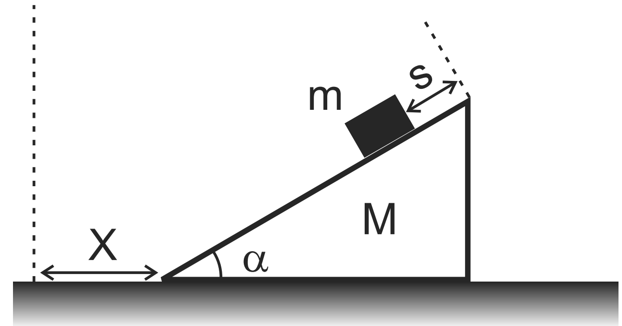
PHY321 Homework Set 11

1. [5 pts] A small block of mass m slides without friction down a wedge-shaped block of mass M and of opening angle α. The triangular block itself slides along a horizontal floor, without friction. Your ultimate goal will be to find the horizontal acceleration $\ddot{X}$ of the triangular block, following the second and third Newton's laws. The cartesian coordinates for m are x and y.



   (a) Consider the forces acting on the mass m and the wedge block. Express the components of the acceleration $\ddot{x}$ and $\ddot{y}$ for m and $\ddot{X}$ for M in terms of the force components.

   (b) The system of equations you wrote cannot be closed without introducing an equation of constraint between x, y and X. Express the equation of constraint in terms of the angle α. Differentiate the equation to get an equation of constraint between the acceleration components.

   (c) Solve the system of equations for the acceleration components, to arrive at $\ddot{X}$.

2. [10 pts] Now solve the preceding problem following the Lagrangian method and using X and distance s traveled by m down the wedge as generalized coordinates.

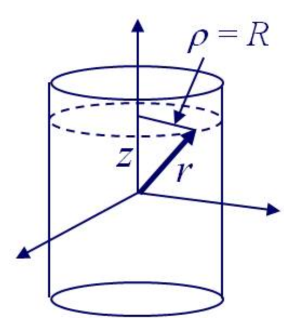
   (a) Express the total potential energy U of the block system in terms of the cartesian coordinates. Reexpress that energy in terms of the generalized coordinates. An arbitrary reference constant in the potential energy is of no relevance.

   (b) Express the total kinetic energy T of m and M in terms of the cartesian coordinates. Reexpress that energy in terms of the generalized coordinates.

   (c) Derive the Lagrange equations for the system. Manipulate them to obtain $\ddot{X}$. Does your result agree with that from the preceding problem?

   (d) Given that there is no external horizontal force component acting on the block system, the center of mass of that system should not accelerate in the horizontal direction. Demonstrate that the lack of acceleration for the center of mass follows from your Lagrange equations.

3. [10 pts] A particle of mass m is constrained to move on the cylindrical surface described in cylindrical coordinates (ρ, ϕ, z) by the constraint equation $\rho = R$. The *only* force acting on the particle is the central force directed towards the origin, $\vec{F} = -k\,\vec{r}$. Your task will be to obtain and solve the Lagrangian equations for the particle, with and without an explicit reference to the constraint equation.



(a) At first introduce the constraint inherently and express the Lagrangian for the particle only in terms of the generalized coordinates z and ϕ, and associated velocities, excluding the possibility of the particle leaving the cylindrical surface.

(b) Obtain and solve the Lagrange equations associated with the coordinates z and ϕ. Describe in words the motion that the particle executes.

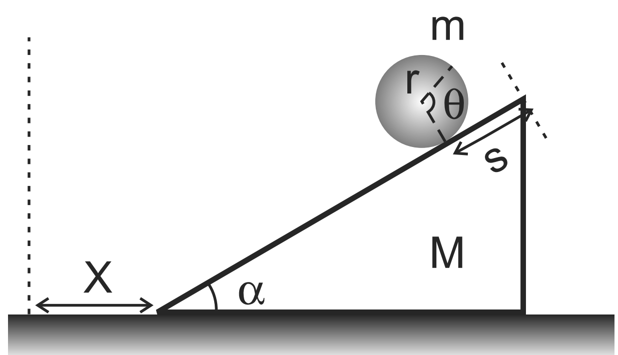
(c) Now restart the problem from scratch and construct the Lagrangian in terms of the coordinates ρ, ϕ and z, allowing at this stage for the possibility of the particle moving outside of the cylindrical surface.

(d) Obtain the Lagrange equations associated with the coordinates ρ, ϕ and z, incorporating explicitly the constraint equation with the accompanying, yet undetermined, multiplier λ.

(e) Solve the Lagrange equations. Arrive at the dependence of the multiplier λ on time, following from the requirement of the particle remaining on the cylindrical surface.

(f) What conclusion can you draw from the Lagrange equations on the normal force that the constraint surface exerts on the particle?

4. [10 pts] This problem is similar to an earlier one within the set. A uniform ball of mass m and radius r rolls down, without slipping, down a wedge-shaped block of mass M and of opening angle α. The triangular block itself slides along a horizontal floor, without friction. Your general task will be to find the horizontal acceleration $\ddot{X}$ of the triangular block, following the Lagrangian method and using X and distance s, traveled by m down the wedge, as generalized coordinates. The cartesian coordinates for the center of the ball are x and y. The angle by which the ball has rotated is θ.



(a) Express the net kinetic energy T of the system in terms of the cartesian coordinates and angle θ. Reexpress that energy in terms of the generalized coordinates. In the latter task, you need to exploit the no-slip condition connecting the angle of rotation θ to the distance traveled s.

(b) Obtain the Lagrangian in terms of the generalized coordinates and the Lagrange equations. Manipulate the equations to arrive at $\ddot{X}$.

(c) How does your answer compare to one obtained in the earlier similar problem? Is the acceleration now larger or smaller than before?

5. [5 pts] (adapted from a graduate qualifying exam) A bead of mass m can slide without friction on a vertical hoop of radius R. The hoop is rotating at constant angular speed ω about a vertical axis passing through the hoop's center.

   (a) Determine the Lagrangian for the bead, using the indicated angle θ as the generalized coordinate, and including the effects of gravity.

   (b) Obtain an equation of motion for the bead in terms of the angle θ.

   (c) Determine any equilibrium angles $\theta_{\mathrm{eq}}(\omega)$ where the bead would remain if there initially at $\dot{\theta} = 0$.