**PHY321 Homework Set 3**

1. [5 pts] Determine the location of the center of mass of a uniform solid cone of base radius $R$ and height $H$.

2. [10 pts] A thin plate with uniform areal density $\sigma$ is bounded by two curves $f(x)$ and $g(x)$ over an $x$-interval $[a, b]$, where $f(x) \geq g(x)$.

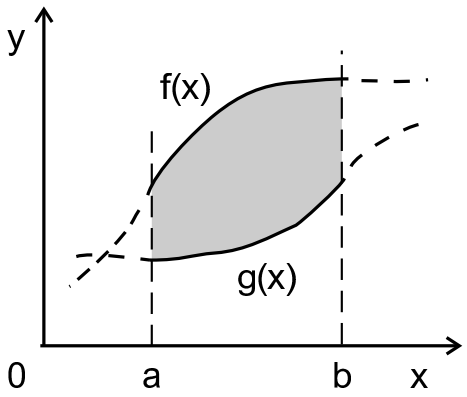
   (a) Derive an expression for the mass of the plate in terms of an integral involving the functions $f$ and $g$.

   (b) Derive an expression for the position $X$ of the center of mass of the plate along the $x$-axis, in terms of integrals involving the $f$ and $g$ functions.

   (c) Derive an expression for the position $Y$ of the center of mass of the plate along the $y$-axis, in terms of analogous integrals.

   (d) Find values for the center of mass coordinates $X$ and $Y$ for a plate bounded by the functions $f(x) = \cos x$ and $g(x) = 0$, over the interval $[-\frac{\pi}{2}, \frac{\pi}{2}]$.

   (e) Find values for the center of mass coordinates $X$ and $Y$ for a plate bounded by the functions $f(x) = \sqrt{x}$ and $g(x) = x^2$, over the interval $[0, 1]$.



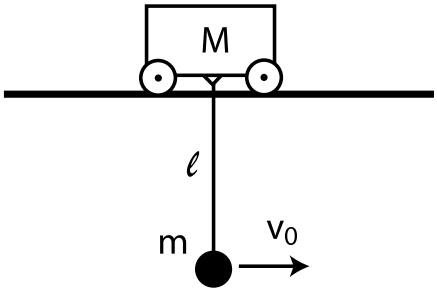

3. [5pts] In testing a missile defense system, a missile is fired from the ground on a trajectory that would directly hit a bunker some distance away. When the missile is at the top of the trajectory, a laser light from the bunker ignites fuel in the missile and the missile disintegrates into two pieces, one twice as massive as the other. The pieces reach the ground nearly simultaneously, 60 m apart from each other.

   (a) By how much does the larger piece miss the bunker? Hint: Consider motion of the center of mass.

   (b) By how much does the smaller piece miss the bunker?

   (c) How important is the information that the fuel ignited at the top of the trajectory?

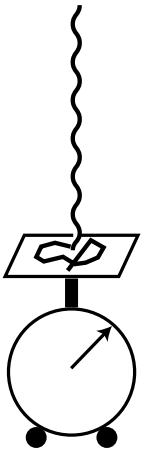
4. [5 pts] Ball of mass $m$ hangs on a string of length $\ell$ straight down from a cart of mass $M$ standing on horizontal rails. The cart can move along the rails without friction. While the cart is at rest, the ball is given a horizontal velocity $v_0$, $v_0 < \sqrt{2g\ell}$, directed along the rails.




   (a) What is velocity of the ball when it reaches its maximal elevation?

   (b) What is elevation $h$ that the ball reaches above its original location?

5. [5 pts] A chain of mass $M$ and length $L$ is suspended vertically with its lower end touching the scale. The chain is released and falls onto the scale. Determine the force read by the scale when the length $x$ of the chain has fallen. Neglect the size of individual links.



6. [5 pts] A centrifugal pump processes $j = 18\,\mathrm{m}^3/\mathrm{min}$ of water, with inlet and outlet velocities of $v = 15\,\mathrm{m/s}$. The impeller rotates in the clockwise direction around the shaft $\mathcal{O}$. The horizontal inlet and outlet are positioned, respectively, at $h = 8.0\,\mathrm{cm}$ below and $H = 20\,\mathrm{cm}$ above $\mathcal{O}$. The pump is attached to the floor by two supports $A$ and $B$, separated by $d = 34\,\mathrm{cm}$ and positioned symmetrically at the two sides of $\mathcal{O}$. The pipes connected to the inlet and outlet act to contain the water, balancing its pressure and together playing no role in holding the pump in place.

   When the impeller is not rotating, but the pump is filled with water, the forces acting on the supports point downward and are both equal to 150 N. Determine net vertical forces onto the two supports, magnitude and orientation, when the pump operates. Hint: Consider the rate at which the angular momentum of the water changes due to the action of the pump.