PHY321 Homework Set 4

1. [5 pts] N people, each of mass m_p, stand on a railway flatcar of mass m_C. They jump off one end of the flatcar with velocity u relative to the car. The car rolls in the opposite direction without friction.

   (a) What is the final velocity of the car if all the people jump at the same time?

   (b) What is the final velocity of the car if the people jump off one at a time? Leave the answer as a sum of terms.

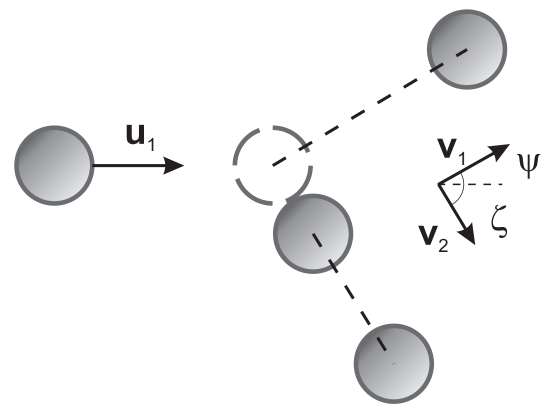
   (c) Does case 1a or 1b yield the largest final velocity of the flat car?

2. [5 pts] A puck of mass of 0.200 kg moving at $u_1 = 3.0\,\mathrm{m/s}$ approaches an identical puck that is stationary on frictionless ice. After the collision, the first puck leaves with a speed v_1 at $\psi = 30°$ relative to the original line of motion; the second puck leaves with speed v_2 at $\zeta = 60°$.

   (a) Determine v_1 and v_2.

   (b) What are the relative speeds of the pucks before and after the collision? Is the collision elastic or inelastic?

   (c) What are the angles and magnitudes the final puck velocities in the CM system of this collision?



3. [5 pts] A block of mass $m_1 = 1.00\,\mathrm{kg}$, moving at a speed $u_1 = 4.00\,\mathrm{m/s}$, collides with another block of mass $m_2 = 10.0\,\mathrm{kg}$ at rest. The lighter block comes to rest after the collision.

   (a) What is the speed of the heavier block after the collision?

   (b) What is the coefficient of restitution for the collision?

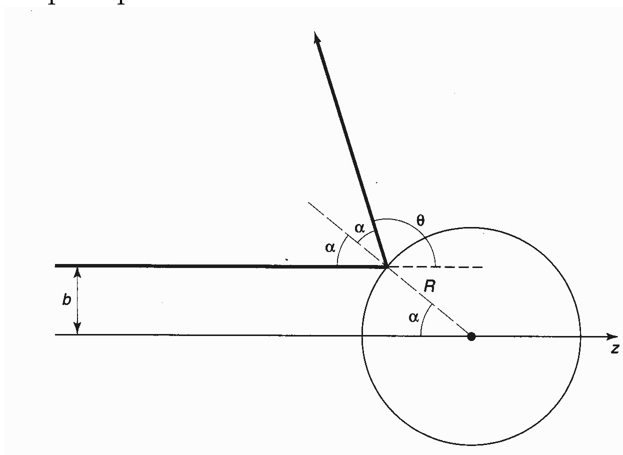
   (c) What is the reduced mass for the system?

   (d) How much of the relative energy has been dissipated for this collision, in absolute terms and as percentage of original CM energy?

4. [5 pts] Determine the differential cross section $\sigma(\theta) \equiv \mathrm{d}\sigma/\mathrm{d}\Omega$ and total cross section σ_t for elastic scattering of a point particle from a strong repulsive potential sphere of radius R:

$$U(r) = \begin{cases} 0, & r < R\,, \\ U_0, & r > R\,, \end{cases}$$

where $U_0 \to \infty$. For a particle scattered from a strong potential, the law of reflection is valid, see the figure. Hint: From geometry work out the relation between the scattering angle θ and the impact parameter b.



5. [5 pts] A rocket of mass m_0 starts its engine in interstellar space. Assuming constant speed u of the exhaust gas relative to the rocket, at what fraction of the original mass is the rocket going to achieve maximal momentum?

6. [10 pts] Consider the problem of a rocket ascending vertically against gravity. The rocket starts from rest and its initial mass is m_0. The rocket's fuel burns at a constant rate α and exhaust gas leaves the rocket at a constant speed u relative to the rocket. A convenient characteristic of a rocket, that is commonly used in place of α, is the initial trust-to-weight ratio $\tau_0 = \alpha\, u / m_0\, g$.

   (a) From the expression for the velocity of the rocket as a function of the remaining mass m,
   $$v = -(m_0 - m)\,\frac{g}{\alpha} + u \ln\left(\frac{m_0}{m}\right),$$
   eliminate α and write the velocity in terms of u, the mass ratio m_0/m and τ_0.

   (b) Demonstrate that, for the lift-off to occur, the rocket must be light enough so that $\tau_0 > 1$.

   (c) Integrate the velocity with respect to time to obtain elevation h of the rocket as a function of the remaining mass m. Note that the integration with respect to time can be easily converted to integration with respect to mass exploiting the linear relation between the two variables. Note further that $\int \mathrm{d}x \, \ln x = x \ln x - x$. Again eliminate α from your result and represent h in terms of τ_0, u, g and the mass ratio m_0/m.

   (d) Consider the case of Ariane 5 rocket with initial mass of $m_0 = 7.77 \times 10^5\,\mathrm{kg}$. During the initial stage-0 of the flight, boosters are used that provide a thrust of $\alpha u = 1.29 \times 10^7\,\mathrm{N}$ and employ solid fuel with exhaust velocity of $u = 3010\,\mathrm{m/s}$. At the end of stage-0, the rocket mass drops to $m = 2.23 \times 10^5\,\mathrm{kg}$. Find τ_0 and mass ratio m_0/m. Use those to determine the expected velocity and elevation of the rocket at the end of stage-0.