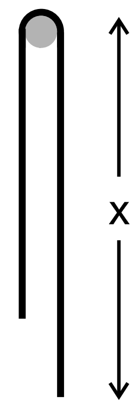
PHY321 Homework Set 5

1. [5 pts] A uniform rope of mass M and length L is hung off a small peg. The rope can slide without friction over the peg.

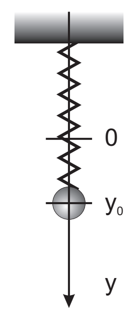
   

   (a) With x denoting the length of rope hanging to the right of the peg, obtain an equation of motion for $x(t)$.

   (b) Solve the equation of motion, if an x_0-stretch of the rope hangs to the right of the peg at $t = 0$ and the rope is then at rest. How does the fate of the rope depend on whether $x_0 > L/2$ or $x_0 < L/2$?

   (c) Find the time t_f that it takes for the rope to fall off the peg. How does this time depend on M?

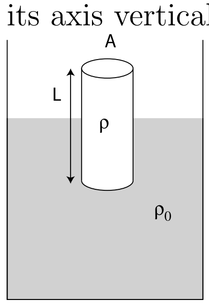
2. [5 pts] A massless spring hangs down from a support, with its lower end at $y = 0$, where the y-axis is vertical and points down. When a small unknown mass is attached to the spring, the lower end of the spring moves down to a position y_0 for the mass being in equilibrium.

   (a) Demonstrate that when the mass is pulled down to a position $y = y_0 + A$ and released from rest, it will execute a simple harmonic motion around y_0.

   (b) Express the period of oscillations of the mass in terms of y_0 and g.

3. [5 pts] [adapted from Graduate School qualifying exam] A large axially symmetric cylinder of length L, cross-sectional area A and of average density ρ is floating with its axis vertical, in a fluid of density ρ_0, $\rho_0 > \rho$.

   (a) Determine the frequency ν_0 of small-amplitude vertical oscillations of the cylinder. Note: Effects of viscosity and fluid adhesion and cohesion may be ignored.

   (b) Compute the frequency for $\rho_0 = 1.00\,\mathrm{g/cm^3}$, $\rho = 0.80\,\mathrm{g/cm^3}$ and $L = 3.0\,\mathrm{cm}$.

4. [5 pts] Two masses m_1 and m_2, connected by a massless spring of neutral length x_0, move freely along the x-axis. For small changes in spring's length, the force produced by the spring is proportional to the change in length, with the coefficient of proportionality k.

   (a) Rely on the third Newton's law and demonstrate directly that the center of mass for m_1 and m_2 moves at constant velocity.

(b) Further, rely on the third Newton's law and demonstrate that for separations close to x_0, the masses execute a simple harmonic motion in their separation. Obtain the angular frequency ω_0 of small oscillations of the masses in relative distance.

5. [5 pts] A simple harmonic oscillator consists of a $m = 15\,\text{g}$ mass attached to a spring with spring constant of $k = 4.0\,\text{N/m}$. The mass is displaced by $A = 3\,\text{cm}$ and released from rest.

   (a) Determine the anticipated natural frequency ν_0 and period T_0 for the motion,

   (b) the total energy,

   (c) and the anticipated maximum speed.

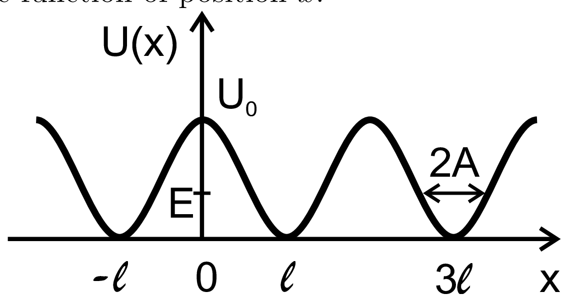
   (d) In the actual measurement of the system, the amplitude of the oscillations is found to decrease to half of the original value after 8.0 s. Determine the parameter β for this motion.

   (e) Find the frequency ν_1 for the damped motion and compare it to the anticipated frequency ν_0.

   (f) Find the decrement of the damped motion, i.e. the fraction by which the amplitude decreases during one period of motion.

6. [5 pts] A particle of mass m moves in one dimension under the influence of a force for which the potential energy is a periodic function of position x:

$$U(x) = U_0 \cos^2\left(\frac{\pi x}{2\ell}\right).$$



For a net energy $0 < E < U_0$, the particle is trapped in the vicinity of one of the minima of the potential energy and its motion is periodic in time.

   (a) Relying on the second Newton's law, obtain an equation of motion for the particle.

   (b) Consider the case of motion where the maximum displacement A of position from one of the minima is small compared to ℓ and expand the force to the lowest order in the displacement of the particle.

   (c) Find an approximate period of motion T_0 of the particle when the maximum displacement is small. Does that period depend on A?

   (d) What is the maximal percentage error in the force when employing the expansion 6b, for $A = 0.1\ell$? What magnitude of percentage error do you correspondingly expect in the period T_0 obtained using the expansion 6b, as compared to the real period T for the particle? If the period T for a given amplitude A were expanded in the powers of A/ℓ, what kind of powers would you expect in the expansion?