Motional EMF

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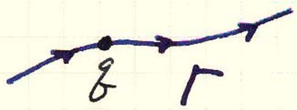
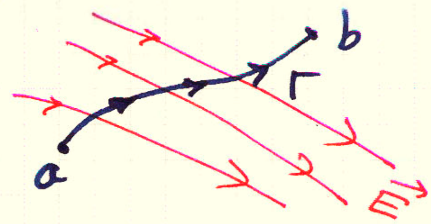
EMF = Electro Motive Force



Define $\quad \text{EMF}(\Gamma) = \int_\Gamma \frac{\vec{F}}{q} \cdot d\vec{\ell} = \frac{W(\Gamma)}{q}$

In words, the electromotive force along a curve Γ is the work per unit charge that would be done on a particle that moves along Γ.

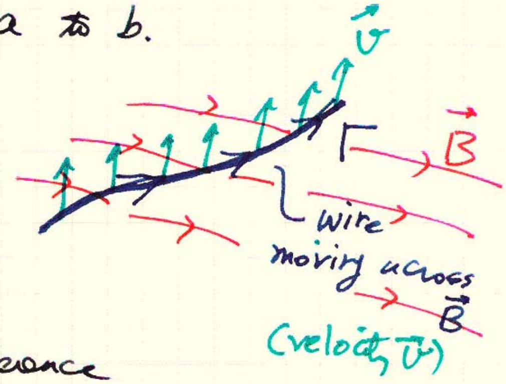
- For the electrostatic force



$$\text{EMF} = \int_\Gamma \vec{E} \cdot d\vec{\ell} = -\Delta V$$

$$= V(a) - V(b)$$, independent of the path from a to b.

- For the magnetic force



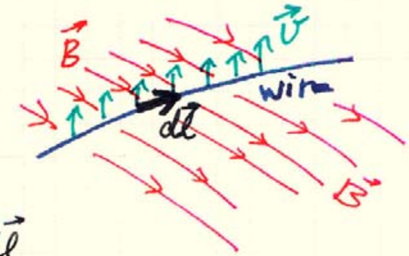
$$\text{EMF} = \int_\Gamma (\vec{v} \times \vec{B}) \cdot d\vec{\ell}$$

which is NOT a potential difference

Three quantities that can be confused:
EMF , electric potential, voltage difference.
They all have the same unit : volt

Motional EMF

If a conducting wire moves in a magnetic field, then there is an EMF along the wire

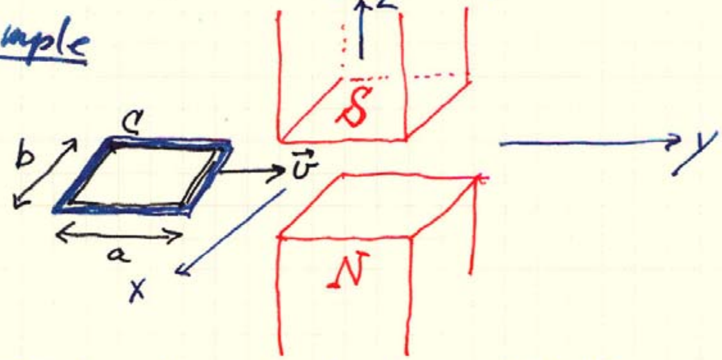


$$\text{EMF} = \int_{\text{wire}} (\vec{v} \times \vec{B}) \cdot d\vec{l}$$

$\vec{v}$ = velocity of the wire segment $d\vec{l}$

The EMF will drive a current $I = \text{EMF}/R$ if the wire is part of a complete conducting loop.

Example



The conducting loop C will be pulled through the gap between the magnet poles.

$$\vec{B} = B_z \hat{k} \quad ; \quad \vec{v} = v_y \hat{j}$$

- As the circuit enters the field, the counterclockwise EMF is

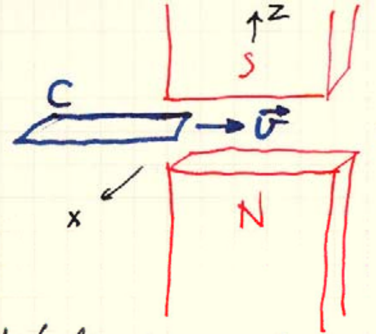
$$\text{EMF} = \int_{\substack{\text{length}\\ b}} \frac{q v_y B_z \hat{i}}{q} \cdot d\vec{l} \quad \text{w/ } d\vec{l} = -\hat{i}\, dl$$

$$\text{EMF} = -v_y B_z b \qquad \text{(counterclockwise)}$$

units: $\frac{m}{s} \, T m = V$ ✓

The counterclockwise **EMF**

$$= -v_y B_z b$$



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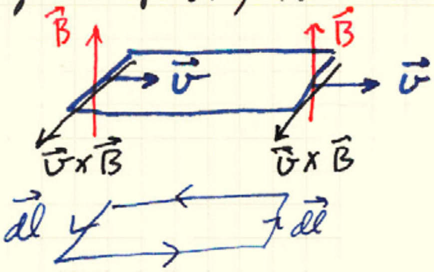
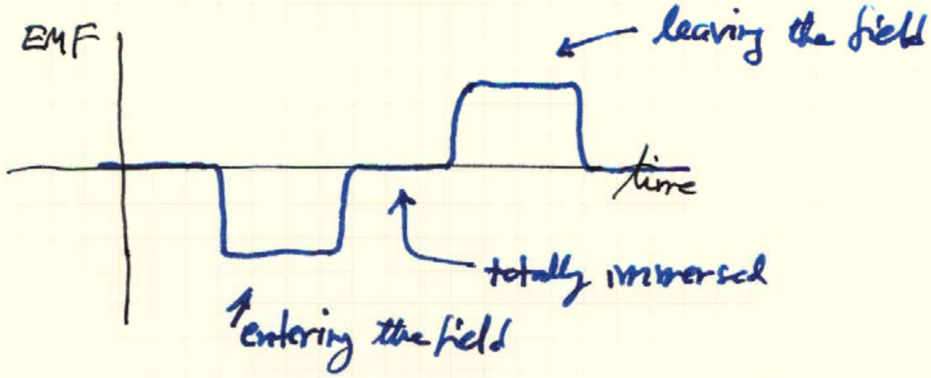
- While the circuit C is totally immersed in the magnetic field, the induced EMF = 0

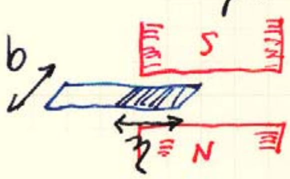


- As the circuit exits the field, the counterclockwise EMF is

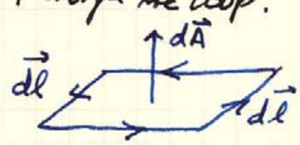
$$\text{EMF} = +\, v_y B_z b \qquad (\text{counterclockwise})$$



Compare the EMF to the <u>magnetic flux</u> through the loop.



$$\Phi = \int \vec{B}\cdot d\vec{A}$$
$$= BA = Bb\eta$$



Relate the Counter-clockwise EMF to the upward flux.

Comparing: $\text{EMF} = -\dfrac{d\Phi}{dt}$ "Motional EMF"

Summary

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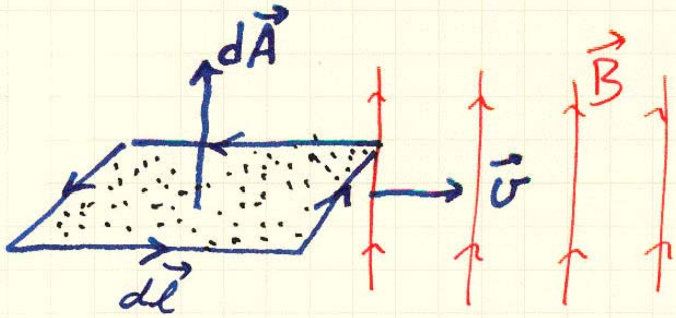


$$EMF = -\frac{d\Phi}{dt}$$

$$\oint_C \vec{E}\cdot d\vec{l} = -\frac{d}{dt}\int_S \vec{B}\cdot d\vec{A}$$

The directions of $d\vec{l}$ and $d\vec{A}$ are related by the Right Hand Rule.

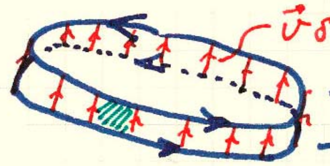
Counterclockwise EMF and upward flux.

General Theorem

If a rigid loop C moves in a magnetic field $\vec{B}$, then the EMF around C is

$$\text{EMF} = -\frac{d\Phi}{dt}$$ where Φ = flux of $\vec{B}$ through any surface bounded by C.

Proof



$\delta\Sigma$ = the area swept out by C (a ribbon)

$$\frac{\delta\Phi}{\delta t} = \frac{\Phi[S(t+\delta t)] - \Phi[S(t)]}{\delta t}$$

upward flux denoted Φ

But $\oint \vec{B}\cdot d\vec{A} = 0$ for any closed surface

(outward flux)

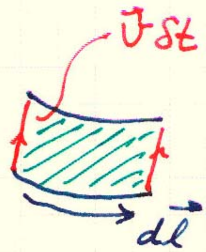
$$\therefore\ \tilde{\Phi}[S(t+\delta t)] + \tilde{\Phi}[S(t)] + \tilde{\Phi}(\delta\Sigma) = 0$$

$$\downarrow \qquad\qquad \downarrow$$

$$\Phi[S(t+\delta t)] - \Phi[S(t)] + \tilde{\Phi}(\delta\Sigma) = 0$$

outward flux is denoted by $\tilde{\Phi}$.

Thus $$\frac{\delta\Phi}{\delta t} = -\frac{\tilde{\Phi}(\delta\Sigma)}{\delta t}$$



Now, $$\tilde{\Phi}(\delta\Sigma) = \oint_C \vec{B}\cdot(d\vec{\ell}\times\vec{v}\,\delta t)$$

$$\vec{B}\cdot(d\vec{\ell}\times\vec{v}) = \epsilon_{ijk} B_i (d\ell)_j v_k = (\vec{v}\times\vec{B})\cdot d\vec{\ell}$$

So

$$\frac{\delta\Phi}{\delta t} = -\oint_C (\vec{v}\times\vec{B})\cdot d\vec{\ell} = -\text{EMF}$$

Q.E.D.

Quiz: The Flip Coil

A coil of wire is placed in a uniform magnetic field, with the normal vector of the coil parallel to the magnetic field vectors. The coil is connected to a galvanometer, which can measure the electric current versus time and the integrated current (i.e., total charge Q) around the coil.

Now the coil is flipped over, i.e., rotated by 180 degrees about the rotation axis. Determine ***the strength of the magnetic field*** from the total charge that passes through the galvanometer.