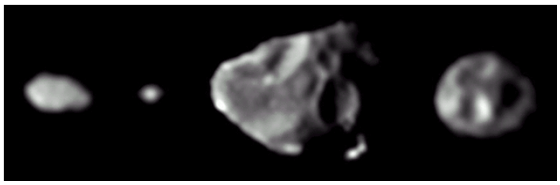
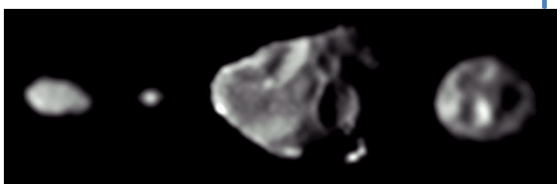
Rings vs. moons–28 Jan



Metis Adrastea Amalthea Thebe

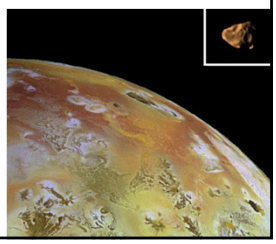
- Why are there no big moons near planets? Why are rings close to planets?
- Differences between Jupiter's inner and outer moons
- Tides
- Roche's limit: If a moon is near its planet, the tidal force of the planet is greater than the gravitational force of the planet: the moon cannot hold together.

The Innermost Moons of Jupiter



Metis Adrastea Amalthea Thebe

- Q5: What holds me together?
  a. Gravity
  b. Atomic bonds between the atoms
- Q6: What holds Io & Metis together? (Think about the shapes of Io & Metis.)
  a. Gravity for both
  b. Bonds for both
  c. Gravity for Io; bonds for Metis
  d. Gravity for Metis; bonds for Io.



Amalthea & Io

Tidal force

- The force of the sun on a rock or a drop of water on Earth is

$$F = -GM_s m/R^2$$

- Consider rocks along the line between the earth and sun. $R = R_{es} + r$.

$$F = -GM_s m(R_{es} + r)^{-2}$$

- Important math trick: If $x \ll 1$, $(1 + x)^n \approx 1 + nx$
- Taylor series: $f(x) = f(x_0) + (x - x_0)\, f'(x_0)$

$$F = -GM_s m{R_{es}}^{-2} + 2GM_s m{R_{es}}^{-3} r$$

- First term is force at center of earth.
- Second term is tidal force.

Tidal force

- Consider a rock along the line between the earth and sun. $R = R_e + r$.

$$F = -GM_s m{R_{es}}^{-2}(1 - \frac{2r}{R_{es}})$$

- First term is force at center of earth.
- Second term is tidal force.

1. To think about the tides, consider the force on a drop of water. What does the first term do to the drop of water? Answer: The first term accelerates the drop to keep it orbiting the sun.
2. Why are there two high tides every day? Answer: At noon (r<0) the tidal force pulls the drop away from Earth. At midnight, the tidal force (r>0) also pulls the drop away from Earth.

- Write the answers on paper. Turn your papers in.

1. If a planet is twice as far from the sun, the ratio of the tidal force to the force on the planet is ___ .
   A. ½
   B. ¼
   C. 2
   D. 4
2. If a planet has twice the radius, the ratio of the tidal force to the force on the planet is ___ .

How big is the tidal force?

- Consider rocks along the line between the earth and sun. $R = R_e + r$.

$$F_{\text{Tidal}}/m = 2(GM_s{R_{es}}^{-2})(\frac{r}{R_{es}})$$

- The term ${GM_s{R_{es}}^{-2}}$ is the acceleration of the earth in its orbit. It must be $\frac{v^2}{R_{es}} = 4\pi^2 R_{es} P^{-2}$.

$$\frac{F_{\text{Tidal}}}{m} = 8\pi^2 P^{-2} r.$$

- At the equator, $\frac{F_{\text{Tidal}}}{m} = 8.\pi^2 \frac{6300km}{(\pi \times 10^7 s)^2} = 5 \times 10^{1+6-14} = 5 \times 10^{-7}\text{m/s}^2$

Roche limit. Approximation #1

- A moon orbits Jupiter. The moon is a fluid having no strength. Consider a mass m on a line joining Jupiter and the moon, where the tidal force is greatest.
- The tidal force on mass m is

$$F_{\text{Tidal}}/m = 2(GM_J {R_{jm}}^{-2})(\frac{r_m}{R_{jm}})$$

- The gravitational force is

$$\frac{F_{moon}}{m} = GM_{moon} r_m^{-2}$$

- If the moon holds itself together, then

$$F_{\text{Tidal}} \leq F_{moon}$$
$$2M_J {R_{jm}}^{-3} \leq M_{moon} r_m^{-3}$$

Roche limit. Approximation #1

- A moon orbits Jupiter. The moon is a fluid having no strength. Consider a mass m on a line joining Jupiter and the moon, where the tidal force is greatest.
- Express in term of mass density ρ.

$$2\rho_j r_j^3 {R_{jm}}^{-3} \leq \rho_{moon}$$

- For the moon to hold itself together, it must be outside the Roche limit:

$$R_{jm} \geq 2^{1/3}(\rho_j/\rho_{moon})^{1/3} r_j$$

- For the case where Jupiter and its moon have the same density,

$$R_{jm} \geq 1.26\, r_j$$

  - The moon must be outside 1.26 of the radius of the planet.