Rings & Moons—31 Jan

- Homework 3 is due Mon, 7 Feb.
- Roche's limit: If a moon is near its planet, inside the Roche limit, the tidal force of the planet is greater than the gravitational force of the planet: the moon cannot hold together.
- Roche limit for nonrotating moon:

$$R_{jm} \geq 1.26\ (\rho_j/\rho_{moon})^{1/3} r_j$$

- Better calculation of Roche limit. Moon rotates.
- Even better calculation: Moon is oblate.
- Compare rings and moons of the Jovian planets.

Roche limit. Approximation #1

- A moon orbits Jupiter. The moon is a fluid having no strength. Consider a mass m on a line joining Jupiter and the moon, where the tidal force is greatest.
- The tidal force on mass m is

$$F_{\text{Tidal}}/m = 2(GM_J {R_{jm}}^{-2})(\frac{r_m}{R_{jm}})$$

- The gravitational force is

$$\frac{F_{moon}}{m} = GM_{moon} r_m^{-2}$$

- If the moon holds itself together, then

$$F_{\text{Tidal}} \leq F_{moon}$$
$$2M_J {R_{jm}}^{-3} \leq M_{moon} r_m^{-3}$$

Roche limit. Approximation #2

- A moon is likely to rotate with the same period as its orbit, which is the case with Earth's moon.

1. If the moon rotates, the Roche limit be __ $1.26r_j$.
   A. Greater than
   B. Less than
   C. Same as
2. Write Newton's 2nd Law (F=ma) for a rotating moon that is barely held together by its own gravity. You may use symbols, rather than finding expressions for the symbols.

Roche limit. Approximation #2

- Better assumption: The moon rotates with the same period as its orbit. Then

$$F_{moon} - F_{tidal} = m\frac{v^2}{r_{moon}} = m\,4\pi^2 r_{moon}/P^2$$

- Substitute Kepler's 3rd Law

$$P^2 = 4\pi^2 R_{jm}^3/(GM_j)$$

- RHS becomes

$$GM_j r_{moon}/R_{jm}^3$$

  – Half of the tidal force.

- Roche's limit is then

$$R_{jm} \geq 3^{1/3}(\rho_j/\rho_{moon})^{1/3} r_j$$

$$3^{1/3} = 1.44$$

Roche's limit. Approximation #3

- Near breakup, the moon is oblate.

1. If the moons is oblate, the force of the moon on mass m is ___ $G\frac{4\pi}{3}m\,\rho_{moon}r_m$ and Roche's limits is ___.
   A. Less than. Greater
   B. Greater than. Greater
   C. Less than, Less.
   D. Greater than. Less

- The force of the moon is less than that of a spherical moon. Result:

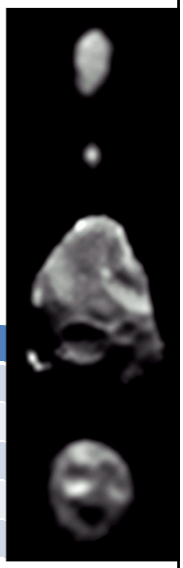
$$R_{jm} \geq 2.44\,(\rho_j/\rho_{moon})^{1/3}r_j$$

Inner moons of Jupiter

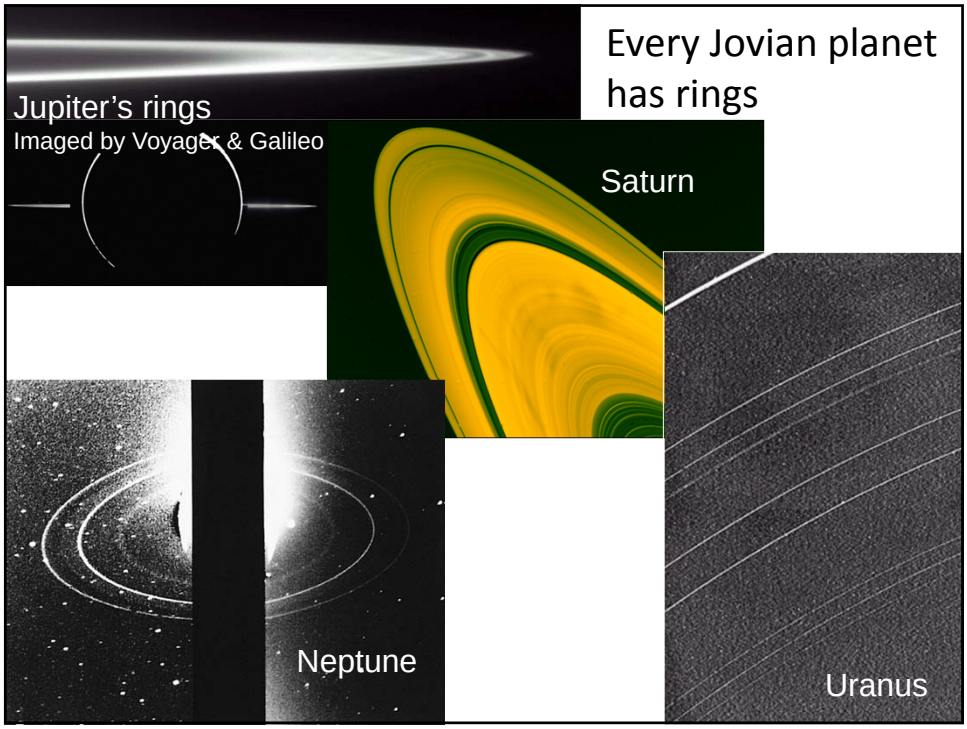
- For moons having the same density as Jupiter, the Roche limit for Jupiter's moons is
  $2.44r_j = 169\text{Mm}$
- R/Roche accounts for the density of Jupiter and the moon.

1. Metis is held together by its own gravity. Amalthea is held together by its own gravity.
   A. T for Metis. F for A
   B. F for Metis. T for A
   C. T for both
   D. F for both

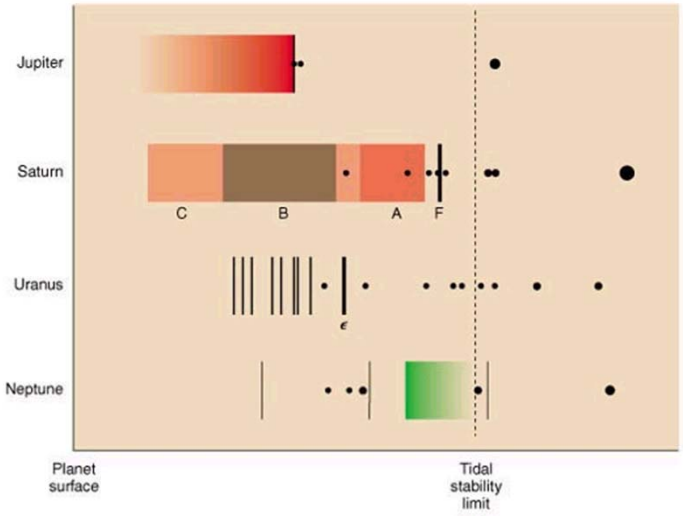
Moon	R[Mm]	R/Roche
Metis	128	0.94
Andrastea	129	0.95
Amalthea	181	0.88
Thebe	222	1.3
Io	422	



Every Jovian planet has rings



Location of the rings (Dots are moons. Lines & bands are rings.)



(Roche limit)