Unbound orbits & position vs time—21 Mar

- Outline
  - Derivation of Kepler's 3rd Law
  - Unbound orbits
  - Example
  - Solving for position vs time
  - Formation of the Oort Cloud. Reading for class on Fri.

Kepler’s 3rd Law

We already derived the law of equal areas. $\frac{1}{2} L = \frac{dA}{dt}$.

$$\frac{1}{2} L P = A$$

where P is the period.

The area of an ellipse is π(semi major axis) (semi minor axis). The semi minor axis is $a\left(1 - e^2\right)^{1/2}$.

$$\frac{1}{2}\left[G M a\left(1 - e^2\right)\right]^2 P = \pi a^2\left(1 - e^2\right)^{1/2}.$$

Solve for P to get

$$\boxed{P^2 = 4 \pi^2 a^3 / (G M)}$$

Translate this from a 1-body to a 2-body problem.

$M \to M_s + m_p$

$a \to$ distance bewteen sun and planet

$P \to$ peiod of the orbit

Unbound orbits

We integrated the equation of motion and got

$$\boxed{r = p/(1 + e\cos\theta)}$$

where $p = L^2/(G M)$

Q: What values of e have we already discussed?

A. $0 < e \leq 1$

B. $e > 1$

C. $e = 1$

Unbound orbits

We integrated the equation of motion and got

$$\boxed{r = p/(1 + e\cos\theta)}$$

where $p = L^2/(G M)$

The case $0 \leq e < 1$

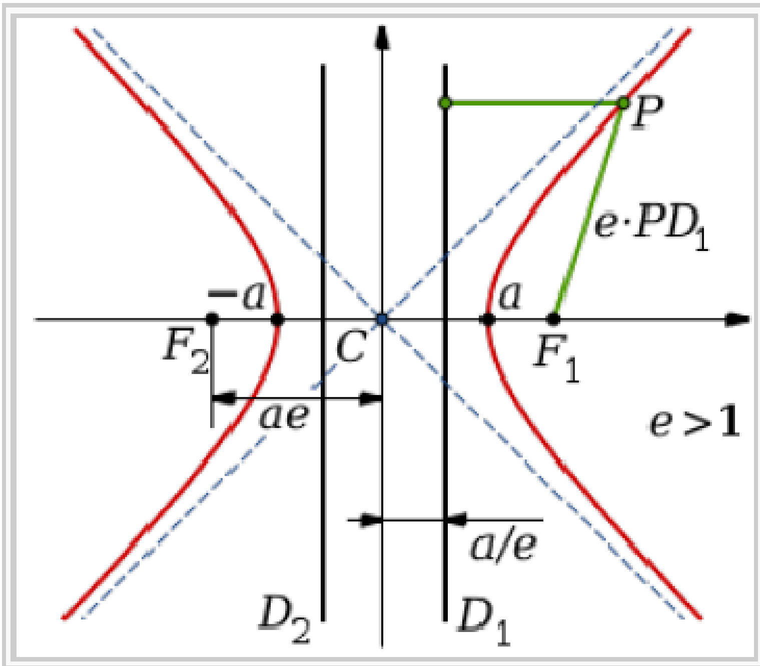
1. The orbit is bound.
2. The orbit is an ellipse The definition of an ellipse: the distance to the sun r and the distance to the other focus r' are related by $r + r' =$ constant
3. $p = a\left(1 - e^2\right)$, where a is the semimajor axis.
4. $E = -G M/(2 a)$

The case $e = 1$

1. The orbit is unbound. For $\cos\theta \to -1$, $r \to \infty$.
2. You can show that the orbit is a parabola with the sun at a focus. The definition of a parabola: the distance from the sun r and the distance to a line (called the directrix) r' are equal.
3. p is twice the perihelion distance.
4. $E = 0$

The case $e > 1$

1. The orbit is unbound. For $e\cos\theta \to -1$, $r \to \infty$.
2. The orbit is a hyperbola. Definition of a hyperbola: the distance to the sun r and the distance to the other focus r' are related by $r' - r =$ constant
3. $p = a\left(e^2 - 1\right)$, where a is the semimajor axis.
4. $E = G M/(2 a)$



http://en.wikipedia.org/wiki/Hyperbola

Example. Runaway stars and planets

A planet is in a circular orbit around a star. The star suddenly explodes and keeps a fraction f of its mass. Find the minimum f so that the planet stays with its star.

2-minute assignment: Write a thought on paper and be prepared to share it with the class.

- **Calculation**

Total energy

$$E = -\frac{1}{2}\,\mathrm{GM}/a$$

$$\mathrm{KE} = +\frac{1}{2}\,\mathrm{GM}/a$$

$$\mathrm{PE} = -\mathrm{GM}/a$$

The explosion does not affect the KE; it does change the PE.
If f of the mass remains,

$$\mathrm{PE} = -f\,\mathrm{GM}/a$$

For planet to leave the star,

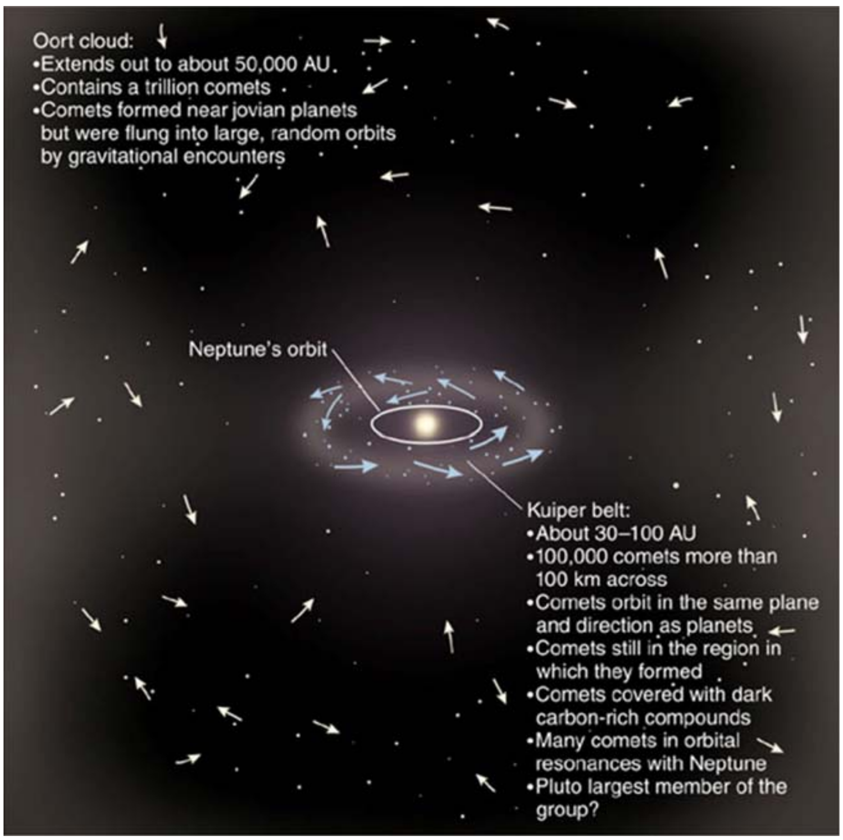
$$\mathrm{KE}+\mathrm{PE}>0$$

$$E = \left(\frac{1}{2} - f\right)\mathrm{GM}/a \geq 0$$

The planet leaves if $f \leq \frac{1}{2}$.

Formation of the Oort Cloud

- Kuiper belt: Short period comets
  - small inclinations
  - prograde orbits.
- Oort Cloud: Long period orbits
  - large inclinations
  - retrograde and prograde orbits
  - Distribution of semimajor axis peaks at 20,000AU



Fernández, J. A., 1997, Icarus, 129, 106, "The Formation of the Oort Cloud and the Primitive Galactic Environment"

- Read up to the paragraph starting with "A passing star" on p109. The paper will be put on angel with a link on the syllabus.
- Notation
  - q is the perhelion distance
  - For a random walk in position with step size L, the average of position^{2}after N steps is $N\,L^2$.

Q: You are blindfolded. You step 1m randomly forward and backward. After 100 steps, how far have you gone?

A. 100m

B. 10m

C. 0m

- Questions to help you read the paper
  - Q1: What is E_{orig} at the bottom of p106? Why does it have units of AU^{-1}?
  - Q2: What is the main point of this paper? Express it in a few sentences.
- Pre-class questions will be due on Fri.