Hwk 4
—18 Feb 2011

Problem 1

(a) The other forces are the gravitational force of Earth and the force exerted by the table.

(b) As the drop moves around the sun, it changes direction. This is an acceleration.

(c) The force that causes the acceleration is the force of the sun on a drop at the center of Earth, which is $-G\,M_s\,R_{es}^{-2}$. The acceleration is

$$a = v^2 / R_{es} = 4\,\pi^2\,R_{es} / P$$

In[43]:=
```
Convert[4 π^2 AstronomicalUnit / Year^2, Meter / Second^2]
```

Out[43]= $\frac{0.00593843\ \text{Meter}}{\text{Second}^2}$

Problem 2

(a) Let the angle of Jupiter from point x by θ_J and the angle of the asteroid be θ_A. At time t, the position of Jupiter and the astreoid are

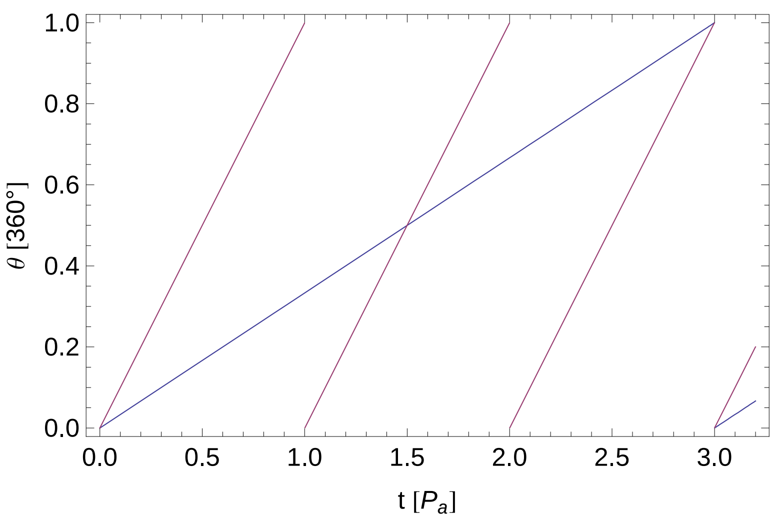
$$\theta_J(t) = 2\,\pi\,t / P_J$$
$$\theta_A(t) = 2\,\pi\,t / P_A.$$

Since they are in a 1:3 resonance, $P_J = 3\,P_A$. They are close when $\theta_J(t) = \theta_A(t) \bmod 2\,\pi$.

In[158]:=
```
plotPosition[pJupiter_, tMin_, tMax_] := Plot[{Mod[t/pJupiter, 1], Mod[t, 1]},
  {t, tMin, tMax}, Frame → True, FrameLabel → {"t [Pa]", "θ [360°]"},
  BaseStyle → {FontSize → Medium, FontFamily → "Helvetica"},
  Ticks → {Automatic, {#, 360 #} & /@ Range[0, 1, .125]}]
```

In[159]:=
```
plotPosition[3, 0, 3.2]
```

Out[159]=



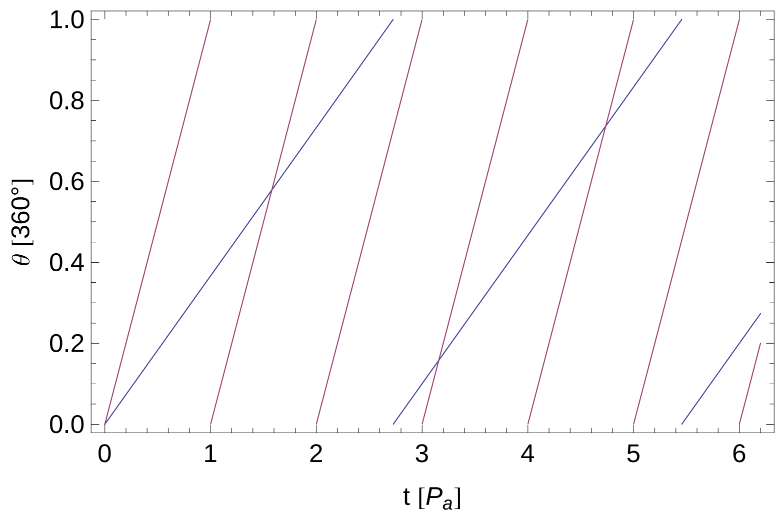
Caption: Position of Asteroid A and Jupiter.

Jupiter and Asteroid A are close after 1.5 P_A. They are back at $\theta = 0$ after 3 P_A.

(b)

In[160]:=
```
plotPosition[30 / 11, 0, 6.2]
```

Out[160]=



Caption: Position of Asteroid B and Jupiter.

Asteroid B and Jupiter are close for the very next time at time t_1. Since the asteroid completed more than one orbit,

$$t_1 / P_B - 1 = t_1 / P_J$$

Solve to get

$$t_1 / P_J = 1/(P_J / P_B - 1)$$

The position is

$$P_J \, t_1 = 1/(P_J / P_B - 1)$$

In[154]:=
```
1 / (30 / 11 - 1)
```

Out[154]= $\frac{11}{19}$

In[101]:=
```
N[%]
```

Out[101]= 0.578947

They are close on subsequent approaches at positions

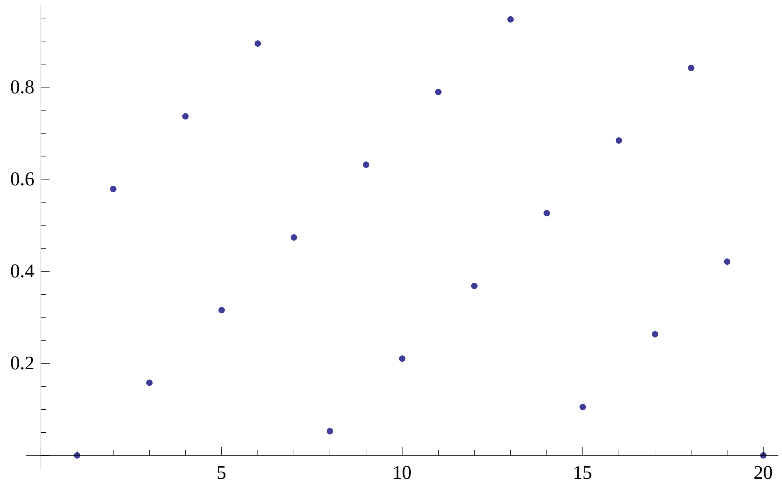
$$2\pi \operatorname{mod}\left(\frac{11}{19} i, 1\right).$$

In[156]:=
```
Mod[11 / 19 #, 1] & /@ Range[0, 19, 1]
```

Out[156]= $\left\{0, \frac{11}{19}, \frac{3}{19}, \frac{14}{19}, \frac{6}{19}, \frac{17}{19}, \frac{9}{19}, \frac{1}{19}, \frac{12}{19}, \frac{4}{19}, \frac{15}{19}, \frac{7}{19}, \frac{18}{19}, \frac{10}{19}, \frac{2}{19}, \frac{13}{19}, \frac{5}{19}, \frac{16}{19}, \frac{8}{19}, 0\right\}$

There are 19 separate points where Jupiter and Asteroid B are close. They are spread over all angles. Therefore Jupiter does not pull on the asteroid in a few directions as it did for a 1:3 resonance.

In[157]:=
```
ListPlot@%
```

Out[157]=


Problem 3

(a) The Earth moves

$$D_E = 2\,\pi\, R_e (t / P_e)$$

In[49]:=
```
dEarth = Convert[2. π AstronomicalUnit Minute / Year, AstronomicalUnit]
```

Out[49]= 0.0000119543 AstronomicalUnit

(b) Using Kepler's 3rd Law, I find the period of the asteroid to be

In[51]:= $2.2^{1.5}$ Year

Out[51]= 3.26313 Year

The asteroid moves

In[52]:=
```
dAsteroid = Convert[2. π 2.2 AstronomicalUnit Minute / %, AstronomicalUnit]
```

Out[52]= 8.05959×10^{-6} AstronomicalUnit

(c) The asteroid moves with repect to Earth by

In[53]:=
```
dAsteroid - dEarth
```

Out[53]= -3.89472×10^{-6} AstronomicalUnit

Since its distance is 1.2AU, the angle that it moved withe respect to the stars is

In[54]:=
```
(dAsteroid - dEarth) / (1.2 AstronomicalUnit)
```

Out[54]= -3.2456×10^{-6}

radians, which is

In[55]:=
```
% 2*^5 ArcSec
```

Out[55]= -0.649119 ArcSec

(d) Now calculate the angle that an asteroid with orbital radius *a* moves in time t.

$$d_A = 2\,\pi\, a\, t / P.$$

Kepler's 3rd Law says $P^2 = a^3$.

$d_A = 2\,\pi\,a^{-1/2}\,t/\text{year}$.

The distance Earth moves is

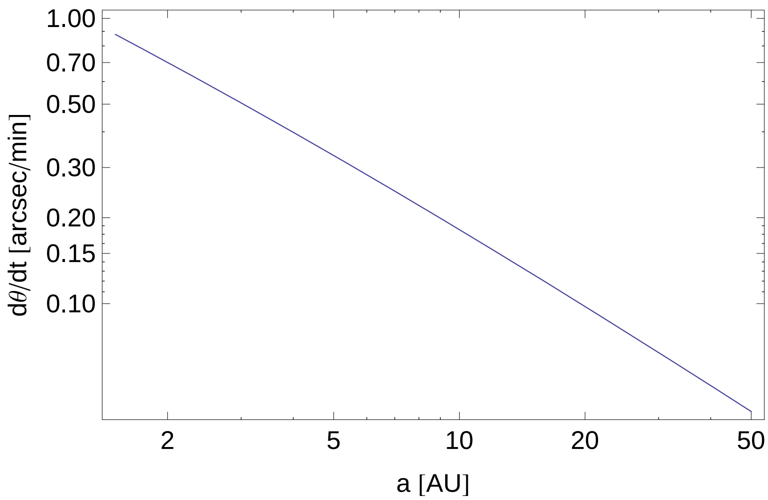
$d_E = 2\,\pi\,t/\text{year}$.

The angle that the asteroid moves with respect to the stars is

$\theta = (d_A - d_E)/(a-1) = 2\,\pi\left(a^{-1/2} - 1\right)\big/(a-1)\,t/\text{year}$.

In[70]:=
```
f[a_] := -2 π (a^-.5 - 1) / (a - 1) 2*^5 Convert[Minute / Year, 1]
```

In[73]:=
```
LogLogPlot[-2 π (a^-.5 - 1) / (a - 1) 2*^5 Convert[Minute / Year, 1],
 {a, 1.5, 50}, Frame → True, FrameLabel → {"a [AU]", "dθ/dt [arcsec/min]"},
 BaseStyle → {FontSize → Medium, FontFamily → "Helvetica"}]
```

Out[73]=


By measuring the motion of an object with respect to the stars, you can find its distance. For example, an object at Pluto's distance moves

In[72]:=
```
60 f[40]
```

Out[72]= 3.09667

arcsec/hour.