

You may not have any books or notes.

You may not borrow a calculator from another student. You may borrow a calculator from the front bench.

Write brief answers. Your time is limited, and the graders do not like to read answers that address off-topic ideas.

Good luck.

Planet	Radius [Mm]	Mass [M <sub>Earth</sub> ]	Semi-major axis [AU]
Mercury	2.44	0.055	0.387
Venus	6.05	0.81	0.723
Earth	6.37	1	1
Mars	3.39	0.11	1.524
Jupiter	69.2	320	5.20
Saturn	57.3	95	9.54

Name	
Signature	
1	/7
2	/6
3	/7
4	/7
5	/6
Total	/33

Force of gravity, gravitational potential energy	$-GMm/R^2, \quad -GMm/R$
Kinetic energy	$mv^2/2$
Kepler's 3 <sup>rd</sup> Law	$P^2 = R^3/M$ using units AU, year, & M <sub>sun</sub> $P^2 = 4\pi^2 R^3 / (GM)$
Tidal acceleration on a mass at r from the center of earth	$2GM_{sun}rR_{earth}^{-3}$
Roche limit	$2.44 R_{planet} (\rho_{planet} / \rho_{moon})^{1/3}$ (Less precise coefficient for nonrotating and rotating moon: 1.26 and 1.44)
Parsec	$3.09 \times 10^{13}$ km
AU	$1.50 \times 10^8$ km
Year	$3.16 \times 10^7$ s
Mean kinetic energy	$\frac{3}{2} kT$ (k=1.38e-23m^2 kg/s^2/K)
Energy and momentum of light	$p = E/c$ (c=3e8m/s)
Proper motion of an object at opposition	$d\theta/dt = 148 \text{arcs/hr} (1 - \sqrt{a}) / (1 - a)$
Magnetic force	$\vec{F} = e \vec{v} \times \vec{B}$
Force of radiation on black object of cross sectional area A	$LA / (4\pi c R^2)$
Area of a circle	$\pi R^2$
Volume of a sphere	$4\pi R^3 / 3$
Eccentricity of an ellipse	(distance between foci)/(major axis)

1. The comet Shoemaker-Levy 9 (SL9) was captured by Jupiter sometime in the 1960s or 1970s in an orbit with a high eccentricity. In 1992 it passed near Jupiter and broke apart. In 1994 it collided with Jupiter.

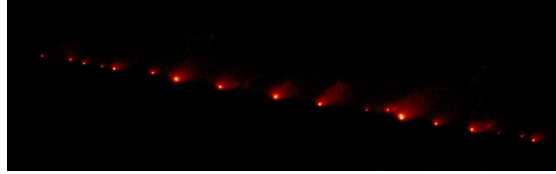


Figure 1 Comet Shoemaker-Levy 9 (NASA, ESA, and H. Weaver and E. Smith (STScI))

- a. (5 pts.) To break apart, how close to Jupiter did it get in 1992? Why did it not break apart before then?

SL9 broke apart because the tidal force of Jupiter became stronger than its self-gravity. It got closer than the Roche limit ( $2.44 \times 69 \text{Mm} = 170 \text{Mm}$  if the densities are the same). It did not break apart before because the orbit did not have such a close approach before.

- b. (2 pts.) After SL9 was captured by Jupiter, why did it violate Kepler's First Law, that its orbit is an ellipse with Jupiter at one focus?

Since the comet feels the gravity of more than one object (Jupiter and the sun, primarily), it does not obey Kepler's laws.

2. Vega has a surface temperature 1.6 times that of the sun and a mass 2.1 times that of the sun. This question asks you to figure out where Vega's Jovian planets might be. (None have been discovered.)

- a. (4 pts.) Are Vega's Jovian planets found farther, closer, or at the same distance as Jupiter (5.2AU)? Explain your reasoning.

Recall how Jovian planets form. Because of the gravity of the proto solar system, the kinetic energy (and temperature) is larger closer in to the center. Jovian planets form in regions where water condenses. All of this happens before the sun starts to shine.

Vega's Jovian planets form farther out because of its higher mass. Temperature is proportional to kinetic energy, which is proportional to potential energy.

$$kT = KE = PE = \frac{GM}{R}$$

For the temperature where water condenses, R is proportional to mass.

- b. (2 pts.) Compute the distance (in AU) beyond which Vega's Jovian may be found.

$$R = 5.2AU \cdot 2.1 = 11AU$$

3. The force of gravity and the force of radiation are equal for black dust with a density of  $1 \text{gm/cm}^3$  and radius  $0.58 \mu\text{m}$ . Black means the dust absorbs the radiation and reemits the energy as infrared radiation equally in all directions.

- a. (2 pts.) Suppose some dust of this density and size came off of a comet nucleus at zero velocity with respect to the comet. What is its path? Explain.

For dust of this size, the forces of gravity and radiation balance. With no force, the path is a straight line with the same velocity as the comet at the time the dust came off the nucleus.

- b. (2 pts.) Suppose some dust of this density and much larger size came off of a comet nucleus at zero velocity with respect to the comet. What is its path? Explain.

For much larger dust, the force of gravity dominates, as it does for the nucleus. Therefore the dust and the nucleus have the same path.

- c. (3 pts.) If dust of this density and size were white, in which direction would it accelerate? Explain. (White means the radiation is reflected and not absorbed.)

The radiation reflects off of the dust and imparts as twice the momentum as it did if it were absorbed. The acceleration is away from the sun.

4. Asteroids in a family have similar proper orbital elements.

- a. (3 pts.) Why do asteroids come in families?

Asteroids in families were originally one asteroid that broke apart because of a collision.

- b. (2 pts.) Why do the family members not have similar osculating (not proper) orbital elements?

The forces of planets (primarily) cause different members of a family to move with changing orbital elements. These effects diminish when averaged over time.

- c. (2 pts.) Besides having similar proper orbital elements, what is likely to be similar for Ida and Dactyl, which are members of the Koronis family?

Since they were part of a larger asteroid, they should have the same composition.



Figure 2 Ida & Dactyl, Galileo, NASA

5. Suppose you found a new planet with a radius of  $5R_E$ , where  $R_E$  is the radius of earth. A reporter from the *State News* asks you, "What is its composition"?

- a. (6 pts.) Write an answer. You will be graded on what you can provide with the limited information and on accuracy. See Fig 3 to help you write an answer.

Since we have no information about the mass, we cannot determine its composition for certain. However, if we *assume* it has the same relationship between size and mass as planets in the solar system, then we can say that it is a mixture of water, methane, and ammonia ices and hydrogen and helium.

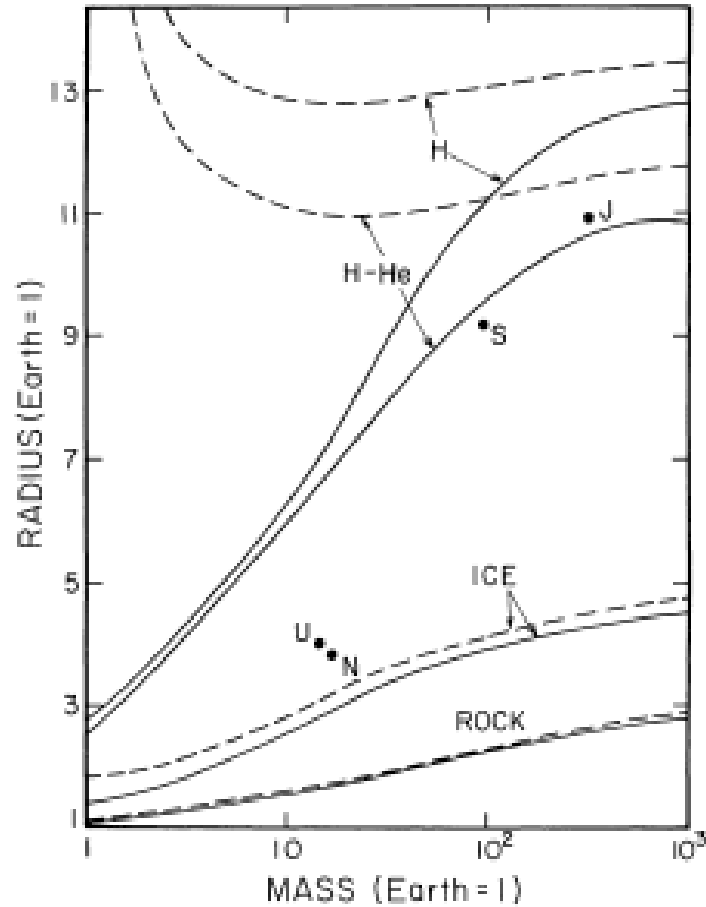


Figure 3 Mass-radius relationship for planets held together by gravity. Solid lines are models with different compositions. (Ice means water, methane, or ammonia.) Ignore the dashed lines. The four Jovian planets are marked. From Stevenson, D. J., 1982, *Ann. Rev. Earth Planet. Sci.* 10, 257