Objectives:
• to investigate whether the velocity of a freely-falling body increases linearly with time
• to calculate an acceleration of a freely falling body from a set of data points

Apparatus: In this experiment you will be given the position of a freely falling body collected by an unmanned spacecraft on Planet X. From the data, you will determine the acceleration due to gravity on the planet’s surface. The data strip you will use will be on the lab handout given to you when you take the practical exam. The time interval between two adjacent marks, \( \tau \), will also be on the handout you will receive when you take the practical exam. A total of 9 data points will be on your data strip. The marks on your tape will be pre-labeled, as point 1 through 9. The position of point 9 is defined as \( y_9 = 0 \) and the time the body was at point 1 is defined as \( t_1 = 0 \) seconds. The quantities needed to analyze the motion are the position \( (y) \), velocity \( (v) \) and time \( (t) \) of the points on your data strip.

Theory: Measuring the distances between any two marks, \( \Delta y \), and knowing the times between the corresponding sparks, \( \Delta t \), it is possible to calculate the average velocity during this interval using the formula

\[
v = \frac{\Delta y}{\Delta t}
\]  

(1)

If \( \Delta t \) is small enough, we can assume that the velocity at any instant within this interval is approximately equal to this average velocity. In the case where acceleration is constant, the instantaneous velocity at the middle of the time interval, \( \Delta t \), is exactly equal to the average velocity of the object during that time interval.

In general, for the motion of a body with a constant acceleration \( a \), the velocity \( v \) is given by the equation

\[
v = at + v_0,
\]  

(2)

where \( v_0 \) is the velocity of the body at \( t = 0 \). Since in our case the body is falling freely,

\[
a = -g,
\]  

(3)

where \( g \) is the magnitude of the acceleration due to gravity on the planet’s surface. The negative sign in front of \( g \) is to indicate that the direction of the acceleration is in the negative direction (i.e. downward). Therefore it follows from (2) that for a freely-falling body

\[
v = v_0 - gt
\]  

(4)

Thus \( g \) can be determined from a plot of \( v \) vs. \( t \), since the slope of any velocity versus time graph is the constant acceleration. The obtained value of \( g \) can then be compared with the known value of the acceleration due to gravity.
Procedure:

A) Put a ruler on your tape such that height \( y = 0 \) (cm) corresponds to point #9 and that height INCREASES as you move towards point #1. Measure the position of each mark with the ruler and write the \( y \) position next to each point. Each point must be labeled with its TOTAL DISTANCE from \( y = 0 \), NOT its distance from the previous point.

B) Transfer the \( y \) positions into the spreadsheet. Assign a reasonable value for the uncertainty in your position measurements (\( \delta y \)). Have Excel calculate the displacement (\( \Delta y \)), the velocity (\( v_y \)) and the uncertainty in the velocity (\( \delta v_y \)) for points during the fall (except for the first and last points). As discussed in the theory section, the instantaneous velocity at time \( t_i \) is obtained by finding the average velocity during the time interval of \( t_{i-1} \) to \( t_{i+1} \) for each of your points, \( v = \frac{\Delta y}{\Delta t} \). In this equation \( \Delta y \) is the difference between the position of the mark FOLLOWING and the position of the mark PRECEDING the mark for which you are trying to calculate a velocity. Similarly, \( \Delta t \) is the time interval between the FOLLOWING mark and the PRECEDING mark, and note that in this procedure \( \Delta t = 2 \tau \) (two time intervals). For example, the instantaneous velocity at \( y = y_2 \) is \( v_2 = \frac{y_3 - y_1}{t_3 - t_1} \).

C) Transfer your data columns into Kaleidagraph and prepare a graph of \( v_y \) vs. \( t \) with error bars and fitted with a best straight line. Include at the bottom of the graph a written description of what is shown in the graph, stating with uncertainties and units the measured slope and intercept for the best straight line.

Questions (include the uncertainty and units for every quantity derived from a measurement)

1. From your graph, what is the acceleration due to gravity on the surface of Planet X?
2. Is the value you measured for the acceleration consistent with the previously measured value that is provided? Explain.
3. What is the initial velocity (\( v_0 \)) of the body?
4. Does the magnitude of the velocity increase linearly with time? Explain.

Checklist

Your lab report should include the following four items:
1) Your spreadsheet
2) The formula view of the spreadsheet
3) The graph of the measured velocities with error bars, and a linear fit to the data.
4) Answers to questions

Uncertainties

To test the compatibility of two measurements, \( d_1 \pm \delta d_1 \) and \( d_2 \pm \delta d_2 \), find \( \Delta = |d_1 - d_2| \), and if \( \delta \Delta < \delta d_1 + \delta d_2 \), the two measurements are compatible.
Spreadsheet for Practical 1:

**Practical 1**

- **Time Interval between data points** $\tau$:
  - $\Delta y(i) = y(i+1) - y(i-1)$
  - Error in $y$: $\delta(y)$
  - Error in $v_y$: $\delta(v_y) = \delta y / \tau$

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Recall:
- Blue = measured values
- Yellow = calculated values