Uncertainties and uncertainty propagation

measure or predict a quantity a

- absolute uncertainty on a is σa
- relative uncertainty on a is σa/a

General formulation:

Given a function *f*(a,b)

The uncertainty on *f* is σ*f*, given by

$$\sigma f = \sqrt{\left(\frac{\delta f}{\delta b}\right)^2 \sigma a^2 + \left(\frac{\delta f}{\delta a}\right)^2 \sigma b^2}$$

Specific equations:

Sums or Differences

When taking the sum or difference of two numbers, add the absolute uncertainties.

f(a,b) = a+b or *f*(a,b) = a-b , then

$$\sigma f = \sqrt{\sigma a^2 + \sigma b^2}$$

Products or Ratios

When taking the product or ratio of two numbers, add the relative uncertainties.

f(a,b) = a*b or *f*(a,b) = a/b , then

$$\frac{\sigma f}{f} = \sqrt{\frac{\sigma a^2}{a^2} + \frac{\sigma b^2}{b^2}}$$

Multiply by a constant or add a constant

When multiplying by a constant, multiply the uncertainty by the same constant.

When adding a constant, the uncertainty is unchanged.

$f(a) = c_1 * a + c_2$, then

$$\sigma f = c_1 \sigma a$$

Comparing two numbers:

We often ask: Does my measurement agree with my prediction?

To answer this question:

1. Determine the uncertainty on the measurement (from the instrument uncertainty or your estimate of how well you can read off your measurement).
2. Determine the uncertainty on the prediction (if necessary doing error propagation, but reasonable estimates are ok). Sometimes you can make an argument that the uncertainty on the prediction will be much smaller than the uncertainty on the measurement. If not sure, check with your TA.
3. Add the two uncertainties in quadrature and check if the two numbers agree within this combined uncertainty.