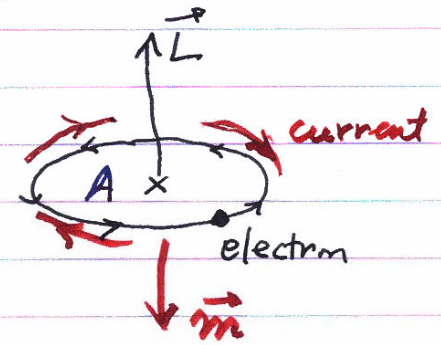
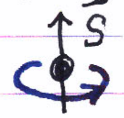
Magnetization and Bound Current 9.2/1

Magnetic moment ($\vec{m}$) and magnetization ($\vec{M}$)



$$\vec{m} = I A \,\hat{n}$$

$$\vec{m} = \frac{1}{2}\int \vec{x} \times \vec{J}(\vec{x})\, d^3x$$

It's a bit more complicated, because an electron also has spin 

and a corresponding spin magnetic moment 

$$\vec{m} = \frac{-e}{2m}\left(\vec{L} + 2\vec{S}\right)$$

Check units: C kg m/s m /kg = A

<u>The essential idea for magnetism in matter</u>

The magnetic field produced by a magnetized object is the same as that of a volume current density $\vec{J}_{\text{Bound}}(\vec{x})$ and surface current density $\vec{K}_{\text{Bound}}(\vec{x})$ given by

$$\vec{J}_B(\vec{x}) = \nabla \times \vec{M} \quad \text{and} \quad \vec{K}_B(\vec{x}) = \vec{M}(\vec{x}) \times \hat{n}$$

($\vec{x}$ on the surface)

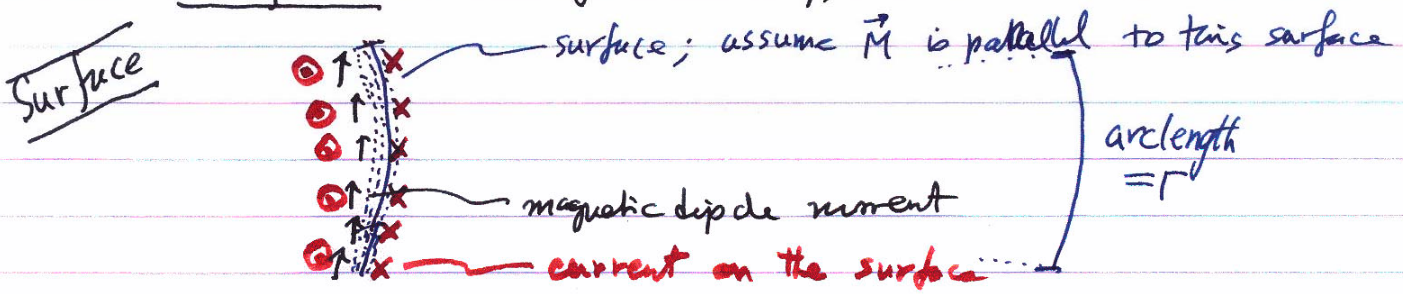
Theorem A sample of matter with magnetization $\vec{M}(\vec{x})$ has an associated magnetic field $\vec{B}_{Matter}(\vec{x})$ which is the same as $\vec{B}$ for a volume current density $\vec{J}_{Bound}(\vec{x})$ and surface current density $K_{Bound}(\vec{x})$,

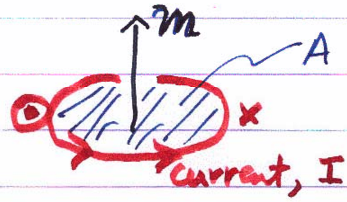
$$\vec{J}_B(\vec{x}) = \nabla \times \vec{M} \quad \text{and} \quad \vec{K}_B(\vec{x}) = \vec{M}(\vec{x}) \times \hat{n}$$

(in the volume) (on the surface)

Proof #1 See Equations 9-7 to 9-14.
/by integration/

Proof #2 (more geometrically)



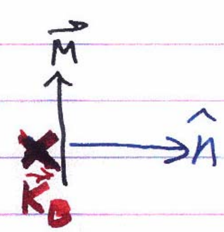


Surface current density K_B

$$K_B \cdot \Gamma = \frac{\delta Q}{\delta t} = \frac{N \cdot I \delta t}{\delta t}$$

$$N = \text{\# of atoms} = n(A\Gamma)$$

Thus $K_B = \dfrac{n A \Gamma I}{\Gamma} = n I A = M$



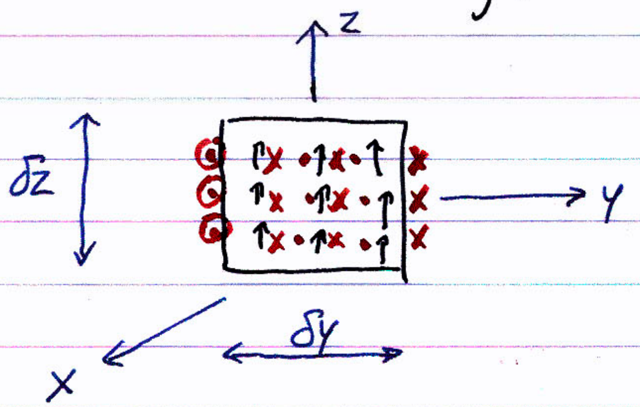
Direction of $\vec{K}_B$ is same as $\vec{M} \times \hat{n}$

$\vec{K}_b = \vec{M} \times \hat{n}$ Q.E.D.

Check units : $\dfrac{A}{m}$ and $\dfrac{Am^2}{m^3}$ ✓

Volume

Consider a small subvolume inside the material. Set up a coordinate system with the z direction parallel to $\vec{M}$



Current density $\perp$ to the yz plane is $J_{Bx}\,\hat{i}$

$$J_{Bx}\,\delta y\,\delta z = \frac{\delta Q}{\delta t}$$

The sign is tricky.

$$J_{Bx}\,\delta y\,\delta z = +\left(I_{atom} N\right)_{y+\frac{\delta y}{2}} - \left(I_{atom} N\right)_{y-\frac{\delta y}{2}}$$

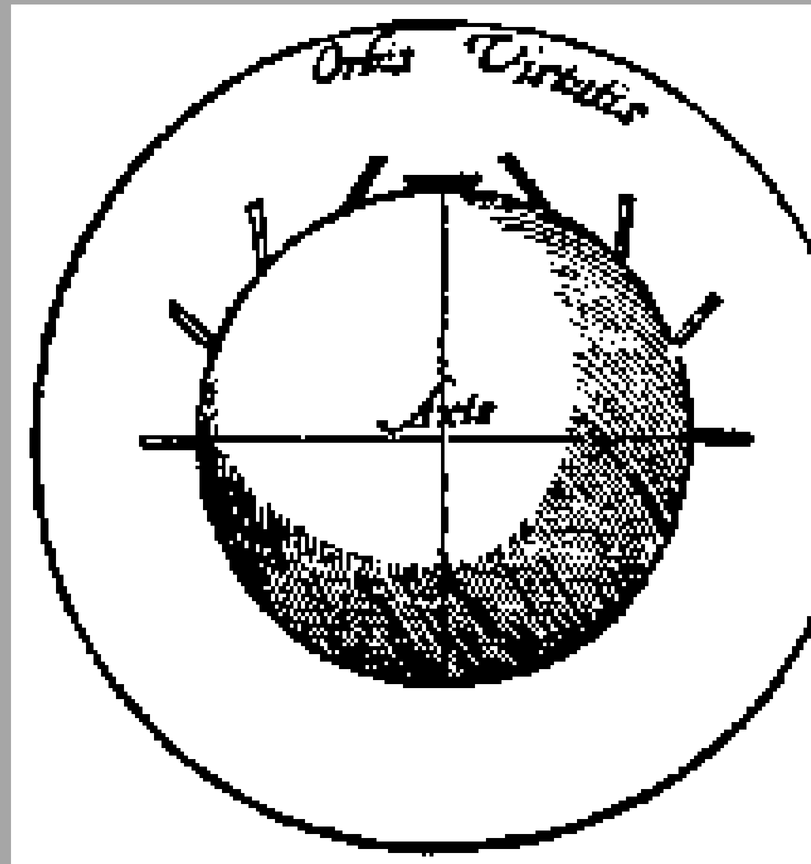
$$= \left(I\,n\,A\,\delta z\right)_{y+\frac{\delta y}{2}} - \left(I\,n\,A\,\delta z\right)_{y-\frac{\delta y}{2}}$$

$$= M_z\left(y+\frac{\delta y}{2}\right)\delta z - M_z\left(y-\frac{\delta y}{2}\right)\delta z$$

$$= \frac{\partial M_z}{\partial y}\,\delta y\,\delta z$$

$$J_{Bx} = \frac{\partial M_z}{\partial y}$$ (assuming $\vec{M}$ is in the z direction)

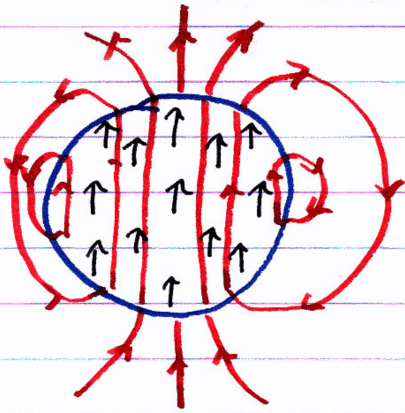
$$\vec{J}_B = \nabla\times\vec{M}$$ $\overline{\text{Q.E.D.}}$



"Terrella" is Latin for "little Earth," the name give William Gilbert to a magnetized sphere with whic demonstrated to Queen Elizabeth I his theory of t Earth's magnetism. By moving a small compass around the terrella and showing that it always poi north-south, Gilbert argued that the same thing, o vastly larger scale, was happening on Earth, and w the only reason why a compass pointed north-sou

Later scientists such as Birkeland used th name "terrella" for magnetized spheres used inside vacuum chambers, together with electron beams, t study the motion of fast charged particles near the Earth. A sophisticated terrella experiment in a vac chamber is currently operated by Dr. Hafez u-Rah at the University of California at Riverside.

Example Determine the magnetic field due to a *uniformly magnetized sphere*

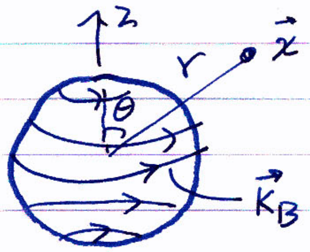


radius $= a$

magnetization $= M\hat{k}$

$$\vec{J}_B = \nabla\times\vec{M} = 0$$

$$\vec{K}_B = \vec{M}\times\hat{n} = M\hat{k}\times\hat{r} = M\sin\theta\,\hat{\phi}$$



$\vec{B} = \nabla\times\vec{A}$ where

$$\vec{A}(\vec{x}) = \frac{\mu_0}{4\pi}\oint_S \frac{\vec{K}_B(\vec{x}')}{|\vec{x}-\vec{x}'|}\,dA'$$

$$\vec{A}(\vec{x}) = \frac{\mu_0 M}{4\pi}\,a^2\,\hat{k}\times\oint_S \frac{\hat{r}'\,d\Omega'}{|\vec{x}-\vec{x}'|}$$

solid angle $dA' = a^2 d\Omega'$

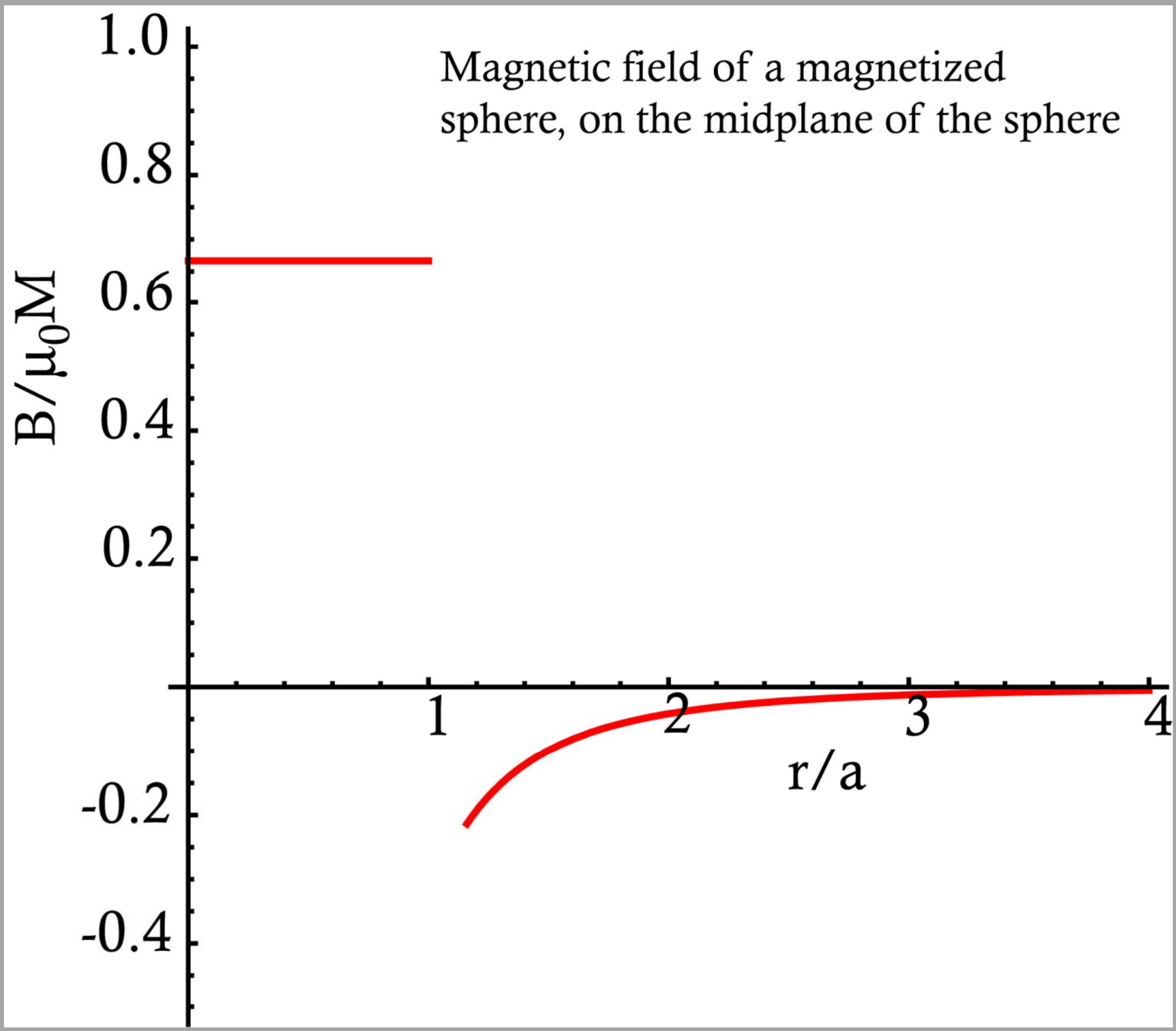
An interesting integral — see Eqs. 8.62–8.65

$$\int \frac{\hat{r}'\,d\Omega'}{|\vec{x}-\vec{x}'|} = \begin{cases} \frac{4\pi}{3}\frac{\vec{x}}{a^2} & \text{if } r<a \\ \frac{4\pi}{3}\frac{\vec{x}a}{r^3} & \text{if } r>a \end{cases}$$

$$\vec{A}(\vec{x}) = \begin{cases} \frac{1}{3}\mu_0 M\,\hat{k}\times\vec{x} & \text{if } r<a \quad \text{(inside)} \\ \frac{1}{3}\mu_0 M\,\frac{\hat{k}\times\vec{x}\,a^3}{r^3} & \text{if } r>a \quad \text{(outside)} \end{cases}$$

$$\vec{B}(\vec{x}) = \begin{cases} \frac{2}{3}\mu_0 M\hat{k} & \text{if } r<a \quad \text{(a uniform field)} \\ \frac{1}{3}\mu_0 M\frac{a^3}{r^3}\left[3\hat{r}(\hat{k}\cdot\hat{r})-\hat{k}\right] & \text{if } r>a \end{cases}$$

(a pure point dipole field)


1.0
0.8
0.6
0.4
0.2
-0.2
-0.4
1
2
3
4
r/a
B/μ0M
Magnetic field of a magnetized
sphere, on the midplane of the sphere

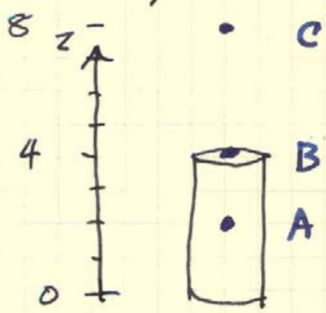
9.2/6

Homework

(a) Determine the magnetic field of a uniformly magnetized cylinder (with radius a and height h) for points on the axis of the cylinder.

Hint First, what is the bound surface current? Then use the Biot-Savart law to determine $\vec{B}$.

(b)



Suppose $a = 1$ and $h = 4$.

(b) Evaluate $\frac{B_z}{\mu_0 M}$ at point A

(c) Evaluate $\frac{B_z}{\mu_0 M}$ at point B

(d) Evaluate $\frac{B_z}{\mu_0 M}$ at point C

Show your work for all parts of the problem.

Demo: magnetometer and bar magnet