Joseph Larmor (1857 – 1942)

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Larmor proposed that the aether could be represented as a homogeneous fluid medium which was perfectly incompressible and elastic. Larmor believed the aether was separate from matter. He united Lord Kelvin's model of spinning gyrostats (e.g., vortexes) with this theory.

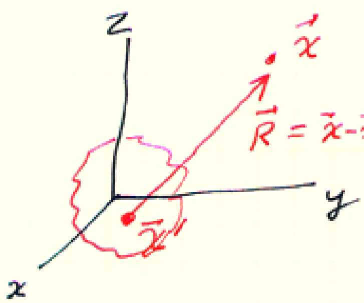
Radiation from a point charge

The theory of radiation

$$\vec{A}(\vec{x},t) = \mu_0 \int \frac{\vec{J}(\vec{x}', t - R/c)}{4\pi R}\, d^3x'$$

$$V(\vec{x},t) = \frac{1}{\epsilon_0} \int \frac{\rho(\vec{x}', t - R/c)}{4\pi R}\, d^3x'$$

} retarded potentials



where $R = |\vec{x} - \vec{x}'|$

Now consider a single charged particle, e.g., an electron.

$$\rho(\vec{x}', t') = q\, \delta^3(\vec{x}' - \vec{x}_q(t'))$$

$$\vec{J}(\vec{x}', t') = q\, \vec{v}(t')\, \delta^3(\vec{x}' - \vec{x}_q(t'))$$

where $\vec{x}_q(t)$ is the position of q as a function of time t.

Note: $\nabla \cdot \vec{J} = -\frac{\partial \rho}{\partial t}$ (continuity equation)

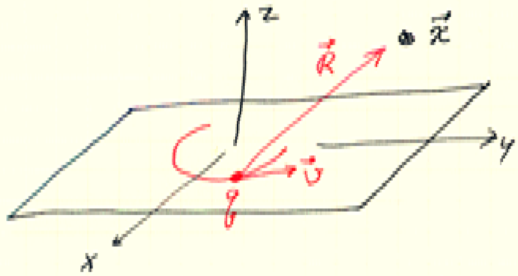
$$\frac{\partial \rho}{\partial t} = q\, \nabla \delta^3(\vec{x} - \vec{x}_q) \cdot \left(-\frac{d\vec{x}_q}{dt}\right) = -q\, \vec{v} \cdot \nabla \delta^3(\vec{x} - \vec{x}_q)$$

$$\nabla \cdot \vec{J} = q\, \vec{v} \cdot \nabla \delta^3(\vec{x} - \vec{x}_q)$$

QED.

Now determine the asymptotic fields, i.e., far from q propagating away from q.

RY/2



$$\vec{R} = \vec{x} - \vec{x}_q(t')$$

$$\vec{J}(\vec{x}', t') = q\,\vec{v}(t')\,\delta^3[\vec{x}' - \vec{x}_q(t')]$$

$$\vec{A}(\vec{x},t) = \mu_0 \int \frac{\vec{J}(\vec{x}', t - R/c)}{4\pi R}\, d^3x' \quad \text{where} \quad R = |\vec{x} - \vec{x}'|$$

The "retarded time" t' is determined by an implicit equation

$$t' = t - \frac{|\vec{x} - \vec{x}_q(t')|}{c}$$

t' depends on t and $\vec{x}$, and on the trajectory of q.

$$\vec{A}(\vec{x},t) = \mu_0 \int \frac{q\,\vec{v}(t - R/c)}{4\pi R}\, \delta^3[\vec{x}' - \vec{x}_q(t')]\, d^3x'$$

where $R = |\vec{x} - \vec{x}'| = |\vec{x} - \vec{x}_q(t')|$

$$\vec{A}(\vec{x},t) = \frac{\mu_0}{4\pi R}\, q\,\vec{v}\left(t - \frac{R}{c}\right) \underbrace{\int \delta^3[\vec{x}' - \vec{x}_q(t')]\, d^3x'}_{\text{integral } \mathcal{I}}$$

$t' = t - \frac{|\vec{x} - \vec{x}'|}{c}$ a function of $\vec{x}'$

You might think $\mathcal{I} = 1$.

But $\mathcal{I} = \dfrac{1}{1 - \hat{R}\cdot\vec{v}/c}$ where $\hat{R} = \dfrac{\vec{x} - \vec{x}_q}{|\vec{x} - \vec{x}_q|}$

i.e. $\hat{R} = \vec{R}/R$

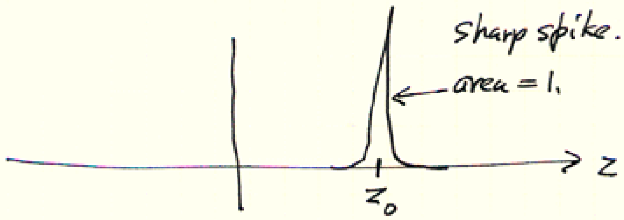
Today we'll consider a non-relativistic particle; i.e., $v \ll c$. Then approximate $\mathcal{I} \approx 1$. So

$$\vec{A}(\vec{x},t) = \frac{\mu_0}{4\pi R}\, q\,\vec{v}(t - R/c) \quad \text{where} \quad R = |\vec{x} - \vec{x}_q(t')|$$

(non-relativistic)

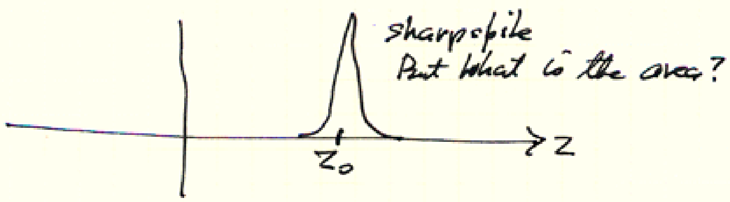
The Dirac delta function

$$\int_{-\infty}^{\infty} \delta(z-z_0)\,dz = 1 \qquad \therefore \int_{-\infty}^{\infty} f(z)\,\delta(z-z_0)\,dz = f(z_0)$$



But now consider $\int_{-\infty}^{\infty} \delta(z-g(z))\,dz$.

The spike occurs at z_0 where $z_0 - g(z_0) = 0$.



Let $\xi = z - g(z)$. Then $d\xi = [1-g'(z)]\,dz$

$$\text{Area} = \int_{-\infty}^{\infty} \delta(\xi)\,\frac{d\xi}{|1-g'(z)|} = \frac{1}{|1-g'(z_0)|}$$

($\xi = 0$ means $z = z_0$)

$$\therefore \int_{-\infty}^{\infty} f(z)\,\delta(z-g(z))\,dz = \frac{f(z_0)}{|1-g'(z_0)|}$$

The radiation fields

$$\vec{A} = \frac{\mu_0 q\, \vec{v}(t - R/c)}{4\pi R}$$ R4/4

$$\vec{R} = \vec{x} - \vec{x}_q(t') \quad \text{and} \quad R = |\vec{R}|$$

- $\vec{B}_{rad}(\vec{x},t)$ = the asymptotic limit of $\nabla \times \vec{A}$

$$= \frac{\mu_0 q}{4\pi R} \left(-\nabla \frac{R}{c} \right) \times \frac{d\vec{v}}{dt}$$

$$= \frac{-\mu_0 q}{4\pi c} \frac{\hat{R} \times \vec{a}}{R} \quad \text{where} \quad \nabla R = \hat{R} = \frac{\vec{x} - \vec{x}_q}{|\vec{x} - \vec{x}_q|}$$

$\boxed{\vec{a} = \text{ACCELERATION}}$

- $\vec{E}_{rad}(\vec{x},t)$ = the asymptotic limit of $-\nabla V - \frac{\partial \vec{A}}{\partial t}$

But easier: $\vec{E}_{rad}(\vec{x},t) = c\, \vec{B}_{rad} \times \hat{R}$

- Poynting vector = radiation intensity

$$\vec{S} = \frac{\vec{E} \times \vec{B}}{\mu_0} = \frac{c}{\mu_0} \left(\frac{\mu_0 q}{4\pi c} \right)^2 \frac{\left[(\hat{R} \times \vec{a}) \times \hat{R} \right] \times \left[\hat{R} \times \vec{a} \right]}{R^2}$$

$$\vec{S} = \frac{\mu_0}{c} \frac{q^2}{16\pi^2 R^2} \underbrace{\left[\vec{a} - \hat{R} a_R \right] \times \left[\hat{R} \times \vec{a} \right]}$$

$$= \hat{R} a^2 - \vec{a} a_R - a_R \left\{ \hat{R} a_R - \vec{a} \right\}$$

$$= (a^2 - a_R^2)\, \hat{R}$$

$$\vec{S} = \frac{\mu_0}{c} \frac{q^2}{16\pi^2 R^2} \left[a^2 - (\vec{a} \cdot \hat{R})^2 \right] \hat{R}$$

The direction is pointing away from the position $\vec{x}_q(t')$ at the earlier time $t' = t - R/c$.

R4/5

$$\vec{S} = \frac{\mu_0}{c}\,\frac{q^2}{16\pi^2 R^2}\left[a^2 - (\vec{a}\cdot\hat{R})^2\right]\hat{R}$$

- The instantaneous power radiated (at time t')

$$P = \oint \vec{S}\cdot d\vec{A} = \int \vec{S}\cdot\hat{R}\, R^2 \sin\theta\, d\theta\, d\varphi$$

$$P = \frac{\mu_0}{c}\,\frac{q^2}{16\pi^2}\left[a^2\cdot 4\pi - a^2\cdot\frac{4\pi}{3}\right]$$

$$P = \frac{\mu_0}{c}\,\frac{2q^2a^2}{3\cdot 4\pi}$$

μ_0

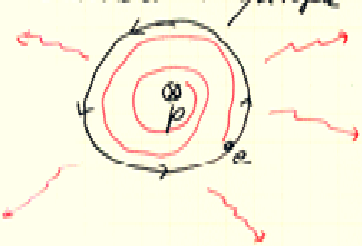
$$\boxed{P = \frac{1}{4\pi\epsilon_0}\,\frac{2q^2a^2}{3c^3}}$$

Larmor's formula
= the power radiat
a nonrelativist

- In the classical theory, if a charged partic
accelerates then it radiates. $P \propto a^2$

- Energy is conserved, so if a charged partic
accelerates then it loses energy.

- The "classical" hydrogen atom is unstable



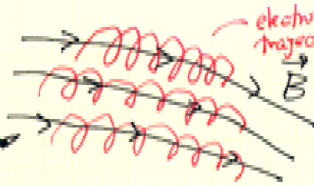
A CLASSICAL UNIVERSE ISN'T REALLY POSSIBLE! ? Intelligent design ?

lifetime = $\frac{r_0}{4c}$

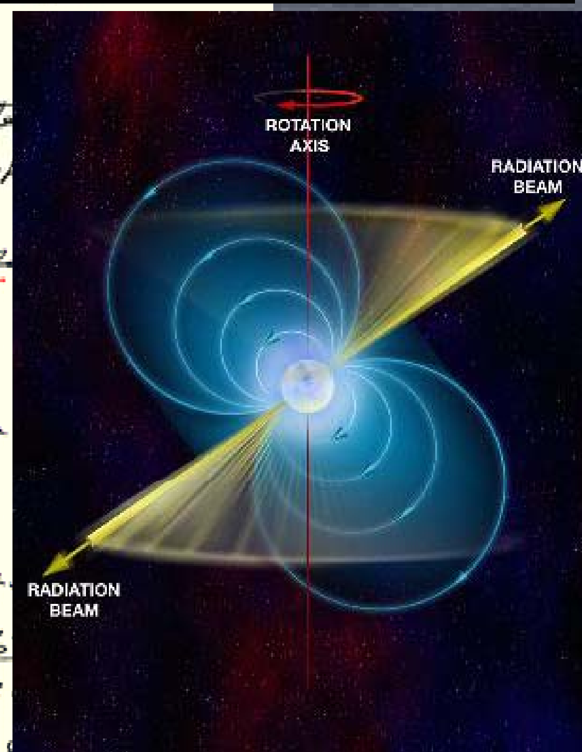
$r_e = \frac{e^2}{4\pi\epsilon_0\, mc^2} = 2.8$

$r_0 \sim 10^{-10}\,\text{m} \Rightarrow \text{lifetime} \sim 10^{-11}\,\text{s}$

- Synchrotron radiation
  - light sources for experiments
  - radio astronomy



Radio waves from a pulsar



Quiz Question

(A) Evaluate $\int_{-\infty}^{\infty} \delta(x-1)\, dx$

(B) Evaluate $\int_{-\infty}^{\infty} \delta(\sqrt{x}-1)\, dx$