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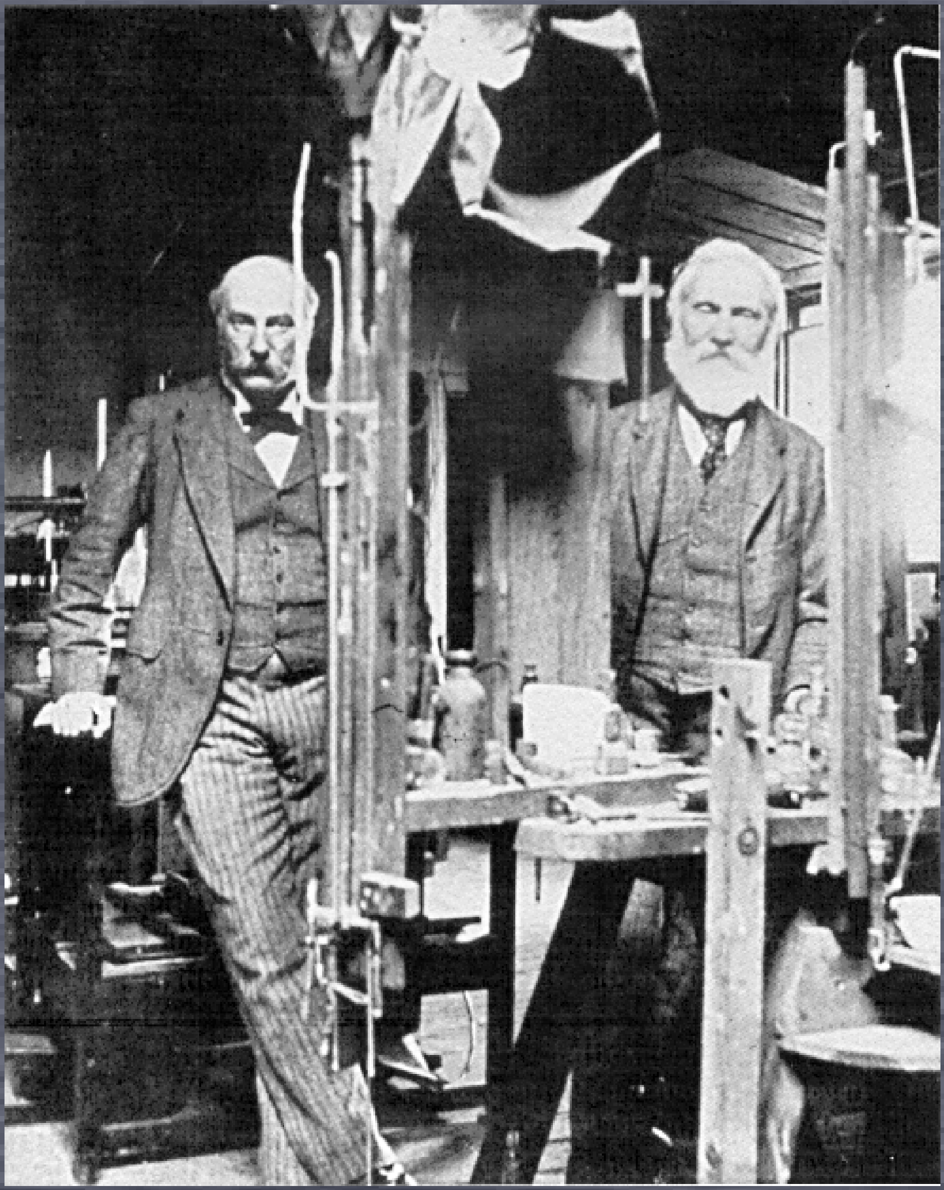
Lord Rayleigh
(family name: John Strutt)
Cavendish Professor at Cambridge (Maxwell's successor)

Lord Kelvin
(family name; William Thompson)
Professor at Glasgow



Rayleigh is perhaps most famous for his discovery the inert gas argon in 1895, work which earned him a Nobel Prize in 1904.

Rayleigh's theory of light scattering, published in 1871, was the first correct explanation of why the sky is blue.

Rayleigh was also interested in Spiritualism and became president of the British Society for Psychical Research, which studied hauntings and seances.

In the late 19^{th} century, Kelvin was regarded widely as the greatest scientific mind of the time. Here are some of his quotations:

"There is nothing new to be discovered in physics now. All that remains is more and more precise measurement."

"X-rays will prove to be a hoax.“

"It seems to me that there must be something in this molecular hypothesis and that as a mechanical symbol, it is certainly not a mere hypothesis, but a reality."

"I never satisfy myself until I can make a mechanical model of a thing. If I can make a mechanical model I can understand it. As long as I cannot make a mechanical model all the way through I cannot understand; and that is why I cannot get the electromagnetic theory ..... But I want to understand light as well as I can, without introducing things that we understand even less of. That is why I take plain dynamics. I can get a model in plain dynamics; I cannot in electromagnetics“.

[The vector] "has never been of the slightest use to any creature.“

"Radio has no future.“

Writing to Niagara Falls Power Company: "Trust you will avoid the gigantic mistake of alternating current.“

"I have not the smallest molecule of faith in aerial navigation other than ballooning or of the expectation of good results from any of the trials we hear of ... I would not care to be a member of the Aeronautical Society."

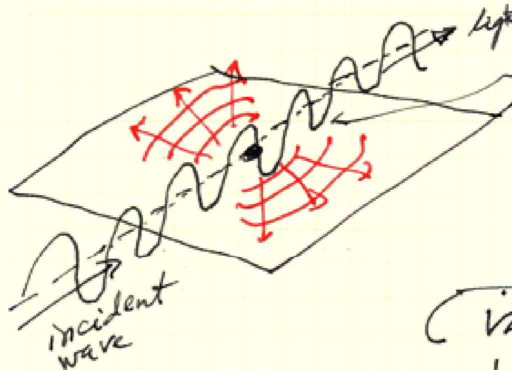
"Overwhelming strong proofs of intelligent and benevolent design lie around us.”

Classical Electron Theory of Light Scattering

Larmor's formula $P = \frac{1}{4\pi\epsilon_0} \frac{2q^2a^2}{3c^3}$ (nonrelativistic)

where $\vec{a}$ = acceleration, P = power.

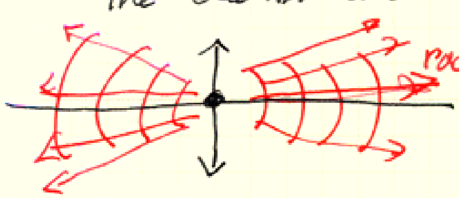
Classical picture of light scattering



$\lambda \gg$ atomic diameter

(visible light: $\lambda = 400$ to $700\,nm$
$d \sim$ few $\times 10^{-10}\,m \sim$ few $\times 0.1\,nm$)

The electron oscillates in a normal direction, depending on polarization.



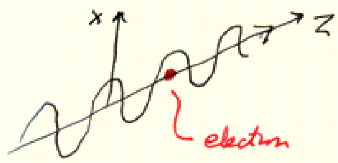
It's a scattering process: the incident wave loses energy as it accelerates the electron; and the electron radiates waves in all directions.

So in the classical theory, we just need to calculate the acceleration.

Acceleration of the electron

$R_5/2$

$$m \frac{d^2\vec{x}}{dt^2} = -K\vec{x} - \gamma \frac{d\vec{x}}{dt} - e\vec{E}_0\, e^{-i\omega t}$$



The steady-state solution is $\vec{x}(t) = x(t)\,\hat{x}$

$$x(t) = \frac{-eE_0\, e^{-i\omega t}}{m(\omega_0^2-\omega^2) - i\gamma\omega} \quad \text{where} \quad \omega_0 = \sqrt{\frac{K}{m}}$$

"natural frequency"

i.e., harmonic oscillations at the driving frequency ω.

The real part is the physical quantity

$$x(t) = A_1 \cos\omega t + A_2 \sin\omega t$$

$$a(t) = -\omega^2 A_1 \cos\omega t - \omega^2 A_2 \sin\omega t$$

$$\langle a^2 \rangle = \omega^4 A_1^2 \langle \cos^2\omega t\rangle + \omega^4 A_2^2 \langle \sin^2\omega t\rangle + 2\omega^4 A_1 A_2 \langle \cos\omega t \sin\omega t\rangle$$

(time average)

$$\langle a^2 \rangle = \frac{\omega^4}{2}(A_1^2 + A_2^2)$$

$$\langle P \rangle = \frac{1}{4\pi\epsilon_0}\, \frac{2e^2\langle a^2\rangle}{3c^3}$$ = average power outgoing in radiation.

$$\langle P \rangle = \frac{e^2\omega^4}{12\pi\epsilon_0 c^3}(A_1^2 + A_2^2)$$

Averaging harmonic functions

$$\langle \cos^2\phi \rangle = \frac{1}{2\pi}\int_0^{2\pi} \cos^2\phi \, d\phi = \frac{1}{4\pi}\left(\phi + \frac{1}{2}\sin 2\phi\right)\Big|_0^{2\pi}$$

$$= \frac{1}{2}$$

$$\cos 2\phi = 2\cos^2\phi - 1$$

$$\langle \sin^2\phi \rangle = \frac{1}{2} \text{ similarly}$$

$$\langle \sin\phi \cos\phi \rangle = \frac{1}{2\pi}\int_0^{2\pi} \sin\phi\cos\phi \, d\phi$$

$$= \frac{1}{2\pi} \; \frac{1}{2}\sin^2\phi\Big|_0^{2\pi} = 0$$

3a

$$x(t) = \frac{-eE_0 e^{-i\omega t}}{m(\omega_0^2 - \omega^2) - i\gamma\omega}$$

$$= \frac{-eE_0(\cos\omega t - i\sin\omega t)\left[m(\omega_0^2-\omega^2) + i\gamma\omega\right]}{m^2(\omega_0^2-\omega^2)^2 + \gamma^2\omega^2}$$

$$\mathrm{Re}\, x = A_1 \cos\omega t + A_2 \sin\omega t$$

$$A_1 = \frac{-eE_0\, m(\omega_0^2-\omega^2)}{m^2(\omega_0^2-\omega^2)^2 + \gamma^2\omega^2}$$

$$A_2 = \frac{-eE_0\, \gamma\omega}{m^2(\omega_0^2-\omega^2)^2 + \gamma^2\omega^2}$$

$$A_1^2 + A_2^2 = e^2E_0^2 \frac{m^2(\omega_0^2-\omega^2)^2 + \gamma^2\omega^2}{\left[m^2(\omega_0^2-\omega^2)^2 + \gamma^2\omega^2\right]^2}$$

$$= \frac{e^2E_0^2}{m^2(\omega_0^2-\omega^2)^2 + \gamma^2\omega^2}$$

$$\langle P\rangle = \frac{e^2\omega^4}{12\pi\epsilon_0 c^3}\left(A_1^2+A_2^2\right) = \frac{e^4 E_0^2\omega^4}{12\pi\epsilon_0 c^3\left[m^2(\omega_0^2-\omega^2)^2+\gamma^2\omega^2\right]}$$

R5/3

Define $r_e = \frac{1}{4\pi\epsilon_0}\frac{e^2}{mc^2}$ "classical radius of the electron" $(= 2.8 \text{ fm})$

$$\langle P\rangle = \frac{4\pi\epsilon_0 c}{3} r_e^2 E_0^2 \frac{\omega^4}{(\omega_0^2-\omega^2)^2+\left(\frac{\gamma\omega}{m}\right)^2}$$

<u>Scattering cross section</u>

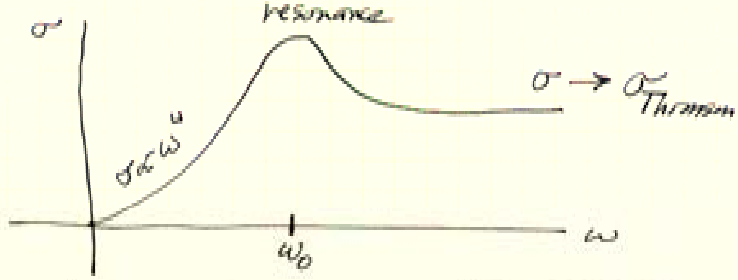
The total cross section is defined by $\sigma = \frac{\langle P\rangle}{\langle S\rangle}$

i.e. $\frac{\text{radiated power}}{\text{incident intensity}}$

Incident intensity $= \frac{EB}{\mu_0} = \frac{E_0^2}{\mu_0 c}\langle\cos^2\omega t\rangle = \frac{E_0^2}{2\mu_0 c} = \frac{1}{2}\epsilon_0 c E_0^2$

$$\sigma = \frac{8\pi r_e^2}{3}\frac{\omega^4}{(\omega_0^2-\omega^2)^2+\left(\frac{\gamma\omega}{m}\right)^2}$$

light scattering cross section in the classical model.



Rayleigh scattering $\omega \ll \omega_0$

Thomson scattering $\omega \gg \omega_0$ but $\hbar\omega \ll mc^2$

$$\sigma_{Rayleigh} = \frac{8\pi r_e^2}{3}\left(\frac{\omega}{\omega_0}\right)^4 \qquad \sigma_{Thomson} = \frac{8\pi r_e^2}{3}$$

$(= 6.6\times 10^{-29}\ m^2)$

The sky is blue
because $\sigma \propto \omega^4$
for light scattering by
atmospheric molecules.
— Lord Rayleigh

Quiz Question

Calculate $\sigma_{violet}/\sigma_{red}$.