Experiment 5
Elastic and Inelastic Collisions

Reading: Bauer&Westfall: Chapter 7 (and 8, for center of mass ideas) as needed

Homework 5: turn in the second week of this experiment.

Assume random and independent errors, and use

$v = L/t \quad p = m\,v \qquad K = \dfrac{p^2}{2m}$

measured: L = 2 ± .06 t = 1 ± .04 m = 2 ± .0001 (negligible uncertainty)

For each question below, give the algebraic formula, then evaluate the numerical uncertainty in value and in % (e.g. find δK, then δK/K in %):

1. Calculate the uncertainty in *v* in terms of the uncertainty in *L* and in *t*.
2. Calculate the uncertainty of *p* in terms of the uncertainty of *v*.
3. Calculate the uncertainty of *K* in terms of the uncertainty of *p*.

1. Goals

1. Study momentum and energy conservation in inelastic and elastic collisions
2. Understand use of Excel in analyzing data
3. Carry out uncertainty calculations of moderate complexity

2. Theoretical Introduction

The following experiment explores the conservation of momentum and energy in a closed physical system (ideally: no interaction of measured objects with rest of universe). As you probably know from the accompanying theoretical course, the conservation of energy and momentum play an important role in physics and their conservation is a consequence of fundamental symmetries of nature.

2.1 Momentum

For a single particle (or a very small physical object), momentum is defined as the product of the mass of the particle and its velocity:

$$\vec{p} = m\,\vec{v} \qquad (1)$$

Momentum is a vector quantity, making its direction a necessary part of the data. To define the momentum in our three-dimensional space completely, one needs to specify its three components in *x, y* and *z* direction. The momentum of a system of more than one particle is the vector sum of the individual momenta:

$$\vec{p} = \vec{p}_1 + \vec{p}_2 + \cdots = m_1\vec{v}_1 + m_2\vec{v}_2 + \cdots \qquad (2)$$

The 2nd Newton's law of mechanics can be written in a form which states that the rate of the change of the system's momentum with time is equal to the sum of the external forces acting on this system:

$$\frac{d\vec{p}}{dt} = \Sigma\vec{F} \tag{3}$$

From here we can immediately see that when the system is closed (which means that the net external force acting on the system is zero), the total momentum of the system is conserved (constant).

2.2 Energy

Another important quantity describing the evolution of the system is its energy. The total energy of a given system is generally the sum of several different forms of energy. Kinetic energy is the form associated with motion, and for a single particle:

$$KE = \frac{mv^2}{2} \tag{4}$$

Here *v* without the vector symbol stands for the absolute value of the velocity,

$$v = \sqrt{v_x^2 + v_y^2 + v_z^2}\,.$$

In contrast to momentum, kinetic energy is NOT a vector; for a system of more than one particle the total kinetic energy is the algebraic sum of the individual kinetic energies of each particle:

$$KE = KE_1 + KE_2 + \cdots \tag{5}$$

Another fundamental law of physics is that the total energy of a system is always conserved. However within a given system, one form of energy may be converted to another (such as potential energy converted to kinetic in the Pendulum experiment). Therefore, kinetic energy alone is often not conserved.

2.3 Collisions

An important area of application of the conservation laws is the study of the collisions of various physical bodies. In many cases, it is hard to assess how exactly the colliding bodies interact with each other. However, in a closed system, the conservation laws often allow one to obtain the information about many important properties of the collision without going into the complicated details of the collision dynamics. In this lab, we will see in practice how the conservation of momentum and total energy relate various parameters (masses, velocities) of the system independently of the nature of the interaction between the colliding bodies.

Assume we have two particles with masses m_1, m_2 and speeds $\vec{v}_{1i}$ and $\vec{v}_{2i}$ which collide, without any external force, resulting in speeds of $\vec{v}_{1f}$ and $\vec{v}_{2f}$ after the collision (*i* and *f* stand for *initial* and *final*). Conservation of momentum then states that the total momentum before the collision $\vec{P}_i$ is equal to the total momentum after the collision $\vec{P}_f$:

$$\vec{P}_i = m_1\vec{v}_{1i} + m_2\vec{v}_{2i}, \qquad \vec{P}_f = m_1\vec{v}_{1f} + m_2\vec{v}_{2f} \qquad \text{and} \qquad \vec{P}_i = \vec{P}_f \tag{6}$$

2.4 Elastic and inelastic collisions

There are two basic kinds of collisions, elastic and inelastic:

2.4.1 In an ***elastic*** collision, two or more bodies come together, collide, and then move apart again with ***no loss in total kinetic energy***. An example would be two identical "superballs", colliding and then rebounding off each other with the same speeds they had before the collision. Given the above example conservation of kinetic energy then implies:

$$\frac{1}{2}m_1 v_{1i}^2 + \frac{1}{2}m_2 v_{2i}^2 = \frac{1}{2}m_1 v_{1f}^2 + \frac{1}{2}m_2 v_{2f}^2 \text{ or } KE_i = KE_f \qquad (7)$$

2.4.2 In an ***inelastic*** collision, the bodies collide and (possibly) come apart again, but now ***some kinetic energy is lost*** (converted to some other form of energy). An example would be the collision between a baseball and a bat. If the bodies collide and stick together, the collision is called *completely inelastic*. In this case, all of the kinetic energy *relative to the center of mass of the whole system* is lost in the collision (converted to other forms).

In this experiment you will be dealing with

a) a *completely inelastic* collision in which all kinetic energy relative to the center of mass of the system is lost, but momentum is still conserved, and

b) a *nearly elastic* collision in which both momentum and kinetic energy are conserved to within a few percent.

2.5 Conservation laws for macroscopic bodies

So far we were talking about the system of point-like particles. However, the conservation of the momentum is also valid for macroscopic objects. This is because the motion of any macroscopic object can be decomposed into the motion of its center of mass (which is a point in space) with a given linear momentum, and a rotation of the object around this center of mass. Then, the conservation of the linear momentum is again valid for the motion of ideal point masses located at the center of the mass of each of the objects. However, some of the linear kinetic energy can be transformed into the rotational energy of the objects, which should be accounted for in a real experiment.

2.6 Kinetic Energy in Inelastic Collisions.

It is possible to calculate the percentage of the kinetic energy lost in a completely inelastic collision; you will find that this percentage depends only on the masses of the carts used in the collision, if one of the carts starts from rest.

After the completely inelastic collision, the carts move together, so that

$$v_{1f} = v_{2f} = v_3$$

The initial *KE* is given by:

$$KE_i = \frac{m_1 v_{1i}^2}{2} + \frac{m_1 v_{2i}^2}{2}. \text{ But, since } v_{2i} = 0$$

$$KE_i = \frac{m_1 v_{1i}^2}{2} \qquad (8)$$

The final *KE* is given by:

$$KE_f = v_3{}^2 \frac{m_1 + m_2}{2} \qquad (9)$$

From conservation of momentum:

$$m_1 v_{1i} + m_2 v_{2i} = (m_1 + m_2)v_3 \qquad \text{or, since } v_{2i} = 0$$

$$m_1 v_{1i} = (m_1 + m_2)v_3 \tag{10}$$

Since the collision is inelastic, the initial *KE* is not equal to the final *KE*. You could use equations (8), (9), and (10) to obtain an expression for $D_K(\%) = \frac{(KE_f - KE_i)}{KE_i}$. Hint: define x = $m_1/(m_1+m_2)$ and use it to eliminate v_3.

3 Experimental setup

We will study the momentum and energy conservation in the following simplified situation:

a) we will look on the collision of only 2 objects;
b) the motion of these objects will be linear and one-dimensional, so that we can choose the reference frame in such a way that only *x*-components of the objects' momenta are non-zero; the sign of these components depends on the direction of the motion;
c) the experimental apparatus can be set up in a way to almost completely eliminate the net external force on the system.

Our objects will be two carts of different masses, with one initially at rest. The carts move on an air track, which ensures that the motion is one-dimensional and reduces the friction between the carts and the surface. The velocities of the carts can be measured with the help of the photogates, which are described in more details below.

Before the beginning of the measurements, spend at least 15 minutes to figure out which external factors can disturb the motion of the carts on the track, and what you should do to reduce or eliminate these factors. Remember, the successful completion of this lab strongly depends on your ability to create an almost closed system. Make a few practice trials to see if you can achieve an unperturbed one-dimensional collision of the carts. Adjust the level of the air track and the power of the air supply if necessary.

Questions for preliminary discussion

3.1 Draw a diagram of all forces acting on each cart when they collide. Which forces will influence the total *P* and *KE* most?
3.2 In our experiment, can we achieve a completely elastic collision? a completely inelastic collision?
3.3 In an inelastic collision in a closed system, can some of the total momentum be lost? Some of the kinetic energy?
3.4 In an elastic collision in a closed system, can some of the total momentum be lost? Some of the kinetic energy be lost?
3.5 Extra Credit: Find a formula for $D_K(\%)$ as a function of x, `as suggested in the end of section 2.6 .
3.6 If kinetic energy is lost, where does it go? Does conservation of energy apply?

Measurements

4. Inelastic collisions

4.1 **Techniques.** In the first part of the lab we make sure that after the collision the carts stick together and move with some velocity common to both masses. Thus, we have to measure the velocity of cart 1 before the collision and the common velocity of the carts 1 and 2 after the collision. For this purpose, we use two photogates (see Figure 1). Each of them allows measuring the time it takes the cart to go through it. The speed is calculated by dividing the length of the fin on the cart 1 by the measured time; but to turn speed into velocity, as required, you have to pick a + direction.

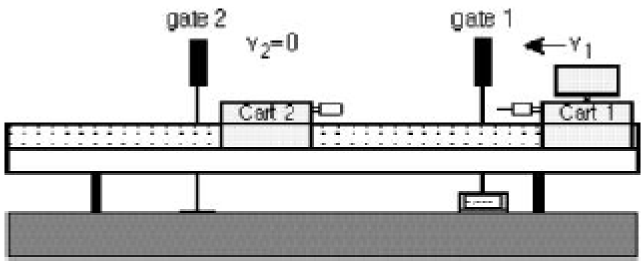

Figure 1: Initial state of the carts before inelastic collision (fin on cart 2 can be removed).
Warning: *your carts should be balanced, unlike the ones in this figure!*

Position cart 2 close to the gate 2 and set the photogate timer to "GATE" mode and the memory switch in "ON" position. In this mode the photogate will display the first time interval measured. Subsequent measurements will not be displayed (only the first one is), but the times are added in the memory. By pushing the "READ" switch you can display the memory contents, which is the sum of all measurements. Example: the initial reading for cart 1 (the time that it took to pass through the gate 1) is 0.300 seconds. Cart 1 collides with cart2 and they go together through the photogate 2 (Figure 2). Suppose it now takes 0.513 seconds. The display will remain at 0.300, but the memory will contain .300 + 0.513=0.813 seconds. To find the second time, you have to subtract the first time from the contents of the memory. Try this out by moving the cart through the gate by hand a few times.

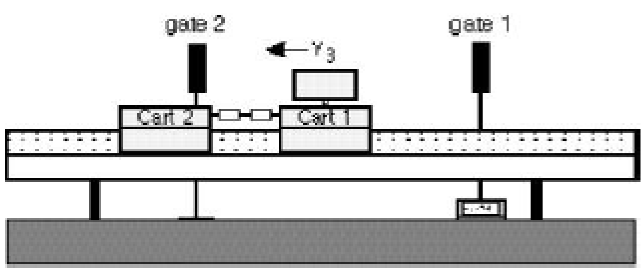

Figure 2: The final state of the carts after inelastic collision.

4.2 **Uncertainty Estimation:** First we test to see whether some of our assumptions are correct (the two timers give the same answer, the track is level, friction is negligible, direction doesn't matter). Perform some trials with a single cart in which no collision occurs. Do this at different speeds, and in different directions. Record the results in your notebook in appropriate tables. Did you conserve momentum? Was there a bias? What time uncertainty would you deduce from these measurements? What conclusions do you draw?

4.3 **Prediction:** Write in your lab book which case below will the change the final Kinetic Energy most. In your report comment on whether your predictions were correct, and if not, why.

4.4 **Data** Do 3 sets of inelastic collisions consisting of 2 trials each. Vary the masses of the carts by adding the masses (small metal disks) to them. In these measurements, use the needle and putty bumpers and measure the initial and final velocities for the following sets of masses:

Trial 1+2: no mass disks on cart 1, 4 mass disks on cart 2;
Trial 3+4: 2 mass disks on cart 1, 2 mass disks on cart 2;
Trail 5+6: 2 mass disks on cart1, no mass disks on cart2.

In each measurement, you need to find all the initial and final masses and velocities, and use them to calculate the initial and final total momentum and kinetic energy. **Make tables in your lab book to organize your recordings.** You may neglect the uncertainty of all masses.

In R:exp6, you will find 2 preset datasheets, Inelastic and Elastic. Open them and save into your section's folder. The spreadsheet has some entries for fractional uncertainties (δDp(%), for example). These should be displayed in % either by multiplying the relevant fraction by 100, or, preferably, by using the % formatting button in Excel. When you have completed the spreadsheets, print them showing the numbers, then save them. There are many uncertainty calculations. You should explain in your lab book or report how you calculated them. An alternative is to re-open your spreadsheet, save it with a new name, use the ctrl-` key to display formulas (handy while checking formulas) and print again, showing the formulas used. It won't be possible to read the formulas unless you adjust the column width, and use page setup to print landscape, to fit to 2 pages wide x 1 tall. You can use Print Preview before printing to check.

4.4 ***Hints*** The correct calculation of the uncertainty of δD(%) is complicated because the initial value for p or K is in both the numerator and denominator. However, for our purposes, it is sufficiently close to use $\boldsymbol{\delta D_p(\%) \approx \delta D/P_i}$ and a similar formula for K.

(**Optional**) If you wish you could add auxiliary cells to help you with uncertainty calculations. For example, you could also calculate δv/v or δP/P.

(**Sanity checks for uncertainties**) *Check* whether your uncertainty calculations are making sense! For independent and random uncertainties, you expect increasing values of absolute and fractional uncertainties as your calculation proceeds (unless your calculation involves fractional powers < 1). This is discussed in the Reference Guide section on Uncertainty Calculations.

5. Elastic collisions

5.1 **Techniques.** In an almost elastic collision, the main difference from the previous part of the lab is that after the collision the carts move separately. The rubber band bumpers allow carts to collide with almost no conversion of the kinetic energy into the other forms of energy.

As before, cart 2 initially stays at rest, and <u>before</u> the collision we have to measure only the velocity of the cart 1 v_{1i} (Figure 3). However, after the collision we have to measure the velocities of <u>both</u> carts, v_{1f} and v_{2f} (Figure 4). Thus, all in all we have to measure three times (t_{1i}, t_{1f}, t_{2f}), while the photogate system can simultaneously measure only <u>two</u> of them.

We can get out of this situation if, <u>after</u> the measurement of the initial time t_{1i} , but <u>before</u> the collision, we reset the timer. You have to make several practice trials to quickly remember and reset the contents of the timer before the carts collide. Then, we can again see the contents of the timer display *and* the memory to find t_{1f} and t_{2f} .

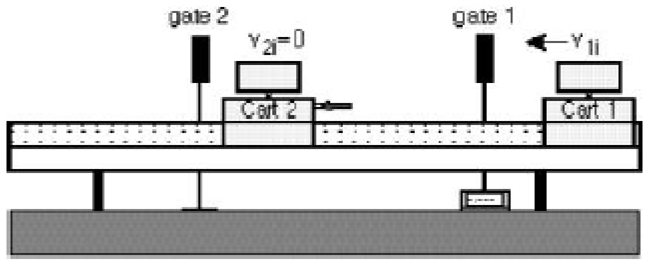

Figure 3: The initial state of the carts before elastic collision.

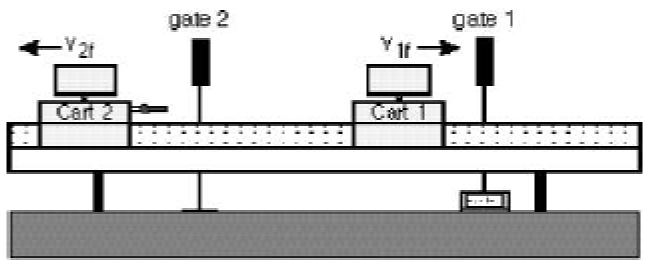

Figure 4: The final state of the carts after elastic collision.

5.2 **Prediction:** Write in your lab book for each case mentioned in section 5.3 below the predicted direction and speed after the collision of the initial-moving cart:

Forward or backward?

Faster or slower than the cart initially at rest?

In your report comment on whether your predictions were correct, and if not, why.

5.3 **Data** Measure the fin length on the two carts. The experiment will be done with cart 2 initially at rest. You will do 6 trials with the following choices of m_1 and m_2 :

Trial 1+2: no mass disks on cart 1, 4 mass disks on cart 2
Trial 3+4: 2 mass disk on cart 1, 2 mass disks on cart 2
Trail 5+6: 4 mass disks on cart 1 no mass disk on cart 2

5.4 **Hints.** Pay attention to the **sign of the velocities,** which depends on the direction of motion of the cart. If one of the carts goes backward with respect to your chosen + direction, how will you make sure the velocity is calculated as negative?

If the percentage change in momentum or kinetic energy before and after the collision is greater than 10%, repeat the measurement more carefully (collide slower/faster, etc.). Since the datasheet is set up it is easy to see whether momentum/energy is better conserved with every trial you do.

If one of the times you measure is too long for the timer to measure, substitute a large number for the time in your spreadsheet.

6 Questions to be discussed

6.1 Create a graph (one for inelastic and one for elastic collisions) of the relative change of the total momentum $D_p(\%) = (P_f - P_i)/P_i$ versus the number (1-6) of the measurement. This is just to show graphically the various values of $D_p(\%)$. Show the uncertainty of $D_p(\%)$ with the help of error bars. Also, show the theoretical prediction for $D_p(\%)$.

6.2 According to the graphs, was the total momentum conserved in the collisions? Use the "two standard deviations" rule to justify your answers. Was there any difference in how well momentum was conserved in elastic vs. inelastic collisions? If the momentum was not conserved, discuss the reason why. What did you do to try to improve the momentum conservation results?

6.3 Did you correctly predict the motion of the carts in the elastic collision? If not, why?

6.4 For the inelastic collision, plot the kinetic energy fractional change, $D_K(\%)$ vs. $x = \dfrac{m_1}{m_1 + m_2}$. Show the error bars for $D_K(\%)$. What are the slope and intercept of a straight line fit to these data? Does a straight line fit these data reasonably? Did you correctly predict the trial with the largest fractional change? Why or why not?

6.5 **Extra Credit**: Draw your predicted line on the graph, and discuss quantitatively the consistency of your inelastic collision data for $D_K(\%)$ with the line you predicted for the theoretical prediction.

6.6 Make and analyze a $D_K(\%)$ graph for the nearly elastic collision (as you analyzed Dp(%) above in steps 6.1-6.3). What was the average loss of the kinetic energy in this part of the experiment? Did you achieve the goal of 10% loss of KE in the nearly elastic collisions? Which potential systematic errors or setup problems is the loss most sensitive to?

6.7 Extra Credit: Use the excel spreadsheets with your data for this question. Save a NEW copy of each sheet before proceeding, and work on the new copy.
a) What happens to your momentum conservation $D_p(\%)$ if you arbitrarily change the fin length in the inelastic collision by a factor of 2? Explain why this is so.
b) Repeat a) for the fractional kinetic energy loss in the inelastic collision.
c) Repeat a) for the elastic collision. Is the result the same as in a)? Why or why not?

6.8 What was the muddiest point of this lab?

7. Systematic Errors

Professional scientists deal with systematic errors in successive levels of sophistication.
Level 1:Think of possible errors. This is one of the hardest parts! That's because doing so means seeing where our *assumptions* break down. The best way is to go through the measurement step by step and identify where things could have been otherwise than we wished. The result is a laundry list of possible systematic effects. Some examples of our assumptions:
Setup: We assumed that the track was straight and level. Carts were balanced.
Timing: The velocities were constant while the carts moved through the gate and until the collision.
Collision: The system of the 2 carts is closed: no momentum (nor energy) enters or leaves the system consisting of the two carts during the collision.

Question 7.1 List ways in which these assumptions might have been violated.

Level 2: Sign of effect. For each violation of our assumptions, try to understand the sign of its effect on our final result. For example, if the carts weren't balanced, then the air track would tend to add momentum of a particular sign to the cart. It often helps to exaggerate (mentally, or by measuring) the effect to see which way it should affect your results.

7.2 What sign would each of your effects have on momentum or energy conservation? Or could it be of either sign?

Level 3: Estimating the size of the effect. Sometimes we can perform a side-experiment and actually measure the effect and correct for it. More commonly, we can only provide a bound "well it should really have been at worst this big". For example, leveling errors didn't give the carts enough extra momentum to go the length of the track in 10 seconds. With the mass, that bounds the change of momentum; that over the smallest initial momentum bounds the % error.

7.3 How big an effect could the air track effect be on your momentum conservation data?
7.4 How big an effect would that have on the energy loss data for inelastic collisions?
7.5 **Extra Credit:** See what you can offer as a bound for the rest of the effects in your list.