Light is very deep…

The First Book of Moses, Called **Genesis**

1 The Creation

1 In the beginning God created the heaven and the earth.
2 And the earth was without form, and void; and darkness *was*
upon the face of the deep. And the Spirit of God moved upon
the face of the waters.
3 ¶ And God said, "**Let there be light**": and there was light.
4 And God saw the light, that *it was* good: and God divided the
light from the darkness.
5 And God called the light Day, and the darkness he called
Night. And the evening and the morning were the first day.

ויהי אור!

… in Hebrew

$$\nabla \cdot \mathbf{E} = 0 \qquad \text{and} \qquad \nabla \cdot \mathbf{B} = 0$$

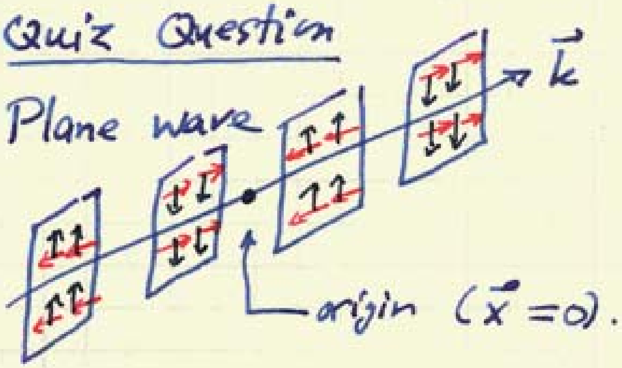
$$\nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t} \qquad \text{and} \qquad \nabla \times \mathbf{B} = \frac{1}{c^2}\frac{\partial \mathbf{E}}{\partial t}$$

… in field equations

Quiz Question

11.2/7

Plane wave



Energy densities in an electromagnetic plane wave .....

(A) Determine $u_E(\vec{x},t)$ at $\vec{x}=0$.

(B) Determine $u_M(\vec{x},t)$ at $\vec{x}=0$.

Express both answers in terms of E_0; $E_0 = |\vec{E}_0|$.

(A) $\vec{E}(\vec{x},t) = \text{Re } \vec{E}_0 e^{i(\vec{k}\cdot\vec{x}-\omega t)} = \vec{E}_0 \cos(\vec{k}\cdot\vec{x}-\omega t)$

$$u_E(\vec{0},t) = \frac{\epsilon_0}{2} E^2(\vec{0},t) = \frac{\epsilon_0}{2} E_0^2 \cos^2(\omega t)$$

(B) $u_M(\vec{0},t) = \frac{1}{2\mu_0} B^2(\vec{0},t) = \frac{1}{2\mu_0} B_0^2 \cos^2(\omega t)$

$\left\{ \text{substitute } B_0 = E_0/c \text{ and } c^2 = \frac{1}{\mu_0\epsilon_0} \right\} \hookrightarrow = \frac{\epsilon_0}{2} E_0^2 \cos^2(\omega t)$

The electric and magnetic field energies are equal.

Exam – Friday Feb 26

Maxwell's equations in matter

- Start with the fundamental equations

(1) $\nabla\cdot\vec{E} = \rho/\epsilon_0$ (2) $\nabla\cdot\vec{B} = 0$

(3) $\nabla\times\vec{E} = -\frac{\partial\vec{B}}{\partial t}$ (4) $\nabla\times\vec{B} = \mu_0\vec{J} + \mu_0\epsilon_0\frac{\partial\vec{E}}{\partial t}$

Now consider the effects of macroscopic matter, i.e., matter with many atoms ($\sim 6\times10^{23}$). We'll treat the charge and current in matter in an average way — a very good approximation because N is so large

$$\text{error} \sim \frac{1}{\sqrt{N}} \sim \frac{1}{\sqrt{6\times10^{23}}} \sim \frac{1}{10^{12}}$$

Could be accurate to 12 decimal places.

- Two of Maxwell's equations do not depend on matter

(2) $\nabla\cdot\vec{B} = 0$ ($\nexists$ magnetic monopoles)

(3) $\nabla\times\vec{E} = -\frac{\partial\vec{B}}{\partial t}$ (a field effect)

The other two depend on sources, i.e., matter, for which we'll make macroscopic approximations.

11.3/2

- <u>Charge density</u>

$$\rho_{MICRO}(\vec{x},t) = \sum_{i=1}^{N} e_i \cdot \delta^3(\vec{x}-\vec{x}_i(t)) \qquad (N > 6 \times 10^{23}$$

sum over all charged (i.e., subatomic) particles

or, in quantum mechanics

$$\rho_{MICRO}(\vec{x},t) = \sum_{i=1}^{N} e_i \left| \Psi_i(\vec{x},t)\right|^2 .$$

Now define the <u>macroscopic</u> <u>average</u>

$$\rho(\vec{x},t) = \frac{1}{\delta V}\int_{\delta V} \rho_{MICRO}(\vec{x}',t)\, d^3x'$$

where δV is a <u>large</u> and <u>small</u> volume at $\vec{x}$.

δV is <u>large</u> compared to an atom,

and <u>small</u> compared to the full system $\Rightarrow \rho(\vec{x},t)$

Recall from Chapter 6, dielectric materials,

$$\rho(\vec{x},t) = \underbrace{\rho_F(\vec{x},t)}_{\substack{\text{Free charge;}\\ \text{all charge that}\\ \text{is not bound charge}}} + \underbrace{\rho_B(\vec{x},t)}_{\substack{\text{Bound charge;}\\ \text{charge of particles that}\\ \text{are bound in atoms}}}$$

$\rho_B = -\nabla\cdot\vec{P}$ function of $\vec{x}$ and t

$\vec{P}$ = polarization = <u>mean</u> <u>dipole moment</u> <u>density</u>

(another macroscopic average)

$\Rightarrow$ the equations for the macroscopic electric field $\vec{E}(\vec{x},t)$.

$$\nabla\cdot\vec{E} = \frac{\rho_F}{\epsilon_0}+\frac{\rho_B}{\epsilon_0} = \frac{\rho_F}{\epsilon_0} - \frac{\nabla\cdot\vec{P}}{\epsilon_0}$$

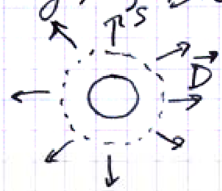
$$\nabla\cdot\left(\epsilon_0\vec{E}+\vec{P}\right) = \rho_F$$

Define the displacement field, $\vec{D} = \epsilon_0\vec{E}+\vec{P}$

Then $\nabla\cdot\vec{D} = \rho_F$

Use the free charge to calculate $\vec{D}(\vec{x},t)$.

E.g., $\oint_S \vec{D}\cdot d\vec{A} = Q_{free}$



But we still need a "constitutive equation" to determine $\vec{E}(\vec{x},t)$, such as

$\vec{D} = \epsilon\,\vec{E}$ or $\vec{P} = \epsilon_0\,\chi_e\,\vec{E}$

permittivity   susceptibility

This is familiar for static fields from Chap. 6.

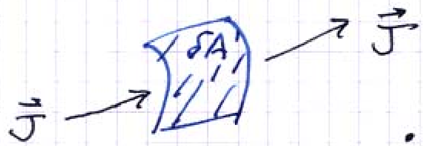
- Electric Current Density   11.3/4

↳ magnitization ; ferromagnets ;
also from electric polarization $\frac{\partial \vec{P}}{\partial t}$

Fundamental equation

$$\nabla \times \vec{B} = \mu_0 \vec{J} + \mu_0 \epsilon_0 \frac{\partial \vec{E}}{\partial t}.$$

Now, what is $\vec{J}(\vec{x}, t)$?



$$\vec{J} \cdot \delta\vec{A} \;\; \delta t = \delta Q$$

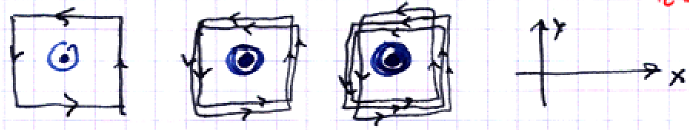
∴ Any process that moves charge contributes to $\vec{J}(\vec{x}, t)$.

3 ~~kinds~~ sources of current

- $\vec{J}_F$ = free current density ; charged particles moving outside atoms
- $\vec{J}_M = \nabla \times \vec{M}$; currents in magnetic dipoles, if $\vec{M}$ varies with position $\vec{x}$

Picture

$$\vec{M}(\vec{x}, t) = \frac{1}{\delta V} \sum_{i=1}^{\delta N} \vec{m}_i$$



M_z increases with x.

$(\nabla \times \vec{M})_y$ is not zero : curl $\vec{M} = -\frac{\partial M_z}{\partial x}\hat{j}$

Do you see the net current in $-y$ direction?

11.3/5

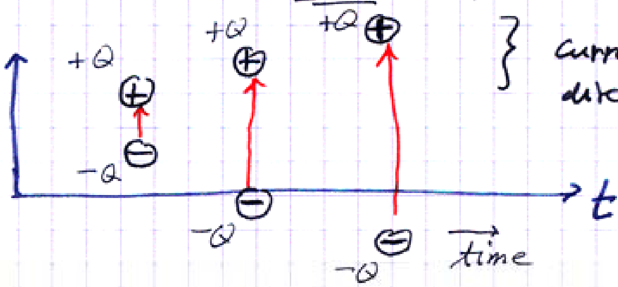
- The 3^{rd} ~~kind~~ source of current

$\vec{P}(\vec{x},t) = \frac{1}{\delta V}\sum_{i=1}^{\delta N} \vec{p}_i$

Polarization Current $\quad \vec{J}_P = \dfrac{\partial \vec{P}}{\partial t}$

= current in electric dipoles if $\vec{P}$ varies in time

Picture



} current in the direction of $\dot{\vec{P}}$

} current in the direction of $\dot{\vec{P}}$

Do you see the currents in the direction of $\dfrac{\partial \vec{P}}{\partial t}$?

Result $\quad \vec{J} = \vec{J}_F + \nabla \times \vec{M} + \dfrac{\partial \vec{P}}{\partial t}$

Theorem The continuity equation is obeyed.

Proof We must have $\nabla \cdot \vec{J} = -\dfrac{\partial \rho}{\partial t}$. Very fundamental!

$$\nabla \cdot \vec{J} = \nabla \cdot \vec{J}_F + 0 + \frac{\partial}{\partial t}(\nabla \cdot \vec{P})$$

$$= -\frac{\partial \rho_F}{\partial t} - \frac{\partial}{\partial t}\rho_B = -\frac{\partial \rho}{\partial t}$$

Q.E.D.

• Now the Ampère-Maxwell equation 11.3/6

$$\nabla\times\vec{B} = \mu_0\left(\vec{J}_F + \nabla\times\vec{M} + \frac{\partial\vec{P}}{\partial t}\right) + \mu_0\epsilon_0\frac{\partial\vec{E}}{\partial t}$$

$$\nabla\times\left(\vec{B}/\mu_0 - \vec{M}\right) = \vec{J}_F + \frac{\partial}{\partial t}\left(\epsilon_0\vec{E}+\vec{P}\right)$$

$$\boxed{\nabla\times\vec{H} = \vec{J}_F + \frac{\partial\vec{D}}{\partial t}}$$

$$\boxed{\begin{aligned}\vec{H} &= \frac{\vec{B}}{\mu_0} - \vec{M}\\ \vec{B} &= \mu_0\left(\vec{H}+\vec{M}\right)\end{aligned}}$$

The displacement current $\frac{\partial\vec{D}}{\partial t}$ is due to both a field effect and a matter effect.

We still need a ~~substitute~~ constitutive equation to relate $\vec{H}$ and $\vec{B}$, such as

$$\vec{B} = \mu\vec{H} \qquad \text{or} \qquad \vec{M} = \mu_0\chi_m\vec{B}$$

permeability susceptibility

or $\vec{B}(\vec{H})$ nonlinear for ferromagnetic materials.

Quiz Question 11/3/7

Maxwell's Equations in glass (ϵ, μ)

$$\nabla \cdot (\epsilon \vec{E}) = 0 \qquad \nabla \cdot \vec{B} = 0$$

$$\nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t} \qquad \nabla \times \left(\frac{\vec{B}}{\mu}\right) = \frac{\partial}{\partial t}(\epsilon \vec{E})$$

$$\boxed{\vec{D} = \epsilon \vec{E} \quad \text{and} \quad \vec{H} = \vec{B}/\mu}$$

(A) Then determine the speed of light, v for light passing through the glass.

(B) Determine the index of refraction, n of the glass.