Energy

Energy

- Energy is a great unifying principle in science.
- All branches of science use the principle of energy.
- We all purchase "energy" every day.

-- or, rather, energy sources

- gasoline
- electric power
- food

- But what *is* energy?

Definitions

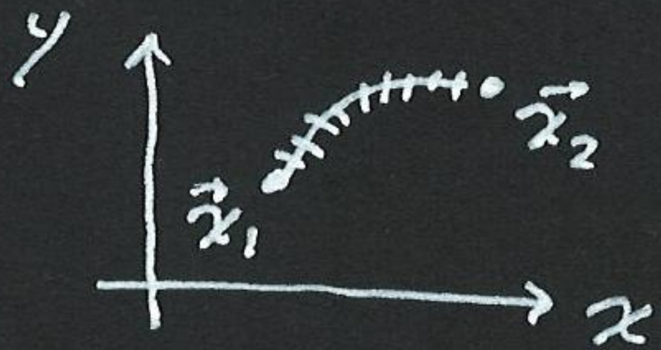
- Energy : the ability to do work.

- So, energy is not a thing or a substance; energy is an "ability."

- Everyday meaning of the words: To work is to accomplish something useful. You need energy to do work.

- Physics meaning of the words: more technical,
  Work = force times distance
  Energy = a quantity defined by an equation

Work



$$W = \sum_{i=1}^{N} \vec{F}_i \cdot (\Delta\vec{x})_i \xrightarrow{\text{calculus}} \int_{\vec{x}_1}^{\vec{x}_2} \vec{F} \cdot d\vec{x}$$

an example of integration.

For a single small step

$$\Delta W = \vec{F} \cdot \Delta\vec{x}$$

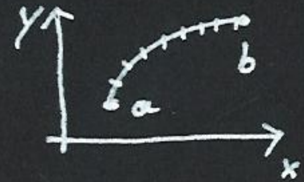
Kinetic Energy

Kinetic energy depends on velocity.

$$K = \frac{1}{2} m v^2$$

The Work-Kinetic energy theorem

A particle moves from a to b, under the influence of a force $\vec{F}$.



(1) Consider one small step

$$\Delta W = \vec{F} \cdot \Delta\vec{x}$$

$$= (m\vec{a}) \cdot (\vec{v}\,\Delta t)$$

$$= m\vec{v} \cdot (\vec{a}\,\Delta t)$$

$$= m\vec{v} \cdot \Delta\vec{v}$$

(2) Compare

$$\Delta K = \tfrac{1}{2} m (\vec{v} + \Delta\vec{v})^2 - \tfrac{1}{2} m \vec{v}^2$$

$$= \frac{m}{2} \Big(2\vec{v} \cdot \Delta\vec{v} + \underbrace{(\Delta\vec{v})^2}_{\text{negligible}} \Big)$$

$$= m\vec{v} \cdot \Delta\vec{v}$$

Thus $\Delta W = \Delta K$ for an arbitrary small step.

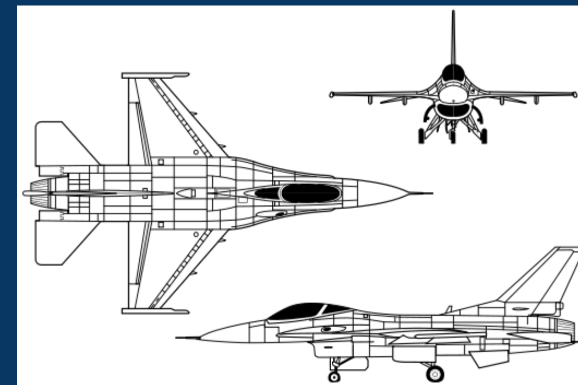
(3) For the full motion from a to b,

$$W = K_b - K_a .$$

$$\boxed{W = \Delta K}$$

An example of kinetic energy

$$K = \frac{1}{2}mv^2$$



F-16
Mass (loaded, takeoff) = 15,000 kg
Max speed = Mach 2 = 2410 km/h
= 670 m/s

Calculate the kinetic energy.

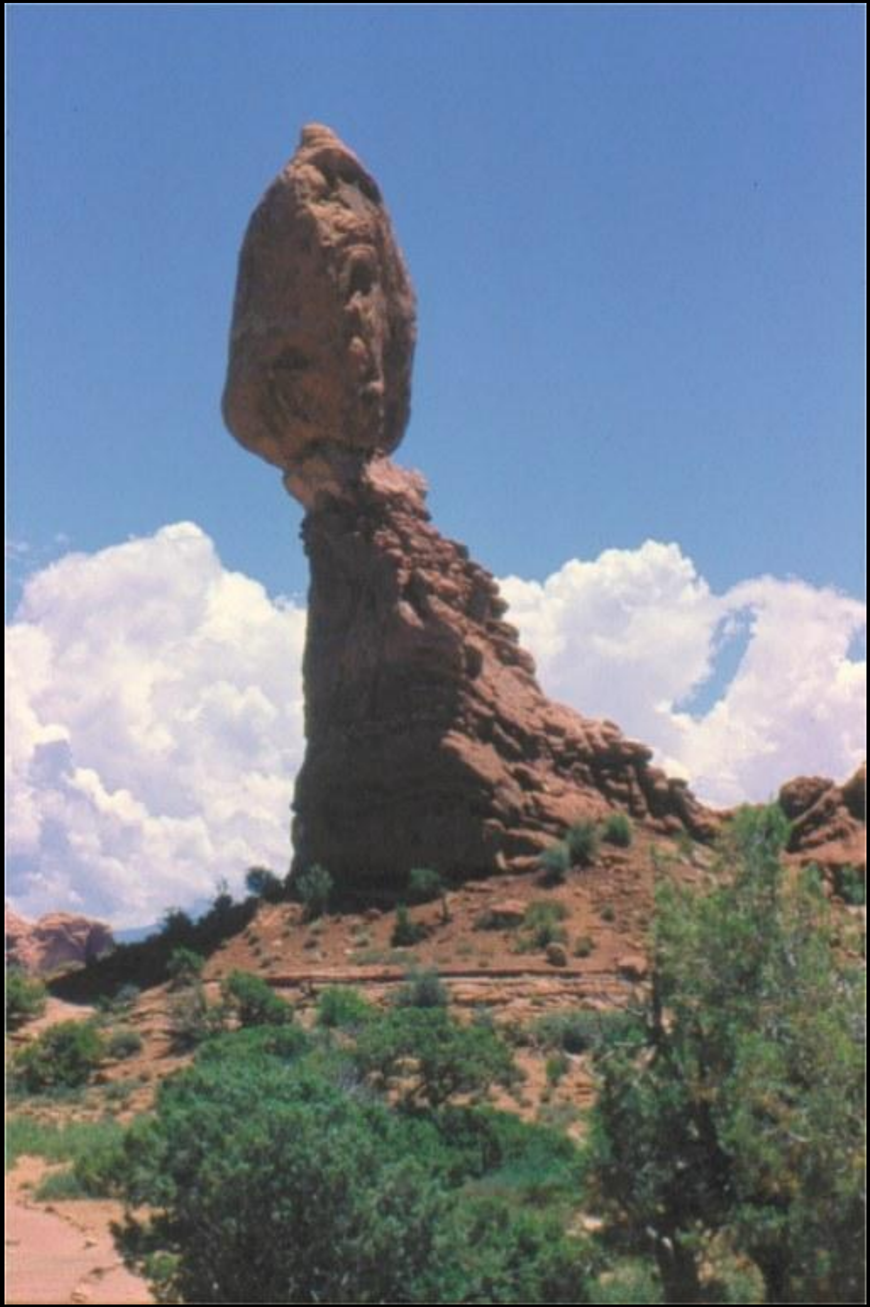
$K = \frac{1}{2}\, m v^2 = \frac{1}{2}\ (15{,}000\ \text{kg})\ (670\ \text{m/s})^2$

$K = 3.37 \times 10^9\ \text{J}$

$1\ \text{J} = 1\ \text{joule} = 1\ \text{kg m}^2/\text{s}^2$

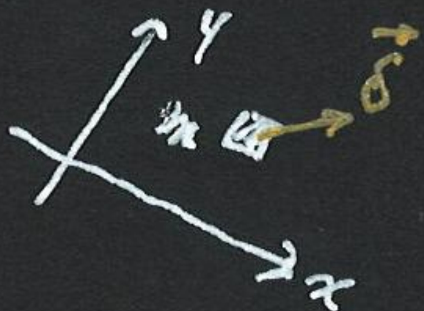
Potential Energy

Nothing is moving here.
But if the rock were to fall, then kinetic energy would appear.
So before the rock falls, there is <u>potential</u> energy.
How much potential energy is there?



Potential Energy

Potential energy depends on position; $U(\vec{x})$



For a conservative force, $\vec{F}(\vec{x})$,

$$U(\vec{x}+\vec{\delta}) - U(\vec{x}) = -\vec{F}\cdot\vec{\delta}$$

—— This defines a function $U(\vec{x})$ ——

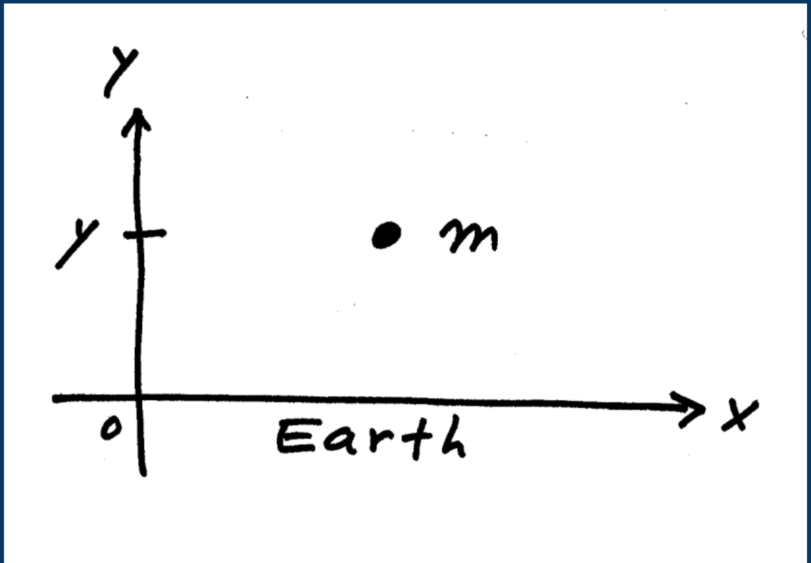
An example of potential energy

$$\Delta U = -\vec{F} \cdot (\Delta \vec{x})$$

A mass *m* in Earth's gravity, near the surface:

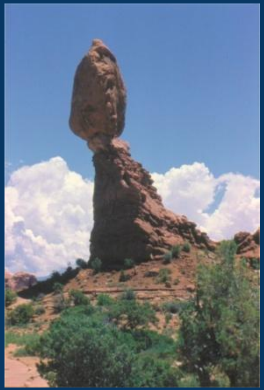
$$dU = -F_y\, dy = +mg\, dy$$

$$U = mgy$$



Equation:

U = mgh where *h* = height

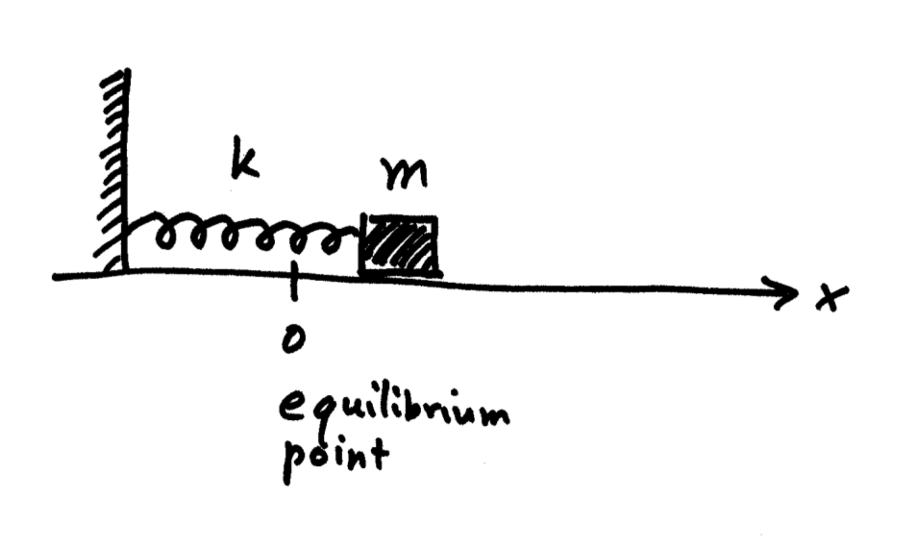


$$\Delta U = -\vec{F} \cdot (\Delta \vec{x})$$

Example of Potential Energy
-- mass on a spring

$$dU = -F\,dx = +kx\,dx$$

$$U = \frac{1}{2}kx^2$$



Equation

$U = ½\, k\, x^2$ where x = displacement from equilibrium

Theorem.

If a particle moves under the influence of a conservative force, then the total energy, $E = K + U$, is a constant of the motion.

Proof.

Consider any small step along the trajectory.

$\Delta K = \Delta W = F \cdot \Delta x$ (work – kinetic energy theorem)

$\Delta U = - F \cdot \Delta x$ (definition of potential energy U)

$\Delta K + \Delta U = 0$

$\therefore$ $K + U$ is constant.

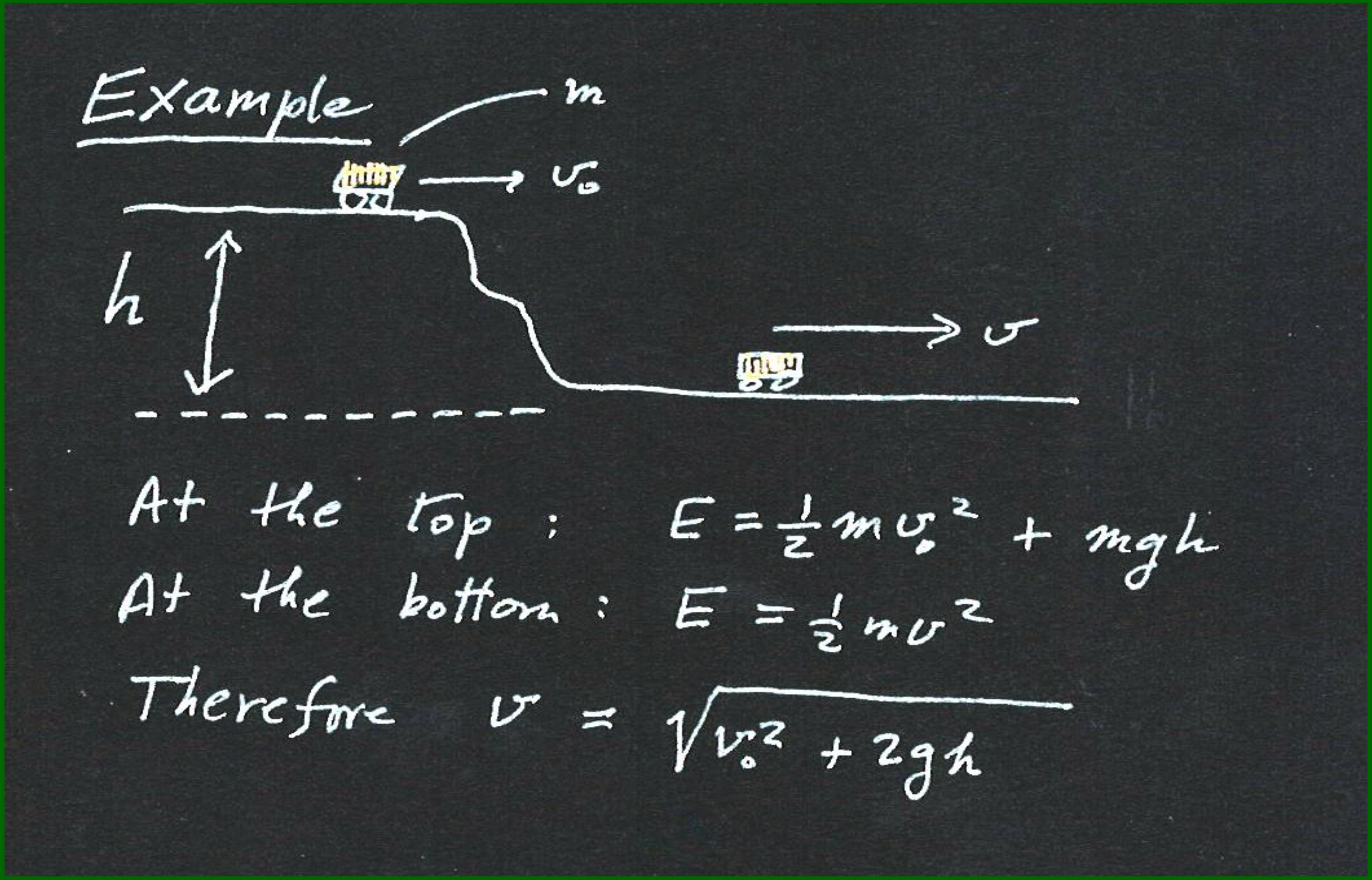
The Law of Conservation of Energy

In any isolated system, the total energy is constant. The total must include all forms of energy.

Energy cannot be created nor destroyed. It can be changed from one form to another, but the total is constant.

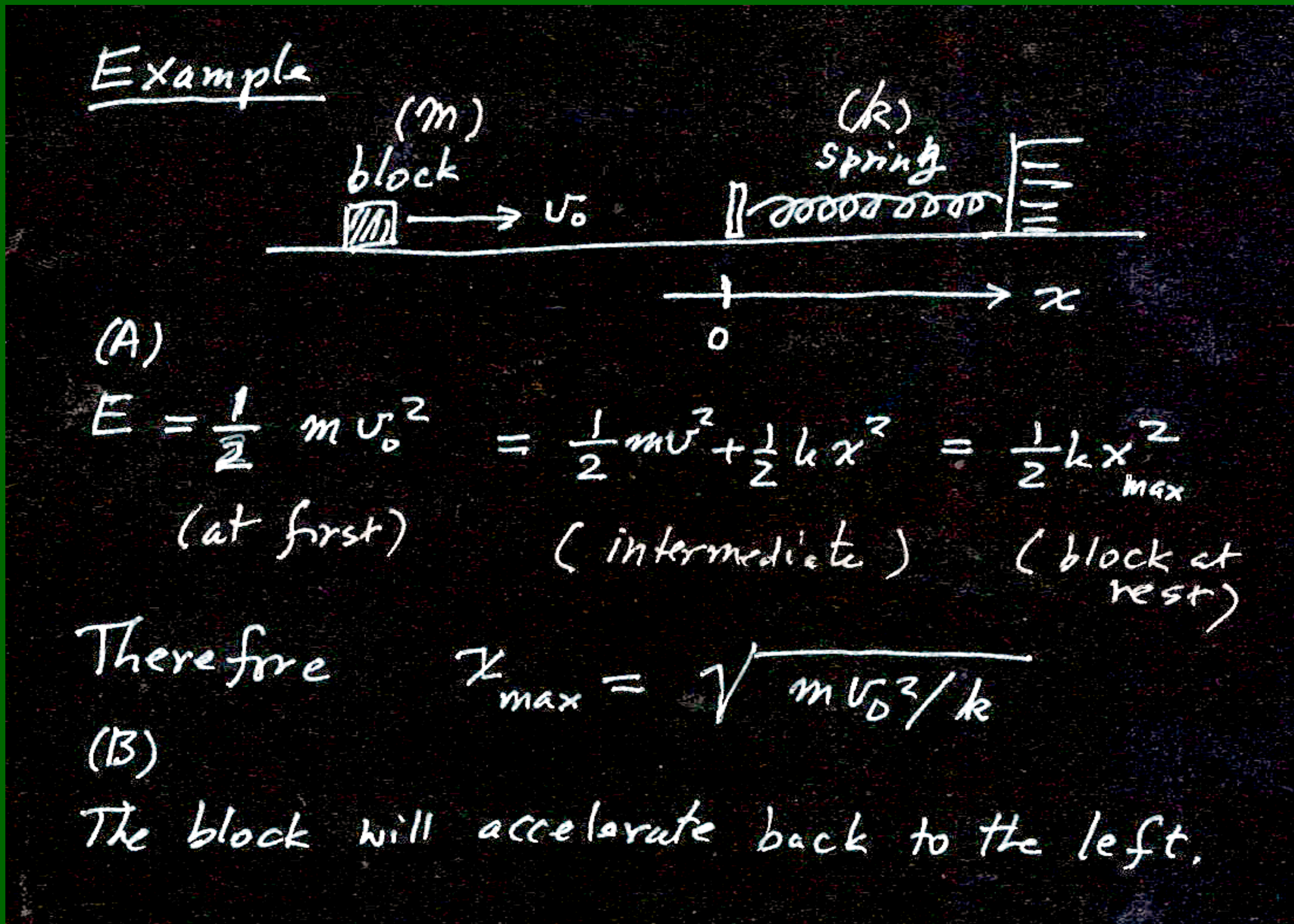
Example

A cart is initially moving with velocity v_0 toward a downward slope; then it rolls down the slope. Calculate the speed of the cart at the bottom of the slope.



Example

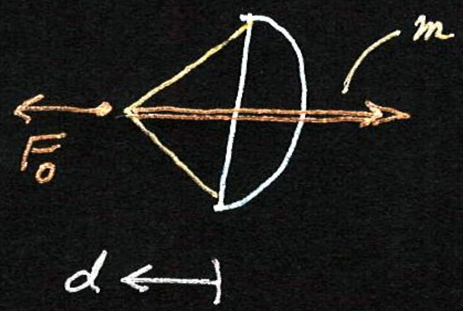
A block slides without friction toward the end of a spring, which is initially at its equilibrium length . (A) How far will the spring be compressed when the block comes to rest? (B) What will happen after the block comes to rest?



Example

The archer pulls the arrow back a distance d; then she momentarily holds her position, exerting a force F_O; then she releases the arrow. Calculate the speed of the arrow as it leaves the bow.

Example



Before release:

$E = \frac{1}{2} k d^2$ and $F_0 = kd$

After release:

$E = \frac{1}{2} m v^2$

Therefore

$$v = \sqrt{\frac{2E}{m}} = \sqrt{\frac{2}{m} \frac{1}{2} \frac{F_0}{d} d^2} = \sqrt{F_0 d / m}$$

Power

$$P = \frac{\Delta E}{\Delta t}$$

I.e., power = rate of energy conversion

= the rate of change of some form of energy

Equivalently, $\Delta E = P\,\Delta t$;

i.e., energy change = power times time.

The unit of <u>power</u> is the watt (W); 1 W = 1 J/s.

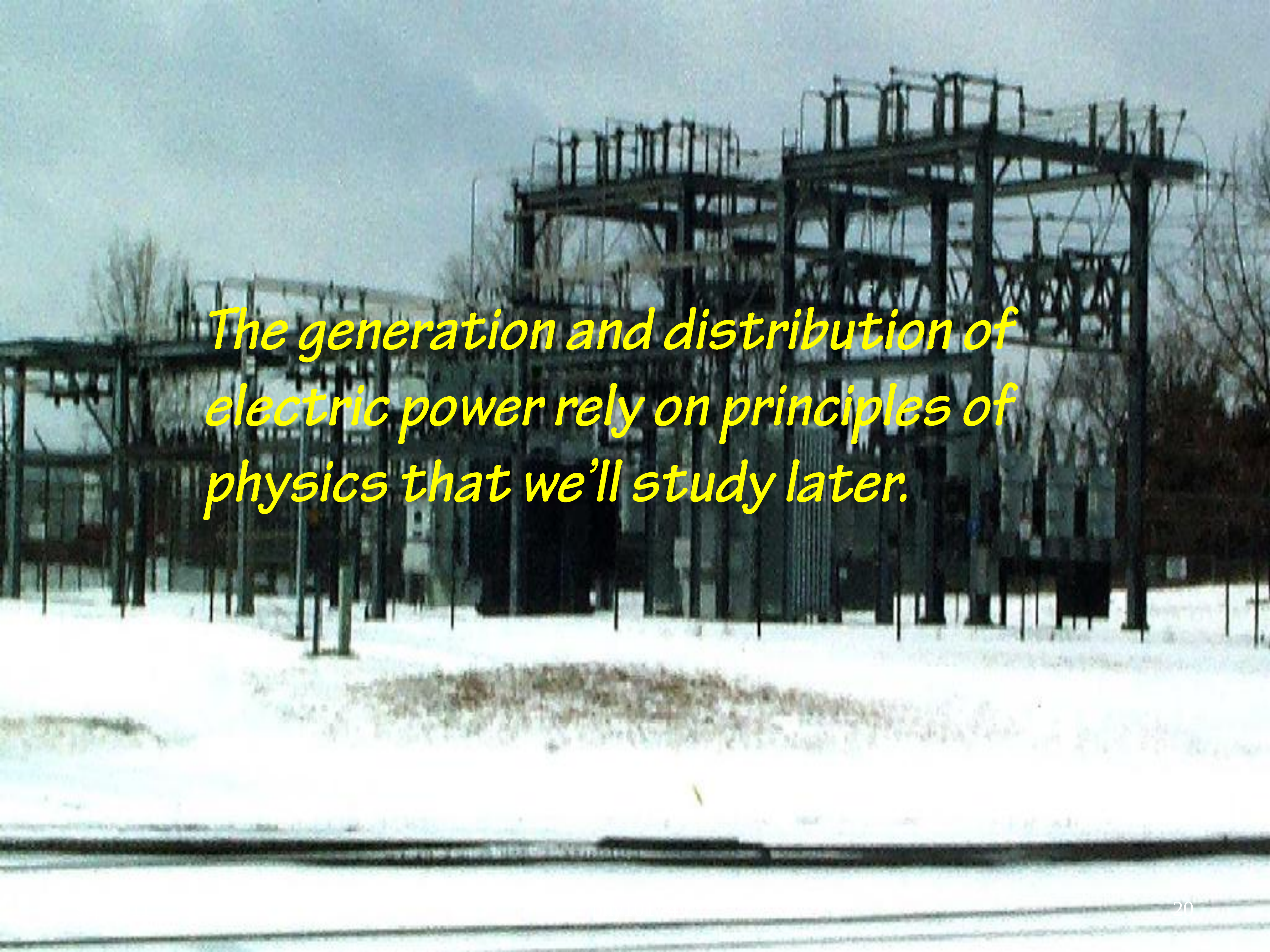
What <u>is</u> energy?

A dictionary definition of "energy" is "the ability to do work;" but that doesn't really answer the question, because "work" is the abstract notion of $F x$.

Energy is an abstract quantity defined by mathematical equations, such as $K = \frac{1}{2} mv^2$ or $\Delta U = - F \Delta x$. It is important because of the conservation law.

Electric power…..
…. a method to distribute energy

Electricity is your friend!
But it can be dangerous,
so be careful!


The generation and distribution of
electric power rely on principles of
physics that we'll study later.

The energy crisis?

Is there an energy crisis?

Not today, but perhaps in the future….

We all depend on energy, e.g., for

- transportation
- heat
- light
- machines
- communication

Where does it come from?

Remember, energy cannot be created nor destroyed!

END