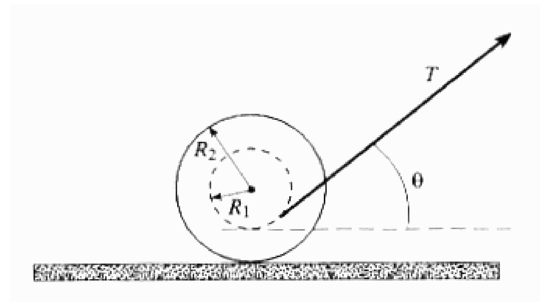
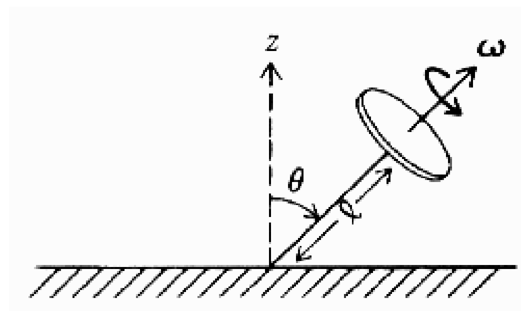
PHY820 Homework Set 10

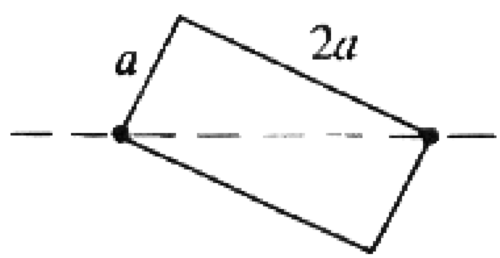
1. [5 pts] A spool rests on a rough table as shown. A thread wound on the spool is pulled with force T at angle θ. (a) If $\theta = 0$, will the spool move to the left or right? (b) Show that there is an angle θ for which the spool remains at rest. (c) At this critical angle find the maximum T for equilibrium to be maintained. Assume a coefficient of friction μ.



2. [5 pts] A heavy axially symmetric gyroscope is supported at a pivot, as shown. The mass of the gyroscope is M, and the moment of inertia about its symmetry axis is I. The initial angular velocity about its symmetry axis is ω. Follow an approximate solution of the equation of motion for the system, under the assumption that ω is very large and obtain the angular frequency Ω of gyroscopic precession. Show that the approximation requires that $\omega \gg \sqrt{g/\ell}$, when ℓ takes the role of an overall size scale with all moments of inertia taken to be roughly $M\,\ell^2$.

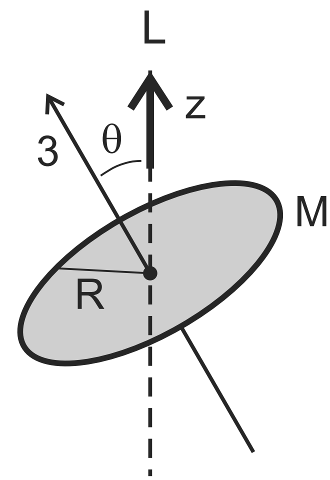


3. [5 pts] A flat rectangular plate of mass M and sides a and $2a$ rotates with angular velocity ω about an axle through two diagonal corners, as shown. The bearings supporting the plate are mounted just at the corners. Follow Euler's equations and find the force on each bearing due to rotation. Only two principal moments of inertia are relevant.



4. [5 pts] Goldstein, Problem 5-29.

5. [10 pts] (a) Within the Lagrangian approach to rotation, use conservation laws to arrive at analytical solutions for the Euler angles as a function of time, of an axially symmetric body precessing uniformly in the absence of torques. Assume that the third body axis coincides with the axis of axial symmetry, $I_1 = I_2 \neq I_3$. Note that the requirement of a constant polar angle in the external frame imposes a condition on the behavior of the effective potential. (b) Use your results to express components of angular velocity $\vec{\omega}$ and angular momentum $\vec{L}$, along the principal axes, in terms of instantaneous Euler angles and generalized velocities. (c) By applying backward rotations, taking you from the body to the external frame, find the components of angular momentum in the external frame. Does the result make sense and why? (d) Assume now that the axially symmetric body ia a flat uniform disk of radius R and mass M. How are the principal moments I_1 and I_3 related to each other? Find the relation between precession velocities of $\vec{\omega}$ in the body frame, $\dot{\Psi}$, and external frame, $\dot{\Phi}$.



6. [10 pts] Investigate the motion of a heavy symmetric top of Sec. 5.7 for the case when the top's rotation axis is vertical. In the latter case, the third external and body axes coincide. By considering the effective potential around $\Theta = 0$, show that the motion is stable or unstable depending on whether $I_3\,\omega_3$ is greater or lesser than $2\sqrt{I_1\,M\,g\,\ell}$. Sketch the effective potential in the two cases. If the top is set spinning in the stable configuration, what is the effect as friction gradually reduces ω_3? *Hint:* Start out from the definitions of p_Ψ and p_Φ and demonstrate that these two momenta become identical when the top is set at $\Theta = 0$. The latter ensures a good behavior of $V'(\Theta)$ around $\Theta = 0$.