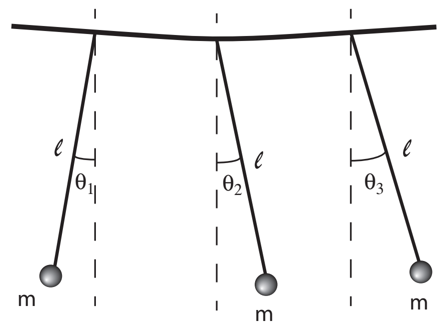
PHY820 Homework Set 13

1. [5 pts] Consider three identical pendula suspended from a slightly yielding support. Because the support is not rigid, a coupling occurs between the pendula, making the potential energy approximately equal to:

$$U \approx \frac{1}{2} m\, g\, \ell\, (\theta_1^2 + \theta_2^2 + \theta_3^2) - \epsilon\, m\, g\, \ell\, (\theta_1\, \theta_2 + \theta_1\, \theta_3 + \theta_2\, \theta_3)\,,$$

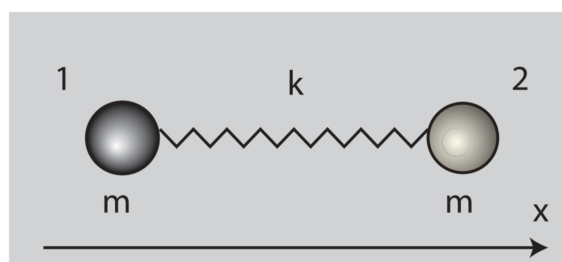
where $\epsilon \ll 1$, while the kinetic energy remains equal to

$$T = \frac{1}{2} m\, \ell^2\, (\dot\theta_1^2 + \dot\theta_2^2 + \dot\theta_3^2)\,.$$



Find the normal frequencies and normal modes for the coupled system. Note: Given the three degrees of freedom, three modes are expected. With the reflection and cyclic symmetries of the system, an individual mode can be expected to be either invariant under a symmetry or get interchanged with another mode. In the latter case, the frequency should not change. After you find the modes, classify their behavior under the symmetries.

2. [10 pts] Consider two identical particles, 1 and 2, of mass m, connected by a massless spring of spring constant k, moving in one dimension, parametrized in terms of x, in a fluid. The particles are subject to drag forces from the fluid, respectively $-b\,\dot{x}_1$ and $-b\,\dot{x}_2$. (a) Ignore any other forces, but the drag and spring forces. Start with the Newton's equations for the particles, turn to the motion of the center of mass and the motion in the relative separation and solve the equations to arrive at the general form of t-dependence for $x_1(t)$ and $x_2(t)$. (b) Next turn to the matrix approach for small oscillations. What eigenvalues would you expect, on the basis of (a), as solutions to the determinant equation? Show that the expected eigenvalues indeed satisfy the determinant equation. Find the amplitude eigenvectors corresponding to the eigenvalues.



3. [10 pts] Goldstein, Problem 6-18.

4. [5 pts] Goldstein, Problem 8-14.

5. [5 pts] The cartesian coordinate system $\mathcal{O}'$ rotates about an inertial system $\mathcal{O}$ at a constant angular velocity $\vec{\omega}$. (a) Consider a particle of mass m moving in a potential $V(\vec{r})$. How is the particle velocity $\vec{v}$ in $\mathcal{O}$ related to the velocity $\vec{v}'$ perceived in $\mathcal{O}'$? Obtain a Lagrangian for the particle in terms of the coordinates $\vec{r}'$ in $\mathcal{O}'$. (b) Obtain the Hamilton's function and the canonical equations of motion for the particle in $\mathcal{O}'$, relying on the coordinates $\vec{r}'$.