PHY820 Homework Set 4

1. [5 pts] Use the Lagrange's equations, in combination with the Hamilton's principle, to find the shortest curve joining two arbitrary points on a cylindrical surface of radius R. Note: You can select coordinates from the cylindrical system such that one of the points is located at $(0, 0)$ and the other at (ϕ_1, z_1).

2. [5 pts] Goldstein, Problem 2-3. Consider three dimensions.

3. [10 pts] A particle is free to move on the surface of a sphere of unit radius under the influence of no forces other than those that constrain the particle to the sphere. It starts at a point q_1 and ends at another point q_2 (without loss of generality, both points may be taken to lie on a meridian of longitude).

   (a) Show that there are many physical paths (i.e. $q(t)$ functions) the particle can take in going from q_1 to q_2 in a given time τ. How many? Under what conditions are there uncountable many?

   (b) Calculate the action for each of two possible paths the particle can take and show that they are not in general equal. Now construct two new, nonphysical paths close to the original ones, going from q_1 to q_2 in the same time τ. This can be done by adding to each physical path a small distortion of the form $\eta_k(t)$ such that $\eta_k(0) = \eta_k(\tau) = 0$. Show that for each of the nonphysical paths the action is greater than it is on the neighboring physical one, thus demonstrating that each physical path minimizes the action locally.

4. [5 pts] Goldstein, Problem 2-12.