Physics 842 – Fall 2012
Classical Electrodynamics II

Problem Set #10 – due Tuesday December 4

1. Debye Relaxation: A simple model for the dielectric relaxation of polar molecules in a gas, liquid, or solid was given by Peter Debye 100 years ago (1912). We divide the response of the molecules into a "fast" part describing the electronic response of the molecules, and a "slow" part describing the hindered molecular rotations. The response function g(t), defined by

$$\vec{P}(t) = \int_0^\infty g(t')\vec{E}(t-t')dt'$$

has the approximate form:

$$g(t) = \chi_{fast}\delta(t) + \chi_{slow}\frac{e^{-t/\tau_D}}{\tau_D}$$

where τ_D is the typical time needed for a molecule to re-orient in the applied electric field. (You calculated χ_{slow} for a system of magnetic dipoles, using statistical mechanics, in problem 1 of Problem Set #6. The answer is the same for electric dipoles, and is $\chi_{slow} = \frac{np_0^2}{3k_BT}$, where p_0 is the magnitude of the permanent molecular dipole moment.)

a) Calculate the dielectric susceptibility, $\varepsilon(\omega)$. Express $\varepsilon(\omega)$ in terms of its value at very high frequency, $\varepsilon_\infty \equiv 1 + 4\pi\chi_{fast}$, and at zero frequency, $\varepsilon_0 \equiv 1 + 4\pi(\chi_{fast} + \chi_{slow})$.

b) Plot ε' and ε" vs. $\omega\tau_D$, first on a linear frequency plot, and then on a log frequency plot. You are welcome to use Mathematica to make the plots, or you can do them by hand.

2. Kramers-Kronig relations: (This problem is taken from Jackson, Chapter 7.)

a) The imaginary part of the dielectric susceptibility for a material is given for ω > 0 by:

$$\varepsilon''(\omega) = \lambda[\theta(\omega-\omega_1) - \theta(\omega-\omega_2)]$$

with $0 < \omega_1 < \omega_2$,where θ(x) is the Heaviside step function.
Calculate the real part of the susceptibility, $\varepsilon'(\omega)$, from the Kramers-Kronig relations, given in equation (82.8) of Landau & Lifshitz.

b) Do the same thing for this case:

$$\varepsilon''(\omega) = \frac{\lambda\gamma\omega}{(\omega_0^2 - \omega^2)^2 + \gamma^2\omega^2}$$

c) For both parts (a) and (b), make rough plots of ε'(ω) and ε"(ω) vs. ω.

Quiz #10

The quiz on Thursday, December 6, will consist of one of the above problems.